Research Contributions
of Emeritus Professor M Safiul Haq

Shahjahan Khan
Department of Mathematics and Computing
University of Southern Queensland
Toowoomba, Queensland, Australia.

Abstract

This brief note highlights most of the original research contributions of Emeritus Professor M Safiul Haq, University of Western Ontario, Canada. An attempt has been made to include his individual as well as joint work with his PhD Students.

Keywords and Phrases: Structural methods/distribuotion, Structural relations, Prediction distribution, Tolerance region, and Bayesian prediction.

1   Introduction

Professor Haq is one of the most successful postgraduate students of the most productive statistician of Canada, Professor D. A. S. Fraser. Professor Haq joined University of Toronto as a graduate student in 1963 to work with Professor Fraser. He completed his Masters in 1964 and Ph.D. in 1966. During this period of time Fraser introduced the theory of *structural inference.* Haq was his first student investigating prediction problems using structural approach. Fraser's first book on Structural Inference (1968) includes part of a chapter with Haq.

2   Research Contributions

It is probably not out of place to say a few words about Structural Method of Statistical Inference. Statistical Inference is concerned with the process of obtaining information about "unknown characteristics and properties of a real world system from background

information and current data from an investigation of the system." Several methods exist for obtaining such information. The classical method, in general, uses the underlying probability model for the data and ignores any further background information about the system. For the Bayesian method, some of the background information is expressed as prior probability distribution of the unknowns. The choice of the prior probability distribution is subjective and the results vary substantially for different choice of priors. For the structural approach, apart from the underlying probability model for the data, the relationship (known as structural relation) between the observations and the unknowns is taken into consideration as background information and is exploited rigorously for obtaining information about the system. The structural relationship is an integral part of the system and ignoring any such background information about the system seems inappropriate. Further, the results obtained by this method are unique. The approach in its present form is known as Structural Inference.

The bulk of Haq's work involves the use of structural method of statistical inference. Using this method he has studied various statistical models which include multivariate models, models with different kinds of correlation structures, prediction problems, tolerance regions, etc. In addition to structural inference, he has also worked on multivariate model buildings. Furthermore, he has been involved in cross-disciplinary research activities with the members of the Faculty of Dentistry, the University of Western Ontario. He has also worked on Bayesian prediction.

Predictive inference primarily concerns about the inference on future observables, (cf. Geisser, 1993). The distribution of the unobserved future response(s), conditional on the realized responses in the form of a random sample, and free from any unknown parameters is called the prediction distribution. This is by nature a Bayesian method and is based on a technique that integrates out the unknown parameters from the joint distribution of the realized and future responses with respect to the posterior distribution of the parameters involved. Fraser (1968), and Fraser and Haq (1969) used the structural distribution of the parameters, as opposed to using the Bayesian posterior distribution based on some pre-selected prior distribution, to derive the prediction distribution for the responses of the multivariate normal model. Haq (1970) derived the exact distribution of the first order serial correlation coefficient using the structural approach. Haq and Khan (1990a) used the structural approach to find the prediction distribution of dependent but uncorrelated responses of the multivariate Student t models. Haq (1982) used the relationship between the responses and their error components to define the *structural relations* between them to find the prediction distribution. This method does not require the derivation of the *structural distribution*, to replace the Bayes posterior distribution, to obtain the prediction distribution. Interestingly, the results from both the *structural relation* and *structural distribution* are identical. Haq's method is straightforward and avoids complicated integration.

Under the supervision of Haq, Ng (1976) derived the generalized structural distribution and prediction distribution. Later Gan (1980) produced prediction distribution for life testing models. Recently, Ng (2000) has obtained the prediction distribution

for the multivariate elliptic models using both Bayesian and classical methods.

In case of the presence of any nuisance parameter in the model, both the structural relation and distribution approaches, require to handle it. Haq (1970), and Haq and Rahman (1971) proposed the marginal likelihood method to handle the problem of such a nuisance parameter for the autoregressive model. Later Haq (1974), and Ng and Haq (1979) used the same technique for the multivariate model with intra-class covariance structure. The same method was applied for equi-correlated responses by Haq (1978). Prediction distribution uses the marginal likelihood estimate of the nuisance parameter to replace the relevant unknown parameter. Khan and Haq (1994) used the marginal likelihood estimate of the autocorrelation coefficient for the predictive inference of the multivariate linear model with Student t errors.

Haq and Kibria (1997) considered the predictive inference for the Gaussian linear model with MA(1) error process. Kibria and Haq (1998) derived the marginal likelihood function of the model parameters and predictive distribution for future responses for the multivariate ARMA(1,1) linear model with multivariate-t error. Kibria and Haq (1999) proposed predictive inference for the linear model with elliptical error distribution.

One of the most popular uses of the prediction distribution is in the construction of tolerance regions (or intervals in one dimension). Guttman (1970), Aitchison and Dunsmore (1976), and Geisser (1993) used the prediction distribution in a varieties of predictive inferences, including defining β-expectation tolerance regions. Such tolerance regions (or intervals) cover a preassigned proportion (β) of future responses on the average. Haq and Rinco (1976) obtained the β-expectation tolerance region for the location-scale normal model and generalized multivariate model with normal errors. Recently, Khan (2002) has proposed an optimal β-expectation tolerance region for the elliptic model.

In the recent years Haq has worked on Bayesian prediction method. Prediction for a compound multivariate linear model using conjugate prior has been introduced by Thabane and Haq (1997, 1998). Thabane and Haq (1999a) used Kullback-Leibler divergence approach to select best population. Recently, Thabane and Haq (2004) introduced the matrix-variate generalized hyperbolic distribution and discussed the applications of the distribution in the Bayesian analysis of the normal multivariate linear model. Very recently Khan (2003) has completed his PhD dissertation on Predictive Inference for Certain Life Testing Models under Haq's supervision. He has also proposed predictive distributions for the two parameter exponential life testing model (cf. Haq et al. 2004).

Among the other areas that Haq worked in is the well known Behrens-Fisher problem (Haq, 1970). The multivariate version of this problem is discussed in Thabane and Haq (1999b), and extended to a class of elliptically symmetric distributions in Thabane and Haq (2002). He also worked in the life testing and reliability problems. Haq and Khan (1990b) proposed the prediction distribution for the simultaneous equation model.

3   Editorial Works

Haq has served as an associate editor of *Foundations of Statistical Inference:* Volume 2 of the Festschrift in Honor of Professor V.M. Joshi's 70th birthday in 1987. He also worked as the editor of a volume on *Recent Advances in Statistics and Actuarial Science:* Festschrift in celebration of Professor M. M. Ali's twenty-five years at Western in 1989. He is the founding chief editor of the International Journal of Statistical Sciences. He has been as associate editor of a number of international journals including the Journal of Statistical Research.

4   Concluding Remarks

The outstanding contribution of Haq in the statistical research has been well recorded in his various research publications. His remarkable scholarship in the subject has been reflected in his brilliant supervision of 8 PhD students and 15 other postgraduate students. He is still very active in joint research with some of his former students and expected to continue producing more superior quality results in the years to come. Here we have included only those papers that focus the major research contributions of Professor Haq. It does not include all the papers that he has published, nor does it list all the papers that his students have published with him as coauthors. For a full list of Professor Haq's publications please refer to his updated curriculum vitae.

References of Haq's Individual and Joint Work

(1969) Structural probability and prediction for the multivariate model, *Jou. Roy. Statist. Soc.*, Series B 31, No. 2, 317-331, with D A S Fraser.

(1969) On prediction from a linear regression model, *Statistiche Hefte*, 10, 111-118.

(1970) Structural analysis for the first order autoregressive stochastic models, *Ann. Math. Statist.*, 41, No. 3, 970-978.

(1970) On a simple linear regression analogue for the Behrens-Fisher problem, *Statistiche Hefte*, 11, 22-29.

(1970) Inference and prediciton for the multilinear model, J. Stat. Res., 4, 93-109, with D.A.S. Fraser.

(1971) Marginal likelihood function of the first order auto correlation of a multilinear model, *Jou. Statist. Res.*, 5, 23-42, with A S M M Rahman.

(1972) On β-expectation tolerance for the location-scale model, *Jou. Stat. Res.*, 6, 52-59, with S Rinco.

(1974) A multivariate model with intraclass covariance structure, *Ann. Inst. Statist. Math.*, 26, 413-420.

(1974) Inference about the difference between two p-component means in the presence of intra-class correlation, *Jou. Statist. Res.*, 8, 1-12.

(1976) On β-expectation tolerance region for a generalized multivariate model with normal error variable, *Jou. Mulitvariate Analysis*, 6, 414-421, with S. Rinco.

(1978) On prediction from the location-scale model with equi-correlated response, *Communications in Statistics*, A7(9), 857-866.

(1978) A class of bivariate distributions including the bivariate logistic., Jou. Mult. Anal. , 8, 404-412, with M.M. Ali and N.N. Mikhail.

(1979) Spiramicin as a selective dental plaque control agent: laboratory data. Jou. of Periodontal Research, vol.14, 55-64, with R. H. Johnson, J. Rozanis, I. D. F. Schofield.

(1979) On marginal likelihood inference for the intraclass correlation coefficient, *Communications in Statistics Theory and Methods,* A8(2), 179-189, with V M Ng.

(1982). Structural relation and prediction for multivariate models. *Statistische Hefte*, 23, 218-228.

(1984) Structural estimation of certain correlation coefficients. *Statistische Hefte*, 25, 211-218, with A.K. Md. E. Saleh.

(1990a). Prediction Distribution for the Linear Model with Multivariate Student-t Error Distribution. *Communications In Statistics, Theory & Methods*, 19, 4705-4712.

(1990b) Structural inference for the parameters of the heteroscedastic simultaneous equation model, *Communications in Statistics, Theory & Methods.* 19, 4713-4729, with S Khan.

(1992) On prediction from a location-scale model with a compound error distribution. *Statistical Papers*, 33, 361-366, with Q M Rahman.

(1993) Prediction distribution for the multilinear model with matrix-T error: the case of the spherical error distribution. *Journal of Applied Statistical Science*, 1, 239-250, with S Khan.

(1994) Predictive inference for the multilinear model with errors having multivariate-t distribution and first order autocorrelation structure. *Sankhya: The Indian Journal of Statistics*, Series B, 56, 95-106, with S Khan.

(1996) Bayesian inference of two-sample hypothesis testing problem using Kullback-Liebler divergency measure. *Journal of Statistical Research,* 30, 101-120, with L Thabane.

(1997) Predictive inference for the linear and multivariate linear models with MA(1) error processes. *Communications in Statistics - Theory & Methods*, 26, 331-353 with B M G Kibria

(1998) Marginal likelihood and prediction for the multivariate ARMA(1,1) linear models with multivariate t error distribution. *Journal of Statistical Research.* 32(1) 71-80 with B M G Kibria.

(1999a) On Bayesian selection of the best normal population using the Kullback-Leibler divergence measure. *Statistika Neerlandica*, 53, 342-360, with L Thabane.

(1999b) An alternative approach to the multivariate Behrens-Fisher problem. *Com-*

munications in Statistics: Simulation & Computation. Vol.28, 243-258, with L Thabane.

(1999) Predictive inference for the elliptical linear model. *Journal of Multivariate Analysis.* 68, 235-249 with B M G Kibria.

(2002) A Note on the Behrens-Fisher Problem for Elliptically Symmetric Distributions. *Student.* 4 (2), 121-125, with L Thabane.

(2004) On the Generalized Matrix-variate Hyperbolic Distribution and Its Application to the Bayesian Analysis of the Normal Linear Model. To appear in *Statistics*, with L Thabane.

(2004) Predictive Distributions for Responses from the two Parameter Exponential Life Testing Model. *Journal of Applied Statistical Science*, 12(2), 137-156 with Hafiz M. R. Khan and Serge B Provost.

Other References

Aitchison J., and Dunsmore, I.R. (1975). *Statistical Prediction Analysis.* Cambridge University Press, Cambridge.

Fraser, D.A.S. (1968). *The Structure of Inference.* Wiley, New York.

Fraser, D.A.S. (1979). *Inference and Linear Models.* McGraw-Hill, New York.

Gan, E.H. (1980). *Prediction Distribution for Life Testing Models.* Unpublished PhD Thesis, Department of Statistical and Actuarial Sciences, UWO, Canada.

Geisser, S. (1993). *Predictive Inference: An Introduction.* Chapman & Hall, London.

Guttman, I. (1970). *Statistical tolerance regions: Classical and Bayesian.* Griffin, London.

Khan, H. M. R. (2003). Bayesian Predictive Inference for Certain Life Testing Models, Unpublished PhD Thesis, Department of Statistical and Actuarial Sciences, UWO, Canada.

Khan, S. (2002). A Note on an Optimal Tolerance Region for the Class of Multivariate Elliptically Contoured Location-Scale Model, Calcutta Statistical Association Bulletin, 53, 125-131.

Ng, V. M. (1976). *Generalized structural distribution, prediction distribution and related topics.* Unpublished PhD Thesis, Department of Statistical and Actuarial Sciences, The UWO, Canada.

Ng, V.M. (2000). A note on predictive inference for multivariate elliptically contoured distributions. *Comm. In Statist. - Theory & Methods,* 29 (3), 477-483.