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Dynamics of Mean Age at Marriage, TFR and NRR in Bangladesh: An Application of VAR Model

Md. Atikur Rahman Khan
Department of Population Science and Human Resource Development
University of Rajshahi
Rajshahi -6205, Bangladesh

Md. Ayub Ali
Laboratory of Growth and Ergonomics
Institute of Human Living Sciences
Otsuma Women's University
Sanban-cho 12, Chiyoda-ku
Tokyo 102-8357, Japan

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Abstract

A vector autoregression (VAR) model was applied to investigate the dynamic interaction between the mean age at marriage for females, total fertility rate (TFR) and Net Reproductive Rate (NRR). Secondary data from Bangladesh Bureau of Statistics were used. The mean age at marriage for females in Bangladesh is increasing over time, whereas the total fertility rate and net reproduction rate are decreasing. Autoregressive models have indicated a negative growth for the variables TFR and NRR. Further, the vector autoregression (VAR) model shows that these three variables, MAMF, TFR and NRR dynamically interact each other.

Keywords and Phrases: Total Fertility Rate, Net Reproductive Rate, Mean Age at Marriage, Autoregressive Model, and Vector Autoregression Model.

AMS Classification: 62-xx, 62-02, 62-07, 62-09, 62F10, 62F03, 62F30, 62H12, 62H15, 62H11, 62J05, 62J07, 62J10, 62J20, 62M10, 91B62, 62P10, 91D20.

1 Introduction

Age at marriage, total fertility rate and net reproductive rate are the important signs of population growth of a country. In Bangladesh, also many scientists have been dealing with these factors (Obaidullah, 1966; Chowdhury, 1996; Islam and Mahmud, 1996; Islam and Ahmed, 1998; Islam et al., 1998). However, the dynamics among the factors are important.

To know the growth of a certain variable over time, trend models are used as well (Gujarati, 1995 and UN, 1967). A number of authors studied the time trend behavior of the variables TFR and NRR using linear, quadratic, semi-logarithmic trend (King, et al., 1991; Gujarati, 1995; and UN, 1997). Sometimes the use of trend may not explain an endogenous variable properly as there might have some significant effects of other exogenous variables or lagged endogenous variables on it. The autoregressive context may be used as well for partial fulfillment of time trend behavior of a variable (Cleary and Hey, 1980; Pankratz, 1991; Hamilton, 1994; and Gujarati, 1995). To confirm the endogeneity of a set of variables, the test of block exogeneity using granger causality upon a multivariate context has been extensively used (Hamilton, 1994). To do this we divide the variables in two sets. Then we fit VAR model using one set as a vector of exogenous variables and compute the determinant of residual covariance, $\left|\hat{\Omega}_{11}\right|$ (see section 2.2). Again, we fit VAR model excluding the vector of exogeneous variables, X_2 and compute the determinant of residual covariance, $\left|\hat{\Omega}_{11}(0)\right|$. But, there may be some other variables like time trend (T) or seasonal dummies that can affect this test of block exogeneity. So, this block exogeneity test can be conducted using another vector of exogenous variables, X_3, which contains T or other relevant variables like seasonal dummies and intervention term. Also, it is possible to find that the desired set of variables is endogenous only if X_3 is included in the VAR model to compute the determinant of residual covariance and are exogenous either. Again, the Mean Age at Marriage for Female (MAMF) may affect the fertility as fecundability varies with respect to time period (Bongaarts and Potter, 1983; Bean and Mineau, 1986; UNFP, 1993; Misra, 1995; and Bhende and Kanitkar, 1997). So, it is possible to exist a relationship between TFR, NRR and MAMF and may be dynamically interact each other (Gujarati, 1995; Pankratz, 1991; Hamilton, 1994; and Johansen, 1996). To know the dynamic interactions of the variables TFR, NRR and MAMF, a VAR model is applicable.

Thus, the purpose of the present study is to investigate the effect of trend variable (T) to test the endogeneity as well as to know the dynamic interaction among the mean age at marriage for females (MAMF), total fertility rate (TFR) and Net Reproductive Rate (NRR) through a VAR model.

2 Methods and Materials

2.1 Autoregressive Models

An autoregressive model of order p (Cleary and Hey, 1980; Pankratz, 1991; Gujarati, 1995; and Hamilton, 1994) is given by-

$$Y_t = \alpha + \sum_{i=1}^{p} \beta_i Y_{t-i} + \gamma t + u_t \tag{1}$$

where, Y_t is the value of the variable Y at time t, α is the intercept term, $\beta_i, (i = 1, 2, \cdots, p)$ are the parameters, Y_{t-i} is the i-th lagged variable, t is the time trend, γ is the coefficient of the time variable t, and u_t is the error term which is a white noise indeed.

2.2 Unit Root Test

Any variable with constant mean and variance over the passage of time is stationary. Unit root test is one of the popular tools for testing stationarity of a variable. The Dickey-Fuller (DF) test (Dickey and Fuller, 1979) is applied to regressions run in the following forms

$$\begin{aligned}
\Delta Y_t &= \delta Y_{t-1} + u_t \\
\Delta Y_t &= \beta_1 + \delta Y_{t-1} + u_t \\
\Delta Y_t &= \beta_1 + \beta_2 t + \delta Y_{t-1} + u_t
\end{aligned}$$

where t is the time or trend variable. In each case the null hypothesis is that there is a unit root, that is, we would like to test $H_0\colon \delta = 0$. The error terms may be autocorrelated and in that case we can use augmented Dickey-Fuller test which tests $H_0\colon \delta = 0$ in the regression line $\Delta Y_t = \beta_1 + \beta_2 t + \delta Y_{t-1} + \sum_{i=1}^{m} \alpha_i \Delta Y_{t-i} + u_t$ where u_t is assumed to be a white noise error term and m depends on the number of observation and the autocorrelation structure in u_t. We have used the critical τ-statistic as MacKinnon (1996). Now, if the absolute value of computed DF or ADF statistic ($|\tau|$) exceeds the $100\alpha\%$ critical τ-value then we can reject the null hypothesis saying that the variable is stationary. If the variable becomes stationary after first differencing then the variable is integrated of order one, I(1).

2.3 VAR Modeling

To apply a VAR model, we have to check the endogenity among the variables. For this, block exogeneity test based on granger causality upon a multivariate context can be performed (Hamilton, 1994). Let us assume that we are interested to fit a VAR model using a vector of variables $\mathbf{Y}$. Now, we partition $\mathbf{Y}$ as $\mathbf{Y} = (Y_1\ Y_2)'$. If all the variables under study are endogenous then they should help to explain the variation of each other, that is, Y_1 and Y_2 will explain the variation of each other. In other word, lagged Y_2, X_2, will serve significant effect on Y_1 and as well as lagged Y_1, X_1, will posses significant effect on Y_2. To test whether X_2 possesses significant effect on Y_1 we have fitted VAR model as follows:

$$Y_1 = A_1X_1 + A_2X_2 + \varepsilon \tag{2}$$

where Y_1 is the set of variables assumed to be endogenous, X_1 is the vector of lagged endogenous variables, and X_2 is the vector of exogenous variables (lagged Y_2). The block exogeneity test leads to test the null hypothesis $H_0 : A_2 = 0$. Thus, under $H_0 : A_2 = 0$ the eq.(2) becomes

$$Y_1 = A_1X_1 + \varepsilon \tag{3}$$

Assume that $\left|\hat{\Omega}_{11}\right|$ and $\left|\hat{\Omega}_{11}(0)\right|$ be the determinant of residual covariance obtained after fitting VAR models in eq.(2) and eq.(3), respectively. Then the test statistic

$$L\left\{log\left|\hat{\Omega}_{11}(0)\right| - log\left|\hat{\Omega}_{11}\right|\right\} \sim \chi^2(n_1n_2p) \tag{4}$$

where p is the order of fitted VAR model, $L = n_1 + n_2$, n_1 and n_2 are the number of variables in X_1 and X_2, respectively.

Now, if the calculated value of the test statistic exceeds the critical value at 5% level of significance we may reject the null hypothesis, that is, the variables in X_2 have significant effect on Y_1. Similarly, we can fit two VAR models just replacing Y_1 by Y_2 and X_1 by X_2 to know whether X_1 have significant effect on Y_2 or not. If in both cases significant effects are achieved then the variables in Y_1 and Y_2 are said to be endogenous and are said to be exogenous either.

The stationarity of the variables is examined by the unit root test (MacKinnon, 1996) and line graph. If the variables are stationary at level we will build VAR model (Sims, 1980 and Hamilton, 1994). Again, if the variables are non-stationary at the level and are cointegrated then VEC models are to be used as well (Johansen, 1991, 1995). But, if the variables are mixed in nature we can build VEC model (Blanchard and Quah, 1989). However, Harvey (1990) explained that the results from the transformed set of variables may lead unsatisfactory results and he proposed to fit VAR model at the level of all the variables. Recently, Khan and Ali (2003b) prove that unrestricted VAR is more applicable than VEC when the variables are mixed in nature. They showed that the unrestricted VAR model is more stable over the population and serve less root mean squared forecast error than the VEC model when the variables are mixed in nature. An unrestricted VAR model (Hamilton, 1994) can be given as-

$$Y_t = A_1Y_{t-1} + A_2Y_{t-2} + \cdots + A_pY_{t-p} + BX_t + u_t \tag{5}$$

where, Y_t is the vector of endogenous variables, $A_i\,(i = 1, 2, \cdots, p)$ is the matrix of the coefficients of Y_{t-i}, X_t is the vector of exogenous variables (trend or seasonal dummies), B is the matrix of coefficient of exogenous variables, and u_t is the vector of innovation. If we add a vector of intercept term in eq.(5) then

$$Y_t = A_0 + A_1 Y_{t-1} + A_2 Y_{t-2} + \cdots + A_p Y_{t-p} + BX_t + u_t \tag{6}$$

Therefore, if we include intercept term and trend component (or seasonal dummies) in eq.(2) then we can write

$$Y_1 = A_0 + A_1 X_1 + A_2 X_2 + A_3 X_3 + \varepsilon \tag{7}$$

where X_3 is the vector of exogenous variables (time trend or seasonal dummies), and X_2 is a vector of variables for which exogeneity with Y_1 is to be tested. Thus, we would like to test H_0: $A_2 = 0$ and under the null hypothesis eq.(7) becomes

$$Y_1 = A_0 + A_1 X_1 + A_3 X_3 + \varepsilon \tag{8}$$

So, to compute $\left|\hat{\Omega}_{11}\right|$ and $\left|\hat{\Omega}_{11}(0)\right|$ we will fit eq.(7) and eq.(8), respectively. Now, if the intercept term and variables in X_3 provide no significant contribution (statistically insignificant examined by the computed t-values) to explain the variation of Y_1 then we fit

$$Y_1 = A_1 X_1 + \varepsilon \tag{9}$$

and compute $\left|\hat{\Omega}_{11}(0)\right|$ from this fitted equation.

2.4 Diagnostic Checking

The diagnostic checking leads to compute RCVPP (Khan and Ali, 2003a) as:

$$\rho^2_{rcv} = \begin{cases} 1 - \frac{(n-1)(n-2)(n+1)}{n(n-k-1)(n-k-2)}(1 - R^2);\ R^2 \geq 1 - \frac{n(n-k-1)(n-k-2)}{(n+1)(n-1)(n-2)},\ n > k+2 \\ 0;\ \text{otherwise.} \end{cases} \tag{10}$$

Also, the stability of the fitted model can be computed as $\tilde{\eta} = 1 - \tilde{\xi}$, where $\tilde{\xi}$ is the shrinkage (Stevens, 1996) can be computed as $\tilde{\xi} = \left|\rho^2_{rcv} - R^2\right|$. $\tilde{\eta} = 0.99$ indicates that over the population the fitted model is 99% stable. Finally, to detect the outliers standardized residuals can be plotted. Standardized residual within ± 3 indicates no outlier (Pankratz, 1991).

2.5 Data

We have taken annual data of MAMF, TFR and NRR for the period 1981 to 1998 from Bangladesh Bureau of Statistics (BBS).

2.6 Numerical Results

Primarily the stationarity of any variable can be detected by observing the line graph (Fig.1(a), Fig. 2(a) and Fig.3(a)). Then we need to perform the unit root test of the variables TFR, NRR, and MAMF. Using the unit root test we have obtained that the variable TFR and NRR are stationary at the level and MAMF is stationary after first difference (Table 1).

Table 1: Unit Root Test

Variables	Specification	DF-Value	MacKinnon Critical Value at 1% level	Stationary at
MAMF	None	-5.153695	-2.7275	First Difference
TFR	None	-3.501475	-2.7158	Level
NRR	None	-2.732991	-2.7158	Level

The variable MAMF is non-stationary at the level.

Since the variables TFR and NRR are stationary at the level so we may build AR model for them. The fitted AR model for TFR is-

$$\widehat{TFR_t} = 5.581236 - 0.139764t + 0.511633TFR_{t-1} \tag{11}$$
$$t - stat. = \ (32.49427)\,(-10.11645)\,(2.437821)$$
$$prob^y. = \ (0.0000)\,(0.0000)\,(0.0000)$$
$$R^2 = 0.975566 \quad \rho^2_{rcv} = \ 0.9719 \quad \tilde{\eta} = 0.99634$$
$$Inverted\,Autoregressive\,Root = 0.51$$

The validity of the model has examined by computing RCVPP, ρ^2_{rcv} (Khan and Ali, 2003a) and the stability of the model has tested using stability level ($\tilde{\eta}$). We observe that the fitted model secured 97.19% of validity and over all population it is 99.634% stable. To detect the outlier we have plotted standardized residuals (Pankratz, 1991) and have found no outlier (Fig. 1(b)). So, the fitted model is good enough to decide about the growth and we can say that if all other factors remain constant the TFR will decrease 13.9764% for the passage of unit time period and one unit decrease in TFR at previous period will divulge the decrease of 0.511633 units at the current period.

Similarly, the fitted model for NRR is:

$$\widehat{NRR_t} = 2.034837 - 0.038127t + 0.617439NRR_{t-1} \tag{12}$$
$$t - stat. = \ (21.49096)\,(-5.283336)\,(2.869094)$$
$$prob^y. = \ (0.0000)\ (0.0000)\ (0.0124)$$
$$R^2 = 0.953010 \quad \rho^2_{rcv} = 0.94596 \quad \tilde{\eta} = 0.992958$$
$$Inverted\,AR\,Root = 0.62$$

The computed coefficient of the stability level ($\tilde{\eta}$) divulge that over the population the fitted AR model is more than 99% stable. Further, the standardized residuals (Fig. 2(b)) show no outlier. Thus, the fitted model is good enough and we can say that if all other factors remain constant the NRR will decrease 3.8127% for unit increase in the time variable. NRR is decreasing over time and so one unit decrease in NRR at the previous year will cause a 61.744% decrease in NRR at the current period.

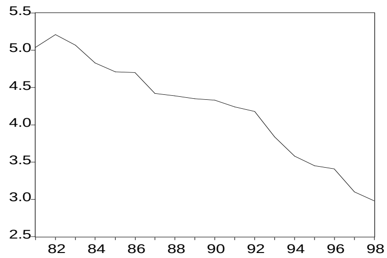

Fig.1(a): Line graph of TFR

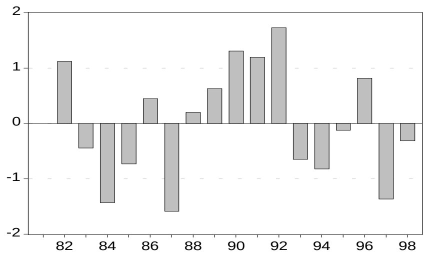

Fig.1(b): Standardized residuals for TFR

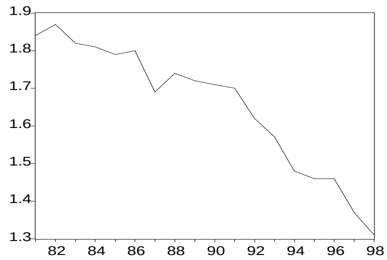

Fig. 2(a): Line graph for NRR

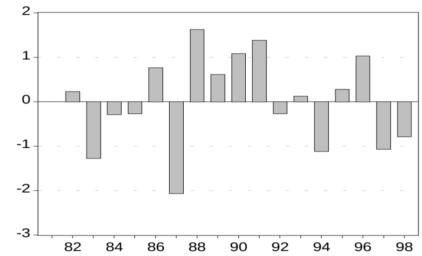

Fig. 2(b): Standardized Residuals for NRR

We have three variables TFR, NRR, and MAMF. Firstly, let $Y_1 = (TFR\ NRR)'$ and X_2 is the vector of lagged MAMF. Then, by fitting eq.(2) and eq.(3) we get $\left|\hat{\Omega}_{11}\right| = 0.00000973$ and $\left|\hat{\Omega}_{11}(0)\right| = 0.0000132$, respectively. The calculated value of test statistic is 0.3974 that is smaller than the tabulated value, $\chi^2(2, 0.05) = 5.99$ and $\chi^2(2, 0.1) = 4.605$, that is, the variable MAMF has no significant effect on pair of variables (TFR, NRR). Similarly, we see that the variables NRR and TFR have insignificant effect on pair of variables (TFR, MAMF) and (NRR, MAMF), respectively (Table 2).

We have proposed a new theme to compute $\left|\hat{\Omega}_{11}\right|$ and $\left|\hat{\Omega}_{11}(0)\right|$ by fitting equation eq.(7) and eq.(8) or eq.(9), respectively. Let $Y_1 = (MAMF\ \ TFR)'$, X_2 is the vector of lagged NRR, and X_3 is a vector of time trend (T). By fitting eq.(7) we get $\left|\hat{\Omega}_{11}\right| =$

Table 2: Test of Endogeneity of variables

Y_1	Y_2	$\lvert\hat{\Omega}_{11}\rvert$	$\lvert\hat{\Omega}_{11}(0)\rvert$	χ^2_{cal}	$\chi^2(n_1 n_2 p)$ $=\chi^2(2,\ 0.1)$
$(TFR\ \ NRR)'$	(MAMF)	0.00000973	0.0000132	0.3974	4.605
$(MAMF\ \ TFR)'$	(NRR)	0.001482	0.003185	0.996784	4.605
$(MAMF\ \ NRR)'$	(TFR)	0.000158	0.000343	1.909911	4.605

Here $n_1 = 2$, $n_2 = 1$, and $p = 1$ for all three tests.

0.000448. We tried to fit eq.(8), but intercept and T were found insignificant. Thus, we have computed $\left|\hat{\Omega}_{11}(0)\right| = 0.002214$ by fitting eq.(9). The computed test statistic is obtained as 5.884243, which is greater than $\chi^2(2,\ 0.1)$ that is, NRR has a significant effect on $Y_1 = (MAMF\ TFR)'$ at 10% level of significance. Similarly, we can show that TFR and MAMF have significant effect on $(MAMF\ NRR)'$ and $(TFR\ NRR)'$, respectively (Table 3). Therefore, we observe that at 10% level of significance MAMF, TFR, and NRR are endogenous. From the results shown in Table 2 and Table 3 it is clear that the endogeneity of a set of variables are affected by some other exogenous variables like time trend or seasonal dummies.

Table 3: Test of Endogeneity of variables including intercept and trend variable (T)

Y_1	Y_2	$\lvert\hat{\Omega}_{11}\rvert$	$\lvert\hat{\Omega}_{11}(0)\rvert$	χ^2_{cal}	$\chi^2(n_1 n_2 p)$ $=\chi^2(2,\ 0.1)$
$(TFR\ \ NRR)'$	(MAMF)	0.00000284	0.000132	4.609	4.605
$(MAMF\ \ TFR)'$	(NRR)	0.000448	0.002214	5.884	4.605
$(MAMF\ \ NRR)'$	(TFR)	0.0000706	0.000343	4.742	4.605

Here $n_1 = 2$, $n_2 = 1$, and $p = 1$ for all three tests.

We have obtained that MAMF is non-stationary at the level, but stationary at first difference. So, the variables under study are mixed in nature. Then, according to Khan and Ali (2003b), a VAR model can be built. Significant effect of linear trend is observed in eq.(11) and eq.(12). So, we need to use linear trend component as an exogenous variable in the VAR model. According to the criterions, SC and AIC, a VAR model with order $p = 1$ is found with maximum stability level (Table 4).

Table 4 divulge that if the dynamic interactions among the endogenous variables are fixed at a particular level then the trend component possesses positive effect on MAMF (increased by 25.125%), but negative effect on TFR (decreased by 6.75%) and NRR (decreased by 1.22%).

Table 4: VAR Model

	$MAMF_t$	TFR_t	NRR_t
$MAMF_{t-1}$	0.694084	0.038273	0.014352
TFR_{t-1}	2.779510	-0.070346	0.077257
NRR_{t-1}	-5.151211	2.647988	1.093240
T	0.251254	-0.067472	-0.012198
R^2	0.831778	0.979006	0.947580
SL	0.84438	0.98058	0.95151

VAR(1) model has the highest SL than any other ordered VAR(p), where SL indicates stability level.

Further, the variance decomposition explains that 100% variation of MAMF is explained by itself at the 1^{st} period and after 1^{st} period the variation of MAMF explained by TFR that increases with lag periods (Table 5). At the 5^{th} lag period the 20.3% and 4.81% variations of MAMF are explained by TFR and NRR, respectively. Again, at the 1^{st} period more than 99% variation of TFR is explained by itself at the one lag period. With the lag periods the variation of TFR explained by the NRR increases in a greater context and at the 5^{th} period 33.54% variation of TFR is explained by NRR. Similarly, at the 1^{st} period more than 8% and 52% variation of NRR are explained by MAMF and TFR, respectively. With the passes of time the variation of NRR explained by MAMF and TFR decreases and at the 5^{th} period only 3.52% and more than 51% variation are explained by MAMF and TRF, respectively.

Further, to know the short-run dynamic interactions we compute impulse response functions (Fig. 3). The response of MAMF shows significant effect at 5% level, as both the line crosses (upward) the zero line. Similarly, the response of TFR and NRR to the shocks of NRR and TFR are significant at 5% level, respectively. All the significant responses on shocks are positive.

Finally, we have forecasted three variables TFR, NRR, and MAMF for the periods 1999 to 2008 (Table 6). In 2008 the mean age at marriage for females, net reproduction rate, and total fertility rate will be 21.23076 years, 0.561056, and 0.222202, respectively. The patterns, increasing in MAMF and decreasing in both TFR and NRR, are in accord with those of others (Islam and Islam, 1993; Sheikh, 1997; Islam and Ahmed, 1998; Hilderink, 2000). The application of VAR model on demographic variables are rare. Therefore, further discussion is kept limited.

Table 5: Variance Decomposition

Variable	Period	Std. Error	MAMF	TFR	NRR
MAMF	1	0.397932	100.0000	0.000000	0.000000
	2	0.533812	89.86413	4.962648	5.173223
	3	0.594018	84.96601	10.31931	4.714678
	4	0.631991	80.13756	15.56458	4.297860
	5	0.664625	74.88937	20.30010	4.810535
TFR	1	0.094209	0.637418	99.36258	0.000000
	2	0.131807	1.406502	76.17161	22.42189
	3	0.156676	1.023395	68.83307	30.14353
	4	0.175116	0.931727	66.32958	32.73869
	5	0.190410	1.121168	65.33917	33.53966
NRR	1	0.037952	8.594847	52.83493	38.57023
	2	0.051608	5.942564	48.26876	45.78868
	3	0.059984	4.420071	48.71180	46.86813
	4	0.066238	3.740035	50.08875	46.17121
	5	0.071484	3.531792	51.49703	44.97117

This table decomposed the variance of one variable by the all other variables.

Table 6: Forecasted MAMF, TFR, and NRR

Variables	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008
MAMF	20.1534	20.2821	20.4088	20.5335	20.6560	20.7762	20.8940	21.0090	21.1214	21.2308
NRR	1.21218	1.15320	1.09115	1.02593	0.95741	0.88549	0.81002	0.73088	0.64794	0.56106
TFR	2.55647	2.34252	2.11814	1.88297	1.63661	1.37864	1.10866	0.82623	0.5309	0.22220

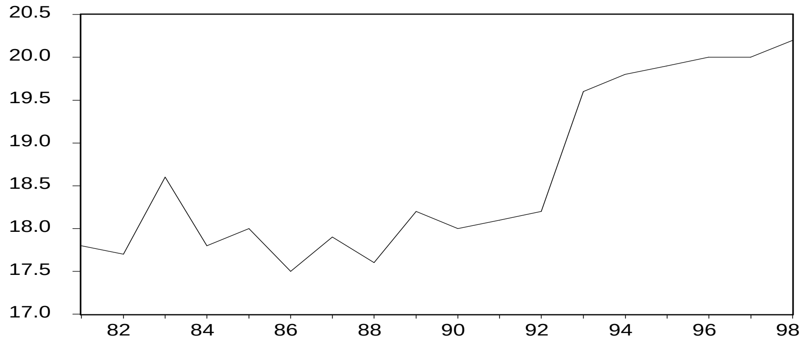

Fig. 3(a): Line graph for MAMF

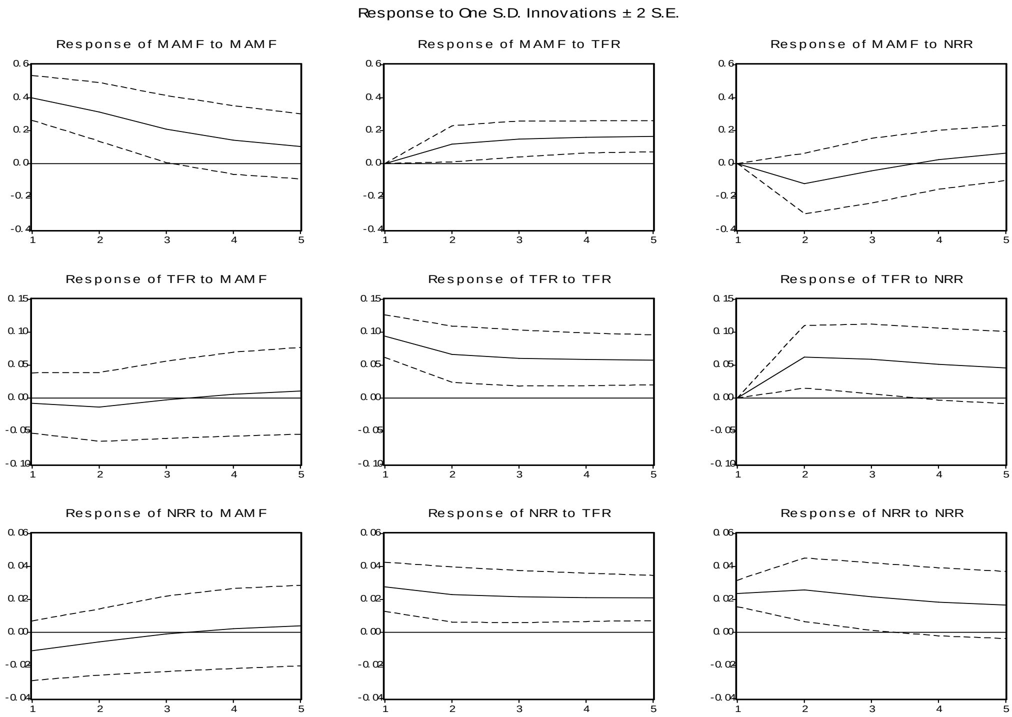

Fig. 3(b): Impulse response functions.

3 Concluding Remarks

The mean age at marriage for females in Bangladesh is increasing over time, whereas the total fertility rate and net reproduction rate are decreasing. Endogeneity of a set of variables are affected by exogenous variables like time trend or seasonal dummies. Over the passage of time mean age at marriage for females, total fertility rate and net reproduction rate are endogenous. These variables are dynamically interacted with each other and there is a positive effect on response to shocks.

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