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Testing equality of means and variances of several Weibull populations

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Abstract

The usual practice for testing homogeneity of several populations in terms of means and variances is first to test for the equality of variances and once this assumption is found to be tenable then to test the equality of means. Singh (1986) proposes a very interesting procedure based on two likelihood ratio statistics for testing homogeneity of several normal (μ, σ^2) populations. Singh uses a method by Fisher (1950) which is based on combining two or more independent tests. Paul and Jiang (2004) generalized Fisher's method for testing homogeneity of several two-parameter populations based on two asymptotically independent likelihood ratio and two asymptotically independent score test statistics. They then obtained specific results to test homogeneity of several normal, several negative binomial and several beta-binomial populations. Based on simulations from these populations they show that Fisher's method of combining two statistics, even when they are only asymptotically independent, does, in general, perform well for testing homogeneity of several two-parameter populations. However, the score test statistics have simple forms, easy to calculate, and have uniformly good level properties. Therefore they recommend Fisher's method based on combining two score test statistics. Note that their conclusion is based on simulation results for the normal and two discrete populations. The purpose of this paper is to investigate the performance of Fisher's method for testing homogeneity of several Weibull populations.

Keywords and Phrases: Fisher's method; Homogeneity tests; Independent tests; Likelihood ratio tests; Simultaneously hypothesis testing; Score tests; Weibull population.

AMS Classification: Primary 62F03; secondary62F05.

1 Introduction

In the usual one-way classification problem of testing the equality of means is based on the assumption that the variances among different groups are homogeneous. But in practice, we often get data which are different not only in means but also in variances. Snedecor and Cochran (1967, p 324) observe that an application of different treatments to otherwise homogeneous experimental units often results in groups that are different not only in means but also in variances. Thus, testing homogeneity of several populations in terms of means and variances is of considerable interest. The usual practice for testing homogeneity of several populations in terms of means and variances is first to test for the equality of variances and once this assumption is found to be tenable then to test the equality of means. Fisher (1950) suggested combining several independent tests. We quote (Fisher, 1950, p 99)

"When a number of quite independent tests of significance have been made, it sometimes happens that although few or none can be claimed individually as significant, yet the aggregate gives an impression that the probabilities are on the whole lower than would often have been obtained by chance. It is sometimes desired, taking account only of these probabilities, and not of the detailed composition of the data from which they are derived, which may be of very different kinds, to obtain a single test of the significance of the aggregate, based on the product of the probabilities individually observed."

Assume that we wish to test a null hypothesis $H_0 : \theta \in \Theta_0$, where Θ_0 is a subset of a parameter space Θ. Suppose we have available p independent tests for testing H_0. We wish to combine these p tests into an overall test for H. Several methods of combining independent tests, including a method by Fisher (1950), are available. None of these procedures are uniformly the most powerful. However, Littell and Folks (1971) have compared Fisher's method with three other well-known methods via exact Bahadur relative efficiency, and have found that Fisher's method is always at least as efficient as the other three methods, and Littell and Folks (1973) have shown that Fisher's method is the most efficient.

Let $T^{(1)}, \cdots, T^{(p)}$ be p independent sequences of test statistics for testing H_0. Then, Fisher's method of combining the independent tests $T^{(1)}, \cdots, T^{(p)}$ is given by $T^{(F)} = -2 \log \prod_i L^{(i)}$, where $L^{(i)} = 1 - F^{(i)}\left(T^{(i)}\right)$ and $F^{(i)}(t) = Pr\{T^{(i)} < t\}$ is the null cumulative distribution function of $T^{(i)}$, and its large values indicate evidence against H_0.

Singh (1986) uses Fisher's method for testing simultaneously the equality of means and the equality of variances of several normal populations. Singh uses a test statistic which is the combination of two independent likelihood ratio statistics and shows that the test statistic is asymptotically optimal in the sense of Bahadur (1967) efficiency. Paul and Jiang (2004) apply Fisher's method to test homogeneity of several two-parameter populations in general. As examples of application they consider the normal, the negative binomial and the beta-binomial distributions. They developed two test statistics, one of which is based on the combination of two asymptotically independent likelihood ratio statistics and the other is based on the combination of two asymptotically independent score test statistics. Thomas, Sinha and Zhou (1993) also worked on combining independent tests in somewhat different contexts.

The score test (Rao, 1947) is a special case of the more general $C(\alpha)$ test (Neyman, 1959) in which the nuisance parameters are replaced by maximum likelihood estimates which are $\sqrt{N}$ consistent estimates (N=number of observations used in estimating the parameters). The score test is particularly appealing as we have only to study the distribution of the test statistic under the null hypothesis which is that of the basic model. Further, the score or the $C(\alpha)$ class of tests have the following advantages: (i) it often maintains, at least approximately, a preassigned level of significance (Bartoo and Puri, 1967), and (ii) it often produces a statistic which is simple to calculate. These tests are robust in the sense that their optimality remains true whatever the form of the distribution assumed for the data under the alternative hypothesis - a property called robustness of optimality by Neyman and Scott (1966). For more discussion on the choice of $C(\alpha)$ or score tests see Breslow (1990) and Paul and Banerjee (1998). The $C(\alpha)$ test has been shown by many authors to be asymptotically equivalent to the likelihood ratio test and to the Wald test (Moran, 1970; Cox and Hinkley, 1974). Potential drawbacks to the use of the likelihood ratio and Wald tests include the fact that both require estimates of the parameters under the alternative hypotheses and often show liberal or conservative behaviour (See, for example, Barnwal and Paul, 1988, Paul, 1989, and Paul and Islam, 1995).

Based on extensive simulations from the normal, negative binomial and the beta-binomial populations, Paul and Jiang (2004) show that Fisher's method of combining two statistics, even when they are only asymptotically independent, does, in general, perform well for testing homogeneity of several two-parameter populations. However, the score test statistics have simple forms, easy to calculate, and have uniformly good level properties. Therefore they recommend Fisher's method based on combining two score test statistics. Their conclusion, however, is based on simulation results for the normal and two discrete populations. In this paper we investigate the performance of Fisher's method for testing homogeneity of several Weibull populations.

In Section 2 we review the likelihood ratio and the score test procedures for testing homogeneity of several two-parameter populations. Specific test statistics are developed in Section 3 for testing homogeneity of several Weibull populations. In Section 4 we report results of the simulation study.

2 Review of Fisher's methods in a general two-Parameter model

2.1 The Likelihood ratio tests

Consider a family of distributions $f(x, \psi, \phi)$, where ψ and ϕ are scalar parameters. We assume that $f(x, \psi, \phi)$ satisfies the usual regularity conditions. Suppose we obtain data

$x_{i1}, x_{i2}, ..., x_{in_i}$ from the ith, $i = 1, ..., k$, population with parameters ψ_i and ϕ_i. Then, the log-likelihood can be written as $l = \sum_{i=1}^{k} l_i$, where $l_i = \sum_{j=1}^{n_i} \log f(x_{ij}, \psi_i, \phi_i)$. Now, let $\Psi = (\psi_1, ..., \psi_k)$, $\Phi = (\phi_1, \ldots, \phi_k)$. Define the parameter spaces $\Omega_0 = \{(\Psi, \Phi) \mid \psi_i = \psi, \phi_i = \phi, i = 1, \ldots, k$, where ψ and ϕ are unspecified$\}$, $\Omega_1 = \{(\Psi, \Phi) \mid \phi_i = \phi, i = 1, \ldots k$, where $\psi_i, i = 1, ..., k$, and ϕ are unspecified$\}$ and $\Omega_2 = \{(\Psi, \Phi) \mid \psi_i$ and $\phi_i, i = 1, \ldots, k$, are unspecified$\}$.

Suppose we wish to test

$H_0 : \psi_i = \psi, \phi_i = \phi, i = 1, \ldots, k$, where ψ and ϕ are unspecified against H_1 : at least two ψ's or two ϕ's are not same.

Then the test by Fisher's method is the combination of two independent tests corresponding to the following hypotheses:

H_0': $\psi_i = \psi, \phi_i = \phi, i = 1, \ldots, k$, where ψ and ϕ are unspecified against H_1': at least two ψ's are not same and $\phi_i = \phi, i = 1, \ldots, k$, where ϕ are unspecified.

and

H_0'': $\phi_i = \phi, i = 1, \ldots, k$, where ϕ is unspecified against H_1'': at least two ϕ's are not same. Under both H_0'' and H_1'', the ψ_i's, $i = 1, ..., k$, are unspecified.

Let $\hat{l_0} = l(\hat{\Omega}_0), \hat{l_1} = l(\hat{\Omega}_1)$ and $\hat{l_2} = l(\hat{\Omega}_2)$ be the maximized log-likelihood under the parameter spaces Ω_0, Ω_1 and Ω_2 respectively. Then, the likelihood ratio statistics for testing H_0' against H_1' and H_0'' against H_1'' are

$$LR_1 = 2(\hat{l_1} - \hat{l_0}) \tag{1}$$

and

$$LR_2 = 2(\hat{l_2} - \hat{l_1}) \tag{2}$$

respectively.

Let $L_1(t_1) = Pr(LR_1 \geq t_1 \mid H_0')$ and $L_2(t_2) = Pr(LR_2 \geq t_2 \mid H_0'')$. Since $L_1(LR_1)$ and $L_2(LR_2)$ are asymptotically independently distributed and they both are asymptotically distributed as $U(0, 1)$, then,

$$M_1 = -2 \log[L_1(LR_1) L_2(LR_2)] \tag{3}$$

is approximately distributed as χ_4^2 under H_0. Thus, we reject H_0 in favor of H_1, if $M_1 \geq \chi_4^2(\alpha)$, where $\chi_4^2(\alpha)$ is the upper $100\alpha\%$ point of the χ^2 distribution with 4 degrees of freedom. For the proof of asymptotic independence of LR_1 and LR_2, see Paul and Jiang (2004).

2.2 The score tests

Reparameterize $\psi_i, i = 1, ..., k$, under H_1', by $\psi_i = \psi + \alpha_i$ with $\alpha_k = 0$. Let $\alpha' = (\alpha_1, ..., \alpha_{k-1})$ and $\omega' = (\psi, \phi)$. Now, define $s_{1i} = \left.\frac{\partial l_i}{\partial \psi}\right|_{\alpha=0}$, $v_{1i} = E\left(-\left.\frac{\partial^2 l_i}{\partial \psi^2}\right|_{\alpha=0}\right)$,

$u_{1i} = E\left(-\frac{\partial^2 l_i}{\partial\psi\partial\phi}\Big|_{\alpha=0}\right)$ and $w_{1i} = E\left(-\frac{\partial^2 l_i}{\partial\phi^2}\Big|_{\alpha=0}\right)$, $i = 1,...,k$. Then, the score test statistic for testing H_0' against H_1' can be written as

$$S_1 = \sum_{i=1}^{k} \frac{\hat{s}_{1i}^2}{\hat{v}_{1i}} + \frac{\left(\sum_{i=1}^{k} \hat{s}_{1i}\frac{\hat{u}_{i1}}{\hat{v}_{1i}}\right)^2}{\sum_{i=1}^{k}\left(\hat{w}_{1i} - \frac{\hat{u}_{1i}^2}{\hat{v}_{1i}}\right)}, \tag{4}$$

where $\hat{s}_{1i} = s_{1i}(\hat{\omega})$, $\hat{v}_{1i} = v_{1i}(\hat{\omega})$, $\hat{u}_{1i} = u_{1i}(\hat{\omega})$ and $\hat{w}_{1i} = w_{1i}(\hat{\omega})$, $i = 1, 2, ..., k$, and $\hat{\omega} = (\hat{\psi}, \hat{\phi})'$ is the maximum likelihood estimate of the nuisance parameter ω under H_0'. For proof see Paul and Jiang (2004).

Now, reparameterize $\phi_i, i = 1, ..., k$, by $\phi_i = \phi + \beta_i$ with $\beta_k = 0$. Let $\beta' = (\beta_1, ..., \beta_{k-1})$. Define $s_{2i} = \frac{\partial l_i}{\partial\phi}\Big|_{\beta=0}$, $v_{2i} = E\left(-\frac{\partial^2 l_i}{\partial\phi^2}\Big|_{\beta=0}\right)$, $u_{2i} = E\left(-\frac{\partial^2 l_i}{\partial\psi_i\partial\phi}\Big|_{\beta=0}\right)$ and $w_{2i} = E\left(-\frac{\partial^2 l_i}{\partial\psi_i\partial\psi_i}\Big|_{\beta=0}\right)$, $i = 1, ..., k$. Then, following similar steps as for S_1, the score test statistic S_2 can be expressed as

$$S_2 = \sum_{i=1}^{k} \frac{\hat{s}_{2i}^2}{\hat{v}_{2i}} + \sum_{i=1}^{k} \frac{\left(\hat{s}_{2i}\frac{\hat{u}_{2i}}{\hat{v}_{2i}}\right)^2}{\left(\hat{w}_{2i} - \frac{\hat{u}_{2i}^2}{\hat{v}_{2i}}\right)}, \tag{5}$$

where $\hat{s}_{2i}$, $\hat{v}_{2i}$, $\hat{u}_{2i}$ and $\hat{w}_{2i}$, $i = 1, 2, ..., k$, are the estimated values of s_{2i}, v_{2i}, u_{2i} and w_{2i}, $i = 1, 2, ..., k$, with the nuisance parameter ξ being replaced by its the maximum likelihood estimate $\hat{\xi}$ under H_0''.

Now, let $L_1(t_1) = Pr(S_1 \geq t_1 \mid H_0')$ and $L_2(t_2) = Pr(S_2 \geq t_2 \mid H_0'')$. Then it follows that

$$M_2 = -2\log[L_1(S_1)L_2(S_2)] \tag{6}$$

is approximately distributed as χ_4^2 under H_0. For proof of asymptotic independence of S_1 and S_2, see Paul and Jiang (2004).

3 Testing equality of means and variances of homogeneity of several Weibull populations

3.1 Fisher's procedure by combining two ratio likelihood statistics

Now let $x_{i1}, ..., x_{in_i}$ be a sample from the Weibull distribution $WB(\psi_i, \phi_i), i = 1, ..., k$, with probability density function

$$Pr(X = x) = \left(\frac{\psi}{\phi}\right)\left(\frac{x}{\phi}\right)^{\psi-1} \exp\left[-\left(\frac{x}{\phi}\right)^{\psi}\right].$$

Note that the mean and variance of X are $\phi\Gamma\left(1+\frac{1}{\psi}\right)$ and $\phi^2\left[\Gamma\left(1+\frac{2}{\psi}\right)-\Gamma^2\left(1+\frac{1}{\psi}\right)\right]$ respectively. Thus, testing the equality of means and equality of variances of the $WB(\psi_i,\phi_i)$ populations, $i=1,...,k$, is equivalent to testing $\psi_i=\psi$ and $\phi_i=\phi$ for all $i=1,...,k$. By using same notation as in section 2.1, the log-likelihood function under Ω is

$l=\sum_{i=1}^k l_i$,

where $l_i=n_i\log\left(\frac{\psi_i}{\phi_i}\right)+(\psi_i-1)\sum_{j=1}^{n_i}\log\left(\frac{x_{ij}}{\phi_i}\right)-\sum_{j=1}^{n_i}\left(\frac{x_{ij}}{\phi_i}\right)^{\psi_i}$, $i=1,...,k$.

The maximum likelihood estimate $\hat{\psi}_i$ of ψ_i, $i=1,...,k$, is obtained by solving the maximum likelihood estimating equation

$$\frac{n_i}{\hat{\psi}_i}+\sum_{j=1}^{n_i}\log x_{ij}-\frac{n_i\sum_{j=1}^{n_i}x_{ij}^{\hat{\psi}_i}\log x_{ij}}{\sum_{j=1}^{n_i}x_{ij}^{\hat{\psi}_i}}=0. \tag{7}$$

Then $\hat{\phi}_i$, $i=1,...,k$, is obtained by

$$\hat{\phi}_i=\left(\frac{\sum_{j=1}^{n_i}x_{ij}^{\hat{\psi}_i}}{n_i}\right)^{1/\hat{\psi}_i}. \tag{8}$$

Similarly, the maximum likelihood estimators $\hat{\psi}_i, i=1,...,k$, and $\hat{\phi}$ of the parameters ψ_i, $i=1,...,k$ and the common parameter ϕ under Ω_1 are obtained by solving the maximum likelihood estimating equations

$$\frac{n_i}{\hat{\psi}_i}+\sum_{j=1}^{n_i}\log\left(\frac{x_{ij}}{\hat{\phi}}\right)-\sum_{j=1}^{n_i}\left(\frac{x_{ij}}{\hat{\phi}}\right)^{\hat{\psi}_i}\log\left(\frac{x_{ij}}{\hat{\phi}}\right)=0, i=1,...,k, \tag{9}$$

and

$$\frac{\sum_{i=1}^k n_i\psi_i}{\phi}+\sum_{i=1}^k\sum_{j=1}^{n_i}\psi_i x_{ij}^{\psi_i}\phi^{-(\psi_i+1)}=0 \tag{10}$$

simultaneously.

The maximum likelihood estimators $\hat{\psi}$ and $\hat{\phi}$ of the common parameters ψ and ϕ under Ω_0 are obtained as

$$\frac{n}{\hat{\psi}}+\sum_{i=1}^k\sum_{j=1}^{n_i}\log x_{ij}-\frac{n\sum_{i=1}^k\sum_{j=1}^{n_i}x_{ij}^{\hat{\psi}_i}\log x_{ij}}{\sum_{i=1}^k\sum_{j=1}^{n_i}x_{ij}^{\hat{\psi}_i}}=0 \tag{11}$$

and

$$\hat{\phi}=\left(\frac{\sum_{i=1}^k\sum_{j=1}^{n_i}x_{ij}^{\hat{\psi}}}{n}\right)^{1/\hat{\psi}}. \tag{12}$$

Now, following (1), (2) and (3), we obtain statistic WBM_1, which is approximately distributed as χ^2_4 under H_0.

3.2 Fisher's procedure by combining two score test statistics

Now, by using the same notation of the general results (4), we obtain the score test statistic WBS_1 for testing H'_0 against H'_1 with

$$s_{1i} = \frac{n_i}{\hat{\psi}} + \sum_{j=1}^{n_i} \log\left(\frac{x_{ij}}{\hat{\phi}}\right) - \sum_{j=1}^{n_i} \left(\frac{x_{ij}}{\hat{\phi}}\right)^{\hat{\psi}} \log\left(\frac{x_{ij}}{\hat{\phi}}\right),$$
$$v_{1i} = \frac{n_i[\pi^2/6+(1-\gamma)^2]}{\hat{\psi}^2},\ u_{1i} = -\frac{n_i(1-\gamma)}{\hat{\phi}},\ \text{and } w_{1i} = n_i\left(\frac{\hat{\psi}}{\hat{\phi}}\right)^2$$

, where γ is Euler's constant, and $\hat{\psi}$ and $\hat{\phi}$ are the maximum likelihood estimates (mle) of ψ and ϕ under H'_0 . The mles of ψ and ϕ are those given in equations (11) and (12).

Similarly, the score test statistic for testing H''_0 against H''_1 is WBS_2 in equation (5) with

$$s_{2i} = \frac{n_i\hat{\psi}_i}{\hat{\phi}} + \sum_{j=1}^{n_i} \hat{\psi}_i x_{ij}^{\hat{\psi}_i} \hat{\phi}^{-(\hat{\psi}_i+1)},$$
$$v_{2i} = n_i\left(\frac{\hat{\psi}_i}{\hat{\phi}}\right)^2,\ u_{2i} = -\frac{n_i(1-\gamma)}{\hat{\phi}} \text{ and } w_{2i} = \frac{n_i[\pi^2/6+(1-\gamma)^2]}{\hat{\psi}_i^2},$$

where $\hat{\psi}_i, i = 1, ..., k$ and $\hat{\phi}$ are the maximum likelihood estimates of $\psi_i, i = 1, ..., k$ and ϕ under H''_0, obtained by solving the maximum likelihood estimating equations (9) and (10) simultaneously.

Again, following (6), we obtain the statistics WBM_2, which is approximately distributed as χ^2_4 under H_0.

4 Simulations

In this section we present results of a simulation study to examine level and power properties of the of the two statistics WBM_1 and WBM_2. In the simulation study we considered K=2, 3, two nominal levels $\alpha = 0.05$ and $\alpha = 0.10$ and equal sample sizes from each population. Each simulation experiment was based on 10,000 samples.

For calculating empirical size, we generated samples from WB (ψ,ϕ) populations with equal ψ's and equal ϕ's. Unequal ψ's and unequal ϕ's were considered for power calculations. Results of the simulations for k=2, $\alpha = 0.05$ and $\alpha = 0.10$ are presented in Table 1 and Table 2. Those for k=3, $\alpha = 0.05$ and $\alpha = 0.10$ presented in Table 3 and Table 4.

According to the results in Table 1 to Table 4, for small sample size ($n < 30$), the statistic WBM_1 shows liberal behaviour, whereas the statistic WBM_2 shows some conservative behaviour. Both statistics show reasonable level properties for large sample size ($n > 50$) The power of the statistic WBM_1 is in general larger than that of the statistic WBM_2. This is because the statistic WBM_1 is liberal and the statistic WBM_2 is conservative. Further, we have extended the simulation experiment to study size adjusted power properties of these two statistics. The empirical 95% quantiles derived from the corresponding size simulation have been used to ensure that each test had approximately the nominal size of 0.05. Empirical quantiles were calculated based on 100,000 replications and empirical power calculations were based on 10,000 replications. In Table 5, we provide empirical power values for $k = 2$, $\psi_1 = \psi_2 = 1.2$, $\phi_1 = \phi_2 = 0.32$, sample sizes are $10, 15, 20, 30, 40, 50$ and for different combinations of the unequal ψ's and unequal ϕ's. Results in Table 5 show that both the size adjusted statistics WBM_1 and WBM_2 have similar power. It seems then that a size correction to either of the two statistics would provide a correct methodology to test equality of means and variances of several Weibull populations.

TABLE 1: Empirical power(%) of different statistics for testing homogeneity of K=2 Weibull populations when data are simulated from $WB(\psi_i, \phi_i)$, $i = 1, 2$; based on 10,000 replications; $\alpha = 0.05$

Sample size	Test Statistic	(ψ_1, ψ_2) (ϕ_1, ϕ_2)					
		(1.2,1.2) (3.2,3.2)	(1.2,1.4) (3.2,3.4)	(1.2,1.6) (3.2,3.6)	(1.2,1.8) (3.2,3.8)	(1.2,2.0) (3.2,4.0)	(1.2,2.2) (3.2,4.2)
10	$WBM1$	7.35	9.14	13.11	19.20	26.88	35.20
	$WBM2$	1.79	2.19	3.70	6.19	10.07	15.46
15	$WBM1$	6.75	9.05	15.60	24.75	36.67	49.91
	$WBM2$	3.22	4.52	8.55	15.47	24.93	36.60
20	$WBM1$	6.65	9.84	17.84	30.79	46.27	61.73
	$WBM2$	3.92	6.23	12.48	23.10	37.35	52.98
30	$WBM1$	5.92	10.74	24.01	43.03	62.76	78.66
	$WBM2$	4.19	8.15	19.69	37.11	57.55	74.34
40	$WBM1$	5.27	11.79	29.71	54.51	76.53	90.33
	$WBM2$	4.17	9.58	26.08	49.84	73.51	88.31
50	$WBM1$	5.82	13.58	35.80	63.84	84.68	95.36
	$WBM2$	4.56	11.71	32.48	60.70	82.40	94.42

TABLE 2: Empirical power(%) of different statistics for testing homogeneity of K=2 Weibull populations when data are simulated from $WB(\psi_i, \phi_i)$, $i = 1, 2$; based on 10,000 replications; $\alpha = 0.10$

Sample size	Test Statistic	(ψ_1, ψ_2) (ϕ_1, ϕ_2)					
		(1.2,1.2) (3.2,3.2)	(1.2,1.4) (3.2,3.4)	(1.2,1.6) (3.2,3.6)	(1.2,1.8) (3.2,3.8)	(1.2,2.0) (3.2,4.0)	(1.2,2.2) (3.2,4.2)
10	WBM1	13.61	15.94	21.35	29.02	38.15	48.02
	WBM2	6.40	8.13	11.99	17.77	24.98	33.39
15	WBM1	12.60	16.05	24.77	36.07	49.63	62.38
	WBM2	8.41	10.94	18.07	28.49	40.91	54.16
20	WBM1	12.27	16.69	27.67	43.04	59.15	73.32
	WBM2	9.20	12.99	22.72	36.99	53.06	67.83
30	WBM1	11.59	18.32	35.14	55.97	73.95	86.95
	WBM2	9.31	15.54	31.18	51.83	70.39	84.51
40	WBM1	10.18	19.81	41.55	66.85	85.30	94.53
	WBM2	8.70	17.61	38.59	64.24	83.69	93.75
50	WBM1	11.05	22.17	48.09	74.92	91.12	97.80
	WBM2	9.87	20.48	45.58	72.96	90.04	97.43

TABLE 3: Empirical power(%) of different statistics for testing homogeneity of K=3 Weibull populations when data are simulated from $WB(\psi_i, \phi_i)$, $i = 1, 2, 3$; based on 10,000 replications; $\alpha = 0.05$

Sample size	Test Statistic	(ψ_1, ψ_2, ψ_3) (ϕ_1, ϕ_2, ϕ_3)					
		(0.12,0.12,0.12) (0.32,0.32,0.32)	(0.12,0.13,0.14) (0.32,0.33,0.34)	(0.12,0.14,0.16) (0.32,0.34,0.36)	(0.12,0.15,0.18) (0.32,0.35,0.38)	(0.12,0.16,0.20) (0.32,0.36,0.40)	(0.12,0.17,0.22) (0.32,0.37,0.42)
10	WBM1	7.87	9.41	12.19	16.8	22.91	30.66
	WBM2	2.53	3.08	4.46	6.99	10.32	15.10
15	WBM1	7.42	8.95	13.38	20.35	29.81	40.05
	WBM2	3.38	4.37	7.52	12.51	20.29	29.55
20	WBM1	6.67	8.77	14.39	24.3	37.09	51.84
	WBM2	4.04	5.10	9.93	17.84	29.05	42.99
30	WBM1	6.26	9.30	18.89	34.41	52.90	70.16
	WBM2	4.52	7.14	15.10	29.73	47.47	65.80
40	WBM1	5.44	9.52	23.34	44.08	66.14	83.22
	WBM2	4.24	7.82	20.25	39.89	62.76	80.77
50	WBM1	5.70	11.74	28.96	54.58	77.04	91.31
	WBM2	4.68	9.70	26.00	51.42	75.00	90.02

TABLE 4: Empirical power(%) of different statistics for testing homogeneity of K=3 Weibull populations when data are simulated from $WB(\psi_i, \phi_i)$, $i = 1, 2, 3$; based on 10,000 replications; $\alpha = 0.10$

Sample size	Test Statistic	(ψ_1, ψ_2, ψ_3) (ϕ_1, ϕ_2, ϕ_3)					
		(0.12,0.12,0.12) (0.32,0.32,0.32)	(0.12,0.13,0.14) (0.32,0.33,0.34)	(0.12,0.14,0.16) (0.32,0.34,0.36)	(0.12,0.15,0.18) (0.32,0.35,0.38)	(0.12,0.16,0.20) (0.32,0.36,0.40)	(0.12,0.17,0.22) (0.32,0.37,0.42)
10	$WBM1$	14.61	16.13	20.41	26.67	34.57	43.09
	$WBM2$	6.60	7.65	10.35	14.51	20.63	27.53
15	$WBM1$	13.31	15.81	21.65	31.01	41.35	53.40
	$WBM2$	7.99	10.10	15.01	22.80	32.76	43.81
20	$WBM1$	12.52	15.66	23.76	35.76	50.17	64.48
	$WBM2$	8.84	11.26	18.21	29.11	43.08	57.87
30	$WBM1$	11.65	16.55	29.12	46.73	65.48	80.11
	$WBM2$	9.37	13.53	25.29	42.46	61.58	76.88
40	$WBM1$	10.61	16.72	34.35	56.69	77.21	89.69
	$WBM2$	8.96	14.59	31.21	53.37	74.85	88.41
50	$WBM1$	11.00	19.39	40.66	66.26	85.42	95.26
	$WBM2$	9.68	17.34	38.12	64.04	84.02	94.77

TABLE 5: Size adjusted empirical power(%) of the statistics WBM_1 and WBM_2 for testing homogeneity of K=2 Weibull populations when data are simulated from $WB(\psi_i, \phi_i)$, $i = 1, 2$; empirical quantiles based on 100,000 replications; empirical size based on 10,000 replications; $\alpha = 0.05$

Sample size	Test Statistic	(ψ_1, ψ_2) (ϕ_1, ϕ_2)					
		(1.2,1.2) (3.2,3.2)	(1.2,1.4) (3.2,3.4)	(1.2,1.6) (3.2,3.6)	(1.2,1.8) (3.2,3.8)	(1.2,2.0) (3.2,4.0)	(1.2,2.2) (3.2,4.2)
10	$WBM1$	4.46	5.85	9.45	14.19	20.56	28.44
	$WBM2$	4.80	6.42	10.38	16.40	24.37	33.03
15	$WBM1$	4.99	6.93	12.47	21.05	31.61	44.39
	$WBM2$	5.11	7.53	13.92	23.94	36.04	49.43
20	$WBM1$	5.39	8.25	15.39	27.33	42.27	57.93
	$WBM2$	5.39	8.11	17.04	30.79	47.01	63.15
30	$WBM1$	5.08	9.42	21.90	40.48	60.13	76.62
	$WBM2$	5.11	10.33	24.27	44.35	65.10	80.76
40	$WBM1$	4.77	10.90	28.10	52.51	75.17	89.40
	$WBM2$	4.55	11.47	30.55	56.67	78.72	91.47
50	$WBM1$	5.16	12.83	34.33	62.31	83.73	95.02
	$WBM2$	5.05	13.93	37.57	66.32	86.54	96.13

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