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On Fixed Critical Value for Preliminary Test Estimator

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Abstract

The performance of pre-test estimator depends on the level of significance. Since the existing max-min rule and minimax regret procedure are computer intensive, we propose one simpler alternative method for optimal level of significance. We perform a numerical comparison among these three methods. Numerical results suggest that the proposed and Brook methods are conservative for fixed sample size, whereas Han and Bancroft is flexible. If the researchers are very conservative about the minimum guaranteed efficiency, they might select our proposed or Brook's method. If they want to have the higher minimum guaranteed efficiency, they should select Han and Bancroft method. However, in later case the researchers have to accept the risk for the higher size of the test. The proposed method is easy to compute compared to Han and Bancroft and Brook methods.

Keywords and Phrases: Mean of F-distribution; Guaranteed Efficiency; Linear Regression; Preliminary Test; Optimal Level of Significance; Quadratic risk, Restricted Estimator, Unrestricted estimator.

AMS Classification: 62F10, 62F03.

1 Introduction

One common problem encountered with general linear regression models is to determine whether to place restrictions on the parameters or not. This leads to a choice of considering either restricted or unrestricted least squares estimator. For selecting either estimator, F-test statistic is used to make the decision. This encourages one to define a pre-test estimator. To describe the problem, consider the following linear regression model, $Y \sim N(X\beta, \sigma^2 I)$, where Y is an $n \times 1$ vector of observations on the dependent variable, which follows a normal distribution with fixed mean vector $X\beta$ and known variance $\sigma^2 I$, β is a $p \times 1$ vector of unknown regression parameters, and X is an $n \times p$ known design matrix of rank p $(n \geq p)$. We are interested to estimate the regression coefficients β when it is *a priori* suspected that β may be restricted to the subspace

$$H_0 : H\beta \quad = \quad h, \tag{1}$$

where H is a $q \times p$ known matrix of full rank $q(< p)$ and h is a $q \times 1$ vector of known constants. The choice of estimator for β whether restricted or unrestricted will depend on the outcome of the test. If we reject the null hypothesis, the unrestricted least squares estimator (URLSE) $\hat{\beta}^{UE} = C^{-1}X'Y$ will be used. Here, $C = X'X$ is called the information matrix. On the other hand, if the null hypothesis is true, the restricted least squares estimator (RLSE) $\hat{\beta}^{RE} = \hat{\beta}^{UE} - C^{-1}H'(HC^{-1}H')^{-1}(H\hat{\beta}^{UE} - h)$ will be used. As a result, one might combine the URLSE and RLSE to obtain a better performance of the estimator in presence of the uncertain prior information $H\beta = h$. This leads to the well known preliminary test least squares estimator (PTLSE) of β given by

$$\hat{\beta}_{\alpha}^{PT} = \hat{\beta}^{UE} - (\hat{\beta}^{UE} - \hat{\beta}^{RE})I(\mathcal{L} < \mathcal{L}_{\alpha}), \tag{2}$$

where $I(A)$ is the indicator function of the set A and $\mathcal{L}_{\alpha}$ is the upper $100\alpha\%$ point of the test statistic

$$\mathcal{L} = \frac{(H\hat{\beta}^{UE} - h)'(HC^{-1}H')^{-1}(H\hat{\beta}^{UE} - h)}{qs^2}, \tag{3}$$

where $s^2 = (n-p)^{-1}(y - X\hat{\beta}^{UE})'(y - X\hat{\beta}^{UE})$ is an unbiased estimator of σ^2. Under the null hypothesis, the test statistic $\mathcal{L}$ is distributed as central F distribution with q and $n-p$ degrees of freedoms. Under the non-null case, it has non-central F distribution with q and $n-p$ degrees of freedom and non-centrality parameter $\frac{1}{2}\Delta$, where

$$\Delta = \frac{\eta'(HC^{-1}H')^{-1}\eta}{\sigma^2},$$

and $\eta = H\beta - h$ is called the departure parameter.

The preliminary test estimation has application in applied econometric analysis. It has been pioneered by Bancroft (1944), followed by Bancroft (1964), Han and Bancroft (1968), Judge and Bock (1978), Benda (1996), Chiou and Han (1999), Han (2002), Kibria and Saleh (2003) and very recently Kibria and Saleh (2005, 2006). A detailed review of the preliminary test estimation procedures is given by Han at al. (1988) and Gilies and Gilies (1993).

It follows from above that the performance of PTLSE depends on the unknown parameter Δ and the size of the test α. Indeed, the choice of α or critical value for F test is an important issue for the users of the PT estimator. Han and Bancroft (1968) proposed the max-min rule based on relative efficiency and Brook (1976) proposed the minimax regret procedure based on risk to determine the optimal significance level for the usual pre-test estimator. Since both procedures are computer intensive, a fixed critical value for PTLSE is proposed in this paper. A numerical comparison among these three procedure are given and discussed their relative merits.

The plan of the paper is as follows. In Section 2, we provide the risk functions of the estimators. The determination of optimal significance level is discussed in section 3. A summary of the paper is added in Section 4.

2 The Risk Analysis

2.1 The Risk Functions

Here, we present the quadratic risk functions of the estimators. Suppose $\hat{\beta}$ denotes an estimator of β, then for a given positive semi definite matrix M, the loss function of the estimator $\hat{\beta}$ is defined as

$$L(\hat{\beta}; M) = (\hat{\beta} - \beta)' M (\hat{\beta} - \beta)$$

and the corresponding risk function of the estimator $\hat{\beta}$ is

$$R(\hat{\beta}; M) = E(\hat{\beta} - \beta)' M (\hat{\beta} - \beta) = tr(U),$$

where U is the mean-squared error matrix of the estimator $\hat{\beta}$. The quadratic risk functions of the proposed estimators are (see Judge and Bock (1978)):

$$\begin{aligned} R(\hat{\beta}^{UE}; M) &= \sigma^2 tr(C^{-1}M), \\ R(\hat{\beta}^{RE}; M) &= \sigma^2 tr(C^{-1}M) - \sigma^2 tr(A) + \eta' D \eta, \\ R(\hat{\beta}^{PT}; M) &= \sigma^2 tr(C^{-1}M) - \sigma^2 tr(A) G_{q+2,n-p}(l_1; \Delta) \\ &+ \eta' D \eta \{2G_{q+2,n-p}(l_1; \Delta) - G_{q+4,n-p}(l_2; \Delta)\}, \end{aligned} \tag{4}$$

where $D = (HC^{-1}H')^{-1}A$, $A = HC^{-1}MC^{-1}H'(HC^{-1}H')^{-1}$, $l_1 = \frac{q}{q+2}F_{\alpha,q,n-p}$, $l_2 = \frac{q}{q+4}F_{\alpha,q,n-p}$ and $G_{a,b}(*; \Delta)$ is the cdf of non-central F distribution with a and b degrees of freedom and non-centrality parameter Δ.

Figure 12: Risk plots for $n = 10$ and different values of p, q and α.

Figure 13: Risk plots for $p = 4$, $q = 2$ and different n and α.

3.1 Han and Bancroft's Method

Here we describe the maximum and minimum (Max & Min) rule proposed by Han and Bancroft (1968) for the optimal choice of the level of significance of the PTLSE for testing the null hypothesis (1). For fixed values of p and q, the relative efficiency of the PTLSE ($\hat{\beta}^{PT}$) compared to the URLSE is a function of α and Δ. Let us denote this relative efficiency by

$$\begin{aligned} E(\alpha, \Delta) &= \frac{R(\hat{\beta}^{UE}, C\sigma^{-2})}{R(\hat{\beta}^{PT}, C\sigma^{-2})} \\ &= \left[1 - \frac{1}{p}\left\{qG_{q+2,n-p}(l_1; \Delta) - \Delta(2G_{q+2,n-p}(l_1; \Delta) - G_{q+4,n-p}(l_2; \Delta))\right\}\right]^{-1} \end{aligned} \quad (5)$$

For known p and q, the relative efficiency is a function of α and Δ. For a given α, $E(\alpha, \Delta)$ is a decreasing function of Δ in the interval $[0, \Delta_{Min}(\alpha)]$ and an increasing function of Δ in the interval $[\Delta_{Min}(\alpha), \infty]$ and $E(\alpha, \Delta) \to 1$ as $\Delta \to \infty$. For $\alpha \neq 0$, it has maximum at $\Delta = 0$ with the value

$$\begin{aligned} E_{Max}(\alpha, 0) &= \left[1 - \frac{q}{p} G_{q+2,n-p}(l_1; \Delta)\right]^{-1} (\geq 1) \\ &= \left[1 - G_{q+2,n-p}(l_1; \Delta)\right]^{-1} (\geq 1) \quad \text{if } p = q. \end{aligned} \tag{6}$$

If we consider the value of $E(\alpha, \Delta)$ at $\alpha = 0$, we have $E(0, \Delta) = [1 - \frac{q}{p} + \Delta]^{-1}$ and $E(0, \Delta) = 1$ when $\Delta = \frac{q}{p}$. Thus, the efficiency is maximum for $0 \leq \Delta \leq \frac{q}{p}$ and selects $\hat{\beta}^{RE}$ as the PTLSE of β.

From Figures 1 and 2, we observed that the PTLSE is not uniformly best compared to URLSE or RLSE. Moreover, if Δ is unknown then one follows the *minimum guaranteed efficiency* procedures proposed by Han and Bancroft (1968) which in turn determine the optimal level of significance for given minimum guaranteed efficiency say E_{Min}. One looks for a suitable α from the set $S_\alpha = \{\alpha | E(\alpha, \Delta) \geq E_{Min}\}$. The PTLSE is chosen for which $E(\alpha, \Delta)$ is maximized over all $\alpha \in S_\alpha$ and Δ. Thus, one solves the equation

$$\min_{\Delta} E(\alpha, \Delta_{Min}(\alpha)) = E_{Min}. \tag{7}$$

From (7), we obtain the optimal significance level α^* for the PTLSE with minimum guaranteed efficiency E_{Min}.

3.2 Brook's Optimal Critical Values

This section discusses the Brook (1976) regret criterion based on quadratic risk function to obtain the optimal critical value, which is also available in Kibria and Saleh (2005). For a given critical value c_α, the risk function of $\hat{\beta}^{PT}$ with $M = \sigma^{-2}C$ is obtained as

$$R(\hat{\beta}^{PT}; \sigma^{-2}C) = p - qG_{q+2,n-p}(c_\alpha; \Delta) + \Delta\{2G_{q+2,n-p}(c_\alpha; \Delta) - G_{q+4,n-p}(c_\alpha; \Delta)\} \tag{8}$$

Notice that

$$R(\hat{\beta}^{UE}; \sigma^{-2}C) = p, \quad \text{and} \quad R(\hat{\beta}^{RE}; \sigma^{-2}C) = p - q + \Delta.$$

Then $\hat{\beta}^{RE}$ is better than $\hat{\beta}^{UE}$ if $\Delta < q$ and $\hat{\beta}^{UE}$ is better than $\hat{\beta}^{RE}$ if $\Delta \geq q$. Since Δ is unknown, we want to have an optimal value of c_α, which provides a reasonable value for the risk function for all Δ.

It is clear that

$$\inf_{c_\alpha} R(\hat{\beta}^{PT}; \sigma^{-2}C) = R(\hat{\beta}^{RE}; \sigma^{-2}C) \quad \text{if} \quad \Delta < q$$

$$= \quad R(\hat{\beta}^{UE};\sigma^{-2}C) \quad \text{if} \quad \Delta \geq q$$

Now, consider the regret function

$$\begin{aligned}
Reg(\Delta, c_\alpha) &= R(\hat{\beta}^{PT};\sigma^{-2}C) - \inf_{c_\alpha} R(\hat{\beta}^{PT};\sigma^{-2}C) \\
&= q(1 - G_{q+2,n-p}(c_\alpha;\Delta)) \\
&- \Delta(1 - 2G_{q+2,n-p}(c_\alpha;\Delta) + G_{q+4,n-p}(c_\alpha;\Delta)) \quad \text{if} \quad \Delta < q \\
&= -qG_{q+2,n-p}(c_\alpha;\Delta) \\
&+ \Delta(2G_{q+2,n-p}(c_\alpha;\Delta) - G_{q+4,n-p}(c_\alpha;\Delta)) \quad \text{if} \quad \Delta \geq q. \qquad (9)
\end{aligned}$$

We find

$$\begin{aligned}
\sup_{0<\Delta<q} Reg(\delta, c_\alpha) &= Reg(\Delta_L, c_\alpha), \\
\sup_{\Delta \geq q} Reg(\delta, c_\alpha) &= Reg(\Delta_U, c_\alpha).
\end{aligned}$$

Then solve for c_α^* for which

$$Reg(\Delta_L, c_\alpha^*) = Reg(\Delta_U, c_\alpha^*), \qquad (10)$$

where Δ_L and Δ_U are the values of Δ for which $Reg(\Delta, c_\alpha)$ is the maximum for $\Delta < q$ and $Reg(\Delta, c_\alpha)$ is the maximum for $\Delta \geq q$, respectively. The relative efficiency corresponding to the optimal c_α^* can be determined from Section 3.1. The optimal critical value c_α^* for different numerator and denominator degrees of freedoms have been tabulated by Brook (1976). We define the PTLSE based on optimal critical value c_α^* as

$$\hat{\beta}^{PT}_{Brook} = \hat{\beta}^{UE} - (\hat{\beta}^{UE} - \hat{\beta}^{RE}) I(\mathcal{L} < c_\alpha^*). \qquad (11)$$

3.3 Fixed Critical Value

As we have seen both Han and Bancroft and Brook methods are computer intensive to obtain an optimal size of the test. Now we define the PT estimator based on fixed critical value, which is easy to calculate. Note that the test statistic

$$\mathcal{L} = \frac{(H\hat{\beta}^{UE} - h)'(HC^{-1}H')^{-1}(H\hat{\beta}^{UE} - h)}{qs^2}$$

has a central F distribution with q and $n-p$ degrees of freedoms under the null hypothesis. The mean of the F distribution is $\mu_F = E(F) = \frac{n-p}{n-p-2}$. We would like to use this mean as a fixed critical value of the test statistic. If we consider $\mu_F = \frac{n-p}{n-p-2}$ as the critical value of the test statistic (2) we obtain conservative size of the test (see Table 1) that works well compared to critical values suggested by Han and Bancroft

Table 1: Probability of F for quantiles μ_F and for different degrees of freedom

n	p	q									
		1	2	3	4	5	8	10	15	20	30
10	2	0.282	0.316	0.330	0.337	0.341	0.347	0.349	0.351	0.351	0.352
	3	0.275	0.308	0.320	0.327	0.330	0.335	0.337	0.338	0.339	0.340
	4	0.267	0.296	0.307	0.313	0.316	0.320	0.321	0.322	0.323	0.323
	5	0.253	0.279	0.288	0.292	0.294	0.297	0.298	0.299	0.300	0.300
	6	0.230	0.250	0.256	0.259	0.261	0.263	0.263	0.264	0.264	0.264
	7	0.182	0.192	0.196	0.197	0.197	0.198	0.198	0.199	0.199	0.199
20	2	0.303	0.346	0.365	0.376	0.383	0.393	0.397	0.401	0.403	0.405
	3	0.302	0.345	0.364	0.374	0.381	0.391	0.394	0.399	0.401	0.403
	4	0.301	0.344	0.362	0.372	0.378	0.388	0.392	0.396	0.398	0.399
	5	0.300	0.342	0.360	0.370	0.376	0.385	0.389	0.393	0.394	0.396
	8	0.295	0.335	0.352	0.360	0.366	0.374	0.377	0.380	0.382	0.383
	10	0.290	0.328	0.343	0.351	0.356	0.363	0.366	0.368	0.369	0.370
	12	0.282	0.316	0.330	0.337	0.341	0.347	0.349	0.351	0.351	0.352
	15	0.253	0.279	0.288	0.292	0.294	0.297	0.298	0.299	0.300	0.300
30	2	0.308	0.354	0.375	0.387	0.394	0.407	0.411	0.417	0.421	0.423
	3	0.308	0.354	0.374	0.386	0.394	0.406	0.410	0.416	0.419	0.422
	5	0.307	0.353	0.373	0.384	0.392	0.404	0.408	0.414	0.417	0.419
	8	0.306	0.350	0.370	0.381	0.389	0.400	0.404	0.409	0.412	0.414
	10	0.304	0.349	0.368	0.379	0.386	0.397	0.401	0.406	0.408	0.410
	15	0.300	0.342	0.360	0.370	0.376	0.385	0.389	0.393	0.394	0.396
	20	0.290	0.328	0.343	0.351	0.356	0.363	0.366	0.368	0.369	0.370
	25	0.253	0.279	0.288	0.292	0.294	0.297	0.298	0.299	0.300	0.300
40	2	0.311	0.358	0.379	0.392	0.400	0.414	0.419	0.426	0.429	0.433
	3	0.311	0.358	0.379	0.391	0.400	0.413	0.418	0.425	0.429	0.432
	5	0.310	0.357	0.378	0.391	0.399	0.412	0.417	0.424	0.427	0.430
	8	0.309	0.356	0.377	0.389	0.397	0.410	0.415	0.421	0.425	0.428
	10	0.309	0.355	0.376	0.388	0.396	0.409	0.413	0.420	0.423	0.426
	15	0.307	0.353	0.373	0.384	0.392	0.404	0.408	0.414	0.417	0.419
	20	0.304	0.349	0.368	0.379	0.386	0.397	0.401	0.406	0.408	0.410
	25	0.300	0.342	0.360	0.370	0.376	0.385	0.389	0.393	0.394	0.396
	30	0.290	0.328	0.343	0.351	0.356	0.363	0.366	0.368	0.369	0.370
	35	0.253	0.279	0.288	0.292	0.294	0.297	0.298	0.299	0.300	0.300
100	1	0.315	0.364	0.387	0.401	0.410	0.426	0.432	0.441	0.446	0.452
	2	0.315	0.364	0.387	0.400	0.410	0.426	0.432	0.441	0.446	0.451
	5	0.315	0.364	0.387	0.400	0.409	0.425	0.431	0.441	0.446	0.451
	10	0.315	0.364	0.386	0.400	0.409	0.425	0.431	0.440	0.445	0.450
	20	0.314	0.363	0.386	0.399	0.408	0.424	0.430	0.439	0.443	0.449
	30	0.314	0.363	0.385	0.398	0.407	0.423	0.428	0.437	0.441	0.446
	40	0.313	0.362	0.384	0.397	0.406	0.421	0.426	0.435	0.439	0.444
	50	0.312	0.360	0.382	0.395	0.404	0.418	0.424	0.431	0.436	0.440

Table 2: Maximum & Minimum Guaranteed Efficiency of $PTLSEs$

n	p	q		α								Brook	Fixed
				2.5%	5%	10%	15%	20%	30%	40%	50%	23.8%	23.0%
6	2	1	E_{Max}	1.81	1.68	1.49	1.37	1.28	1.16	1.09	1.05	1.23	1.24
			E_{Min}	0.29	0.40	0.54	0.64	0.72	0.82	0.89	0.94	0.76	0.75
			Δ_{Min}	12.69	8.96	6.28	5.11	4.45	3.64	3.22	2.95	4.09	4.15
				2.5%	5%	10%	15%	20%	30%	40%	50%	20.5%	28.2%
10	2	1	E_{Max}	1.77	1.63	1.44	1.33	1.25	1.14	1.08	1.04	1.24	1.16
			E_{Min}	0.39	0.49	0.61	0.70	0.76	0.85	0.91	0.95	0.76	0.84
			Δ_{Min}	8.03	6.31	4.93	4.24	3.82	3.31	3.01	2.83	3.79	3.40
				2.5%	5%	10%	15%	20%	30%	40%	50%	18.8%	30.1%
18	2	1	E_{Max}	1.74	1.60	1.42	1.31	1.23	1.13	1.07	1.04	1.25	1.13
			E_{Min}	0.45	0.54	0.65	0.72	0.78	0.86	0.92	0.95	0.77	0.86
			Δ_{Min}	6.55	5.41	4.42	3.91	3.58	3.16	2.92	2.77	3.64	3.16
				2.5%	5%	10%	15%	20%	30%	40%	50%	20.4%	31.6%
12	4	2	E_{Max}	1.84	1.73	1.57	1.45	1.37	1.24	1.16	1.10	1.36	1.23
			E_{Min}	0.48	0.58	0.69	0.76	0.81	0.87	0.92	0.95	0.81	0.88
			Δ_{Min}	11.69	9.35	7.36	6.37	5.71	4.90	4.36	4.00	5.68	4.81
				2.5%	5%	10%	15%	20%	30%	40%	50%	17.9%	34.4%
20	4	2	E_{Max}	1.82	1.70	1.54	1.43	1.34	1.22	1.15	1.09	1.37	1.18
			E_{Min}	0.56	0.64	0.73	0.79	0.83	0.89	0.93	0.95	0.82	0.91
			Δ_{Min}	9.05	7.61	6.31	5.59	5.14	4.51	4.12	3.82	5.32	4.33
				2.5%	5%	10%	15%	20%	30%	40%	50%	16.8%	35.2%
28	4	2	E_{Max}	1.81	1.69	1.53	1.42	1.33	1.22	1.14	1.09	1.38	1.17
			E_{Min}	0.58	0.66	0.74	0.80	0.84	0.89	0.93	0.96	0.82	0.92
			Δ_{Min}	8.33	7.12	5.98	5.35	4.96	4.39	4.03	3.76	5.20	4.18
				2.5%	5%	10%	15%	20%	30%	40%	50%	15.8%	36.2%
64	4	2	E_{Max}	1.80	1.68	1.51	1.40	1.32	1.21	1.14	1.09	1.39	1.16
			E_{Min}	0.61	0.68	0.76	0.81	0.85	0.90	0.93	0.96	0.82	0.92
			Δ_{Min}	7.58	6.58	5.62	5.11	4.75	4.24	3.91	3.67	5.02	4.03
				2.5%	5%	10%	15%	20%	30%	40%	50%	13.6%	38.3%
30	6	4	E_{Max}	2.62	2.38	2.05	1.83	1.67	1.45	1.30	1.20	1.88	1.32
			E_{Min}	0.63	0.70	0.78	0.83	0.86	0.91	0.94	0.96	0.82	0.94
			Δ_{Min}	11.93	10.37	8.93	8.12	7.55	6.76	6.22	5.77	8.30	6.28
				2.5%	5%	10%	15%	20%	30%	40%	50%	11.7%	39.7%
66	6	4	E_{Max}	2.59	2.34	2.01	1.80	1.64	1.43	1.29	1.19	1.93	1.30
			E_{Min}	0.67	0.74	0.81	0.85	0.88	0.92	0.95	0.96	0.82	0.94
			Δ_{Min}	10.52	9.35	8.21	7.55	7.06	6.40	5.95	5.56	7.94	5.95
				2.5%	5%	10%	15%	20%	30%	40%	50%	11.0%	40.1%
126	6	4	E_{Max}	2.58	2.33	2.00	1.79	1.63	1.42	1.29	1.19	1.95	1.28
			E_{Min}	0.69	0.75	0.81	0.85	0.88	0.92	0.95	0.97	0.82	0.95
			Δ_{Min}	10.10	9.02	7.97	7.33	6.91	6.31	5.86	5.50	7.82	5.86

(1968) and Brook (1976). The size of the test for different degrees of freedom and quantiles are presented in Table 1.

We propose the PTLSE based on critical value μ_F as

$$\hat{\beta}^{PT}_{Fixed} = \hat{\beta}^{UE} - (\hat{\beta}^{UE} - \hat{\beta}^{RE}) I(\mathcal{L} < \mu_F). \quad (12)$$

The proposed estimator is to change the way that researchers undertake the preliminary test.

Table 2 compares $\hat{\beta}^{PT}_{\alpha}$, $\hat{\beta}^{PT}_{Brook}$ and $\hat{\beta}^{PT}_{Fixed}$ under the quadratic risk function and minimum and maximum guaranteed efficiency criteria. The last two columns of the table provide the maximum and minimum guaranteed relative efficiencies for optimal critical values provided by Brook (1976) and fixed critical value, respectively. For given q and $n-p$, one enters the table and looks for the smallest relative efficiency E_{Min} he/she wishes to accept. For example, suppose $q = 2$, $n - p = 12$ and the experimenter wishes to have an estimator with a minimum guaranteed efficiency of 0.75. From the table, we recommend him/her to select $\alpha = 0.15$, corresponding to $\hat{\beta}^{PT}$, because such a choice of α would yield an estimator with a minimum efficiency of 0.76 and a maximum efficiency of 1.45. Note that with this condition the minimum guaranteed efficiency of $\hat{\beta}^{PT}$ using Brook's optimal critical value is 0.81 with a maximum efficiency of 1.36. By fixed critical value, the minimum guaranteed efficiency is 0.88 with a maximum efficiency of 1.23.

4 Summary

In this paper, we have compared the methods of Han Bancroft (1968) and Brook (1976) along with a proposed fixed critical value for obtaining an optimal significance level to formulate a PTLSE. To determine the Han Bancroft's level one has to specify a value which is the smallest relative efficiency the investigator is willing to accept. However, the Brook's level balances the loss and gain to determine a level based on a regret function. Since, a theoretical comparison among these three methods is hard to make, a numerical comparison has been performed.

From Table 2, it is observed that the minimum guaranteed efficiency by Brook's method vary between 0.76 and 0.82 and the maximum efficiency vary between 1.23 and 195.0 for α $(0.11 < \alpha < 0.24)$. The minimum guaranteed efficiency by Fixed critical method vary between 0.75 and 0.95 and the maximum efficiency vary between 1.24 and 1.32 for α $(0.23 < \alpha < 0.40)$. The corresponding minimum and maximum guaranteed efficiencies by Han Bancroft method's are between 0.29 and 0.97 and 1.05 and 2.59, respectively, for $0.025 < \alpha < 0.50$. Both Fixed and Brook methods are conservative for fixed sample size, whereas Han Bancroft method is flexible because a higher minimum guaranteed efficiency can be chosen to determine the significance level. In either method, the researchers have to take some risk. If the researchers are concerned or very conservative about the minimum guaranteed efficiency, they might select Fixed or Brook's method. However, if they are willing to accept higher size of the test but want to have higher guaranteed minimum efficiency, they should select Han Bancroft's method. The proposed method is easy to compute compared to Han and Bancroft or Brook's method.

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