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Estimation of Quantiles of Half Logistic Distribution Using Generalized Ranked-Set Sampling

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Abstract

The ranked-set sampling technique has been generalized so that more efficient estimator may be obtained. The generalized ranked-set sampling technique is applied in the estimation of quantiles of the half logistic distribution. Three estimators are proposed. These are minimum variance unbiased estimator, simple estimator and ranked-set sample estimator. Coefficients, variances and relative efficiencies are tabulated. The estimators are compared to the best linear unbiased estimator of the quantiles.

Keywords and Phrases, Half logistic distribution, Order statistics, Linear estimation, Generalized ranked-set sampling, Ranked-set sampling.

AMS Classification: 62G30.

1 Introduction

In applied statistics, experimenters often encounter situations where the actual measurements of the sample observations are difficult to make due to constraints in cost, time and other factors. However, ranking of the potential sample data is relatively easy. In these situations, McIntyre (1952) advocated the use of ranked-set sampling. He applied the ranked-set sampling technique in assessing the yields of pasture plots without actually carrying out the time-consuming process of mowing and weighing the hay for a large number of plots. Since then, the technique has been studied and applied to several areas of applied research. Takahasi and Wakimato (1968) and Dell and Clutter (1972) studied theoretical aspects of this technique on the assumption of perfect judgment ranking and imperfect judgment ranking respectively. Patil, Sinha and Taillie (1993) studied the same technique when the sample is from a finite population. Patil, Sinha and Taillie (1994) have reviewed various aspects of the ranked-set sampling. Also, Bohn (1996) discussed the application of this technique in nonparametric procedures.

In this paper the ranked-set sampling technique has been generalized so that more efficient estimators may be obtained. The generalized ranked-set sampling technique is applied in the estimation of quantiles of the half logistic distribution. Three estimators are proposed. These are generalized ranked-set minimum variance unbiased estimator (GR-MVUE), simple estimator (SE) and ranked-set sample estimator (RSS). Coefficients, variances, and relative efficiences are derived. The estimators are compared to the best linear unbiased estimators (BLUE) of the quantiles.

In generalized ranked-set sampling, first a set of N elements is randomly selected from a given population. The sample is ordered without making actual measurements. The unit identified with the N_1 rank is accurately measured. Next, a second set of N elements is randomly selected from the population. Again the units are ordered and the unit with the N_2 rank is accurately measured. The process is continued until N set of N elements is selected. The units are again ordered and the unit with N_N rank is accurately measured. The ordered sample of the N sets can be represented as follows:

$$
\begin{array}{cccccc}
\text{Set 1} & X_{(11)} & X_{(12)} & \cdots & X_{(1N)} \\
\text{Set 2} & X_{(21)} & X_{(22)} & \cdots & X_{(2N)} \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
\text{Set N} & X_{(N1)} & X_{(N2)} & \cdots & X_{(NN)}
\end{array}
$$

The generalized ranked-set sample of size N consists of units which are accurately measured i.e. $(X_{(1N_1)}, X_{(2N_2)}, \cdots, X_{(NN_N)})$ where $1 \le N_i \le N$ and $1 \le i \le N$. The generalized ranked-set sample actually includes the usual ranked-set sample which is obtained when $N_1 = 1$, $N_2 = 2$, $\cdots$, $N_N = N$.

2 Estimators

2.1 Best Linear Unbiased Estimator of Quantiles

Let the random variable X have a half logistic distribution with probability density function $(f(x))$ and cumulative distribution function $(F(x))$

$$f(x) = \frac{2}{\sigma}\frac{\exp\{-(x-\mu)/\sigma\}}{[1+\exp\{-(x-\mu)/\sigma\}]^2}$$

$$F(x) = \frac{[1 - \exp\{-(x-\mu)/\sigma\}]}{[1 + \exp\{-(x-\mu)/\sigma\}]}, \quad x > \mu, \quad \sigma > 0,$$

where μ and σ are the location and the scale parameters.

The quantile function of the distribution is defined as

$$Q(\xi) = \mu + \sigma \ln \frac{(1+\xi)}{(1-\xi)}, \quad 0 \leq \xi \leq 1$$

The best linear unbiased estimator (BLUE) of the quantile function Q(ξ) is

$$\hat{Q}(\xi)_{BLUE} = \hat{\mu} + \hat{\sigma} \ln \frac{(1+\xi)}{(1+\xi)}$$

where the location and scale parameters are estimated by their respective BLUEs.

The coefficients of the best linear unbiased estimators (BLUEs) for the location and scale parameters based on complete and censored samples have been tabulated by Balakrishnan and Puthenpura (1986) and Balakrishnan and Wong (1994).

2.2 Estimator of Quantiles based on generalized ranked-set sampling

Let

$$\begin{aligned} Z_{(iN_j)} &= (X_{(iN_j)} - \mu)/\sigma \\ \alpha_{(iN_j)} &\equiv \mathrm{E}\{Z_{(iN_j)}\} \\ \omega_{(iN_jN_j)} &\equiv \mathrm{Var}\{Z_{(iN_j)}\}, \quad i = 1, 2, \cdots, N, j = 1, 2, \cdots, N. \end{aligned}$$

Therefore $\mathrm{E}(X_{iN_j})) \equiv \mu + \sigma\alpha_{(iN_j)}$ and $\mathrm{Var}(X_{(iN_j)}) \equiv \omega_{(iN_jN_j)}\sigma^2$.

Let $\alpha_S = (\alpha_{(1N_1)}, \alpha_{(2N_2)}, \cdots, \alpha_{(NN_N)})^T$ where T implies the transpose

$$\begin{aligned} 1^T &= (1, \cdots, 1) \\ S &= \{N_1, N_2, \cdots, N_N\} \\ X_S &= (X_{(1N_1)}, X_{(2N_2)}, \cdots, X_{(NN_N)})^T \end{aligned}$$

and $\mathrm{Var}(X_S) = \Omega_k\sigma^2$, where Ω_k is a $N \times N$ diagonal matrix with $\omega_{(iN_iN_i)}$ as the (i,i)th element.

Balakrishnan (1985) has tabulated the values of $\alpha_{(iN_i)}$ and $\omega_{(iN_iN_i)}$ for the half logistic distribution. Then $\mathrm{E}(X_S) = \mu 1 + \sigma\alpha_S = A_S\theta$

where $A_S^T = \begin{pmatrix} 1 & 1 & \ldots & 1 \\ \alpha_{(1N_1)} & \alpha_{(2N_2)} & \ldots & \alpha_{(NN_N)} \end{pmatrix}$ and $\theta^T = (\mu, \sigma)$.

Least squares estimator of θ is obtained by applying the Gauss and Markov theorem (Sarhan and Greenberg (1962)). Then $\hat{\theta}_S = (A_S^T\Omega_S^{-1}A_S)^{-1}A_S^T\Omega_S^{-1}X_S$ and $\mathrm{Var}(\hat{\theta}_S) = (A_S^T\Omega_S^{-1}A_S)^{-1}\sigma^2$, where superscript -1 implies inverse.

Therefore, based on generalized ranked-set sample $X_S$with S = $\{N_1, N_2, ..., N_N\}$ the estimator for the quantile function is

$$\hat{Q}(\xi)_S = \hat{\mu}_S + \hat{\sigma}_s \ln \frac{(1+\xi)}{(1+\xi)}$$

where

$$\hat{\mu}_S = \sum_{i=1}^{N} \frac{(T_{1S} - \alpha_{(iN_i)} T_{3S})}{(T_{1S} T_{2S} - T_{3S}^2)} (X_{(iN_i)}/\omega_{(iN_iN_i)})$$

$$\hat{\sigma}_S = \sum_{i=1}^{N} \frac{(\alpha_{(iN_i)} T_{2S} - T_{3S})}{(T_{1S} T_{2S} - T_{3S}^2)} (X_{(iN_i)}/\omega_{(iN_iN_i)})$$

$$T_{1S} = \alpha_s^T \cdot \Omega_S^{-1} \cdot \alpha_s) = \sum_{i=1}^{N} \alpha_{(iN_i)}^2 / \omega_{(iN_iN_i)}$$

$$T_{2S} = (1^T \cdot \Omega_S^{-1} \cdot 1) = \sum_{i=1}^{N} 1/\omega_{(iN_iN_i)}$$

$$T_{3S} = (1^T \cdot \Omega_S^{-1} \cdot \alpha_S) = \sum_{i=1}^{N} \alpha_{(iN_i)}/\omega_{(iN_iN_i)}$$

The variances of the estimator is given by

$$V(\hat{Q}(\xi)_S = \frac{\sigma^2}{(T_{1S} T_{2S} - T_{3S}^2)} \left[T_{1S} + T_{2S} \ln^2 \frac{(1+\xi)}{(1-\xi)} - 2T_{3S} \ln \frac{(1+\xi)}{(1-\xi)} \right]$$

2.3 Generalized Ranked-Set Minimum Variance Unbiased Estimators

Generalized ranked-set minimum variance unbiased estimator (GR-MVUE) is obtained from the generalized ranked-set estimator when all possible choices of S are considered. The best choice of S is the one which gives the minimum variance of the estimator. This S is denoted by $S_{GR-MVUE}$. The estimator is denoted by $\hat{Q}(\xi)_{GR-MVUE}$. Table 1 provides ranks $S_{GR-MVUE}$, variance and coefficients of the estimator for N = 2(1)10 and ξ = .05,0.1,0.5,0.9,.95.

2.4 Simple Estimators (SE)

The simple estimators are obtained from the generalized ranked-set estimators as follows:

When sample size is even, $S = \{N_1, N_2, \cdots, N_N\}$ where $N_1 = N_2 = \cdots = N_{N/2} = k_1$ and $N_{N/2+1} = \cdots = N_N = k_2$ then $S = \{k_1, \cdots, k_1, k_2, \cdots, k_2\}$. Then from generalized ranked-set estimator, the simple estimator for even sample sizes is

$$\hat{Q}(\xi)_{SE} = \sum_{i=1}^{N/2} B_i(N)_{SE} X_{(ik_1)} + \sum_{j=N/2+1}^{N} B_j(N)_{SE} X_{(jk_2)}$$

where

$$B_i(N)_{SE} = 2\left(\alpha_{(ik_2)} - \ln\frac{(1+\xi)}{(1-\xi)}\right) / \left(N(\alpha_{(ik_2)} - \alpha_{(ik_1)})\right),\ i = 1, \cdots, N/2$$

$$B_j(N)_{SE} = -2\left(\alpha_{(jk_1)} - \ln\frac{(1+\xi)}{(1-\xi)}\right) / \left(N(\alpha_{(jk_2)} - \alpha_{(jk_1)})\right),\ j = N/2+1, \cdots, N$$

SE $= \{k_1, \cdots, k_1, k_2, \cdots, k_2\}$.

$$\begin{aligned}\mathrm{Var}(\hat{Q}(\xi)_{SE}) &= \sum_{i=1}^{N/2} 4\left(\alpha_{(ik_2)} - \ln\frac{(1+\xi)}{(1-\xi)}\right)^2 \omega_{(ik_1k_1)} / \left(N(\alpha_{(ik_2)} - \alpha_{(ik_1)})\right)^2 \\ &+ \sum_{j=N/2+1}^{N} 4\left(\alpha_{(jk_1)} - \ln\frac{(1+\xi)}{(1-\xi)}\right)^2 \omega_{(jk_2k_2)} / \left(N(\alpha_{(jk_2)} - \alpha_{(jk_1)})\right)^2\end{aligned}$$

When sample size is odd, $S = \{N_1, N_2, \cdots, N_N\}$ where $N_1 = N_2 = \cdots = N_{(N+1)/2} = k_1$ and $N_{(N+1)/2+1} = N_N = k_2$ then $S = \{k_1, \cdots, k_1, k_1, k_2, \cdots, k_2\}$. Then from generalized ranked-set estimators, the simple estimator for odd sample sizes is

$$\hat{Q}(\xi)_{SE} = \sum_{i=1}^{(N-1)/2+1} B_i(N)_{SE} X_{(ik_1)} + \sum_{j=(N-1)/2+2}^{N} B_j(N)_{SE} X_{(jk_2)}$$

where

$$\begin{aligned}B_i(N)_{SE} &= 2\left(\alpha_{(ik_2)} - \ln\frac{(1+\xi)}{(1-\xi)}\right) / \left((N+1)(\alpha_{(ik_2)} - \alpha_{(ik_1)})\right), \\ & \quad i = 1, \cdots, (N-1)/2+1 \\ B_j(N)_{SE} &= -2\left(\alpha_{(jk_1)} - \ln\frac{(1+\xi)}{(1-\xi)}\right) / \left((N-1)(\alpha_{(jk_2)} - \alpha_{(jk_1)})\right), \\ & \quad j = (N-1)/2+2, \cdots, N\end{aligned}$$

$$SE = \{k_1, \cdots, k_1, k_2, \cdots, k_2\}$$

.

$$\begin{aligned}\text{Var}(\hat{Q}(\xi)_{SE}) = & \; 4 \sum_{i=1}^{(N-1)/2+1} \left(\alpha_{(ik_2)} - \ln \frac{(1+\xi)}{(1-\xi)} \right)^2 \omega_{(ik_1k_1)} / ((N+1)(\alpha_{(ik_2)} - \alpha_{(ik_1)}))^2 \\ & + 4 \sum_{i=(N-1)/2+2}^{N} \left(\alpha_{(jk_1)} - \ln \frac{(1+\xi)}{(1-\xi)} \right)^2 \omega_{(jk_2k_2)} / ((N-1)(\alpha_{(jk_2)} - \alpha_{(jk_1)}))^2\end{aligned}$$

Table 2 provides ranks S_{SE}, variance and coefficients of the estimator for $N = 2(1)10$ and $\xi = .05, 0.1, 0.5, 0.9, .95$. The ranks were obtained by computing variance for each choice of S, such that by choosing different values of k_1 and k_2, $S = \{k_1, \cdots, k_1, k_2, \cdots, k_2\}$. Rank, which resulted in minimum variance of the estimator, is identified as S_{SE}.

2.5 Ranked-set Sample Estimators (RSS)

The ranked-set estimators (RSS) for μ and σ are obtained from the generalized ranked-set estimators when $S = \{1, 2, \cdots, N\}$. This choice of ranks is denoted by S_{RSS}. The estimator is denoted by $\hat{Q}(\xi)_{RSS}$. Table 3 provides ranks, variance and coefficients of the estimator for $N = 2(1)10$ and $\xi = .05, 0.1, 0.5, 0.9, .95$.

3 Comparisons

In this section comparison has been made between generalized ranked-set minimum variance unbiased estimator (GR-MVUE), simple estimator (SE), ranked-set sample estimator (RSS) and the best linear unbiased estimator (BLUE). When comparing variances, Table 4 shows that GR-MVUE is more efficient than BLUE and RSS and SE for $N \geq 3$ and $\xi = .05, 0.1, 0.5, 0.9$ and 0.95. It also shows that SE are more efficient than BLUE for $N \geq 4$ and $\xi = .05, 0.1, 0.5, 0.9$ and 0.95. SE are more efficient than RSS for $N \geq 3$ and $\xi = .05$ and 0.1. When $\xi = .5, .9$ or .95 and $N \geq 6$ SE is more efficient than RSS. Table 4 also shows that RSS are more efficient than BLUE for $N \geq 5$ and $\xi = .05, 0.1, 0.5, 0.9$ and 0.95.

Table 1: Coefficients for computing $\hat{Q}(\xi)_{GR-MVUE}$ [In Column 4, $U = Var(\hat{Q}(\xi)_{GR-MVUE})$]

N	$S_{GR-MVUE}$	ξ	$\frac{U}{\sigma^2}$	1	2	3	4	5	6	7	8	9	10
2	{1, 2}	0.95	9.3760	-1.3553	2.3553								
3	{1, 3, 3}	0.95	2.3976	-0.6937	0.8469	0.8469							
4	{1, 4, 4, 4}	0.95	1.1517	-0.4445	0.4815	0.4815	0.4815						
5	{1, 5, 5, 5, 5}	0.95	0.7026	-0.3061	0.3265	0.3265	0.3265	0.3265					
6	{1, 6, 6, 6, 6, 6}	0.95	0.4853	-0.2153	0.2431	0.2431	0.2431	0.2431	0.2431				
7	{1, 6, 7, 7, 7, 7, 7}	0.95	0.3604	-0.2613	0.3086	0.1905	0.1905	0.1905	0.1905	0.1905			
8	{1, 7, 7, 7, 7, 7, 7, 7}	0.95	0.2698	-0.6654	0.2379	0.2379	0.2379	0.2379	0.2379	0.2379	0.2379		
9	{1, 8, 8, 8, 8, 8, 8, 8, 8}	0.95	0.2057	-0.5649	0.1956	0.1956	0.1956	0.1956	0.1956	0.1956	0.1956	0.1956	
10	{1, 9, 9, 9, 9, 9, 9, 9, 9, 9}	0.95	0.1634	-0.4865	0.1652	0.1652	0.1652	0.1652	0.1652	0.1652	0.1652	0.1652	0.1652
2	{1, 2}	0.9	5.0970	-0.7695	1.7695								
3	{1, 3, 3}	0.9	1.3756	-0.3032	0.6516	0.6516							
4	{1, 4, 4, 4}	0.9	0.6824	-0.1238	0.3746	0.3746	0.3746						
5	{1, 5, 5, 5, 5}	0.9	0.4253	-0.0229	0.2557	0.2557	0.2557	0.2557					
6	{5, 6, 6, 6, 6, 6}	0.9	0.2647	0.1129	0.1774	0.1774	0.1774	0.1774	0.1774				
7	{6, 7, 7, 7, 7, 7, 7}	0.9	0.1919	0.2570	0.1238	0.1238	0.1238	0.1238	0.1238	0.1238			
8	{7, 7, 8, 8, 8, 8, 8, 8}	0.9	0.1501	0.1921	0.1921	0.1026	0.1026	0.1026	0.1026	0.1026	0.1026		
9	{8, 8, 8, 9, 9, 9, 9, 9, 9}	0.9	0.1213	0.1659	0.1659	0.1659	0.0837	0.0837	0.0837	0.0837	0.0837	0.0837	
10	{9, 9, 9, 9, 9, 10, 10, 10, 10, 10}	0.9	0.0983	0.1201	0.1201	0.1201	0.1201	0.1201	0.0799	0.0799	0.0799	0.0799	0.0799
2	{1, 2}	0.5	0.3451	0.7344	0.2656								
3	{1, 2, 3}	0.5	0.1653	0.4958	0.3234	0.1808							
4	{1, 3, 3, 3}	0.5	0.0938	0.3982	0.2006	0.2006	0.2006						
5	{1, 3, 3, 4, 4}	0.5	0.0622	0.3159	0.1930	0.1930	0.1491	0.1491					
6	{1, 4, 4, 4, 4, 4}	0.5	0.0434	0.2698	0.1460	0.1460	0.1460	0.1460	0.1460				
7	{1, 4, 5, 5, 5, 5, 5}	0.5	0.0321	0.3180	0.1348	0.1094	0.1094	0.1094	0.1094	0.1094			
8	{1, 5, 5, 5, 5, 5, 6, 6}	0.5	0.0248	0.2702	0.1096	0.1096	0.1096	0.1096	0.1096	0.0908	0.0908		
9	{1, 6, 6, 6, 6, 6, 6, 6, 6}	0.5	0.0196	0.2883	0.0890	0.0890	0.0890	0.0890	0.0890	0.0890	0.0890	0.0890	
10	{1, 1, 7, 7, 7, 7, 8, 8, 8, 8}	0.5	0.0159	0.2101	0.2101	0.0790	0.0790	0.0790	0.0790	0.0659	0.0659	0.0659	0.0659
1	{1,2}	0.1	1.2760	1.4660	-0.4660								
2	{1,1,3}	0.1	0.2156	0.5936	0.5936	-0.1871							
3	{1,1,1,4}	0.1	0.0730	0.3665	0.3665	0.3665	-0.0994						
4	{1,1,1,1,5}	0.1	0.0327	0.2644	0.2644	0.2644	0.2644	-0.0578					
5	{1,1,1,1,1,6}	0.1	0.0172	0.2068	0.2068	0.2068	0.2068	0.2068	-0.0339				
6	{1,1,1,1,1,1,7}	0.1	0.0101	0.1698	0.1698	0.1698	0.1698	0.1698	0.1698	-0.0187			
7	{1,1,1,1,1,1,1,8}	0.1	0.0065	0.1440	0.1440	0.1440	0.1440	0.1440	0.1440	0.1440	-0.0083		
9	{1, 1, 1, 1, 1, 1, 1, 1, 9}	0.1	0.0045	0.1251	0.1251	0.1251	0.1251	0.1251	0.1251	0.1251	0.1251	-0.0008	
10	{1, 1, 1, 1, 1, 1, 1, 1, 1, 3}	0.1	0.0032	0.1064	0.1064	0.1064	0.1064	0.1064	0.1064	0.1064	0.1064	0.1064	0.0422
2	{1,2}	0.05	1.5125	1.5479	-0.5479								
3	{1,1,3}	0.05	0.2680	0.6209	0.6209	-0.2418							
4	{1,1,1,4}	0.05	0.0954	0.3814	0.3814	0.3814	-0.1443						
5	{1,1,1,1,5}	0.05	0.0448	0.2744	0.2744	0.2744	0.2744	-0.0974					
6	{1,1,1,1,1,6}	0.05	0.0245	0.2140	0.2140	0.2140	0.2140	0.2140	-0.0702				
7	{1,1,1,1,1,7,7}	0.05	0.0145	0.2105	0.2105	0.2105	0.2105	0.2105	-0.0263	-0.0263			
8	{1,1,1,1,1,1,8,8}	0.05	0.0092	0.1734	0.1734	0.1734	0.1734	0.1734	0.1734	-0.0202	-0.0202		
9	{1,1,1,1,1,1,1,9,9}	0.05	0.0062	0.1474	0.1474	0.1474	0.1474	0.1474	0.1474	0.1474	-0.0158	-0.0158	
10	{1,1,1,1,1,1,1,1,1,9}	0.05	0.0044	0.1151	0.1151	0.1151	0.1151	0.1151	0.1151	0.1151	0.1151	0.1151	-0.0360

Table 2: Coefficients for computing $\hat{Q}(\xi)_{SE}$ [In Column 4, $V = Var(\hat{Q}(\xi)_{SE})$]

N	S_{SE}	ξ	$\frac{V}{\sigma^2}$	1	2	3	4	5	6	7	8	9	10
2	{1,2}	0.95	9.3760	-1.3553	2.3553								
3	{1,1,3}	0.95	4.6316	-0.3469	-0.3469	1.6937							
4	{1,1,4,4}	0.95	1.6996	-0.2222	-0.2222	0.7222	0.7222						
5	{1,1,1,5,5}	0.95	1.3899	-0.1020	-0.1020	-0.1020	0.6531	0.6531					
6	{1,1,1,6,6,6}	0.95	0.8044	-0.0718	-0.0718	-0.0718	0.4051	0.4051	0.4051				
7	{1,1,1,1,6,6,6}	0.95	0.6872	-0.2001	-0.2001	-0.2001	-0.2001	0.6001	0.6001	0.6001			
8	{1,1,1,1,7,7,7,7}	0.95	0.4431	-0.1664	-0.1664	-0.1664	-0.1664	0.4164	0.4164	0.4164	0.4164		
9	{1,1,1,1,1,8,8,8,8}	0.95	0.3909	-0.1130	-0.1130	-0.1130	-0.1130	-0.1130	0.3912	0.3912	0.3912	0.3912	
10	{1,1,1,1,1,9,9,9,9,9}	0.95	0.2828	-0.0973	-0.0973	-0.0973	-0.0973	-0.0973	0.2973	0.2973	0.2973	0.2973	0.2973
2	{1,2}	0.9	5.0970	-0.7695	1.7695								
3	{1,1,3}	0.9	2.7199	-0.1516	-0.1516	1.3032							
4	{1,1,4,4}	0.9	1.0215	-0.0619	-0.0619	0.5619	0.5619						
5	{1,1,1,5,5}	0.9	0.8506	-0.0076	-0.0076	-0.0076	0.5115	0.5115					
6	{5,5,5,6,6,6}	0.9	0.4306	0.0376	0.0376	0.0376	0.2957	0.2957	0.2957				
7	{6,6,6,6,7,7,7}	0.9	0.3112	0.0643	0.0643	0.0643	0.0643	0.2477	0.2477	0.2477			
8	{7,7,7,7,8,8,8,8}	0.9	0.1786	0.0960	0.0960	0.0960	0.0960	0.1540	0.1540	0.1540	0.1540		
9	{8,8,8,8,8,9,9,9,9}	0.9	0.1348	0.0995	0.0995	0.0995	0.0995	0.0995	0.1256	0.1256	0.1256	0.1256	
10	{9,9,9,9,9,10,10,10,10,10}	0.9	0.0983	0.1201	0.1201	0.1201	0.1201	0.1201	0.0799	0.0799	0.0799	0.0799	0.0799
2	{1,2}	0.5	0.3451	0.7344	0.2656								
3	{1,1,3}	0.5	0.1996	0.3497	0.3497	0.3006							
4	{2,2,3,3}	0.5	0.1015	0.3513	0.3513	0.1487	0.1487						
5	{2,2,2,4,4}	0.5	0.0655	0.2170	0.2170	0.2170	0.1745	0.1745					
6	{2,2,2,5,5,5}	0.5	0.0461	0.2149	0.2149	0.2149	0.1184	0.1184	0.1184				
7	{3,3,3,3,5,5,5}	0.5	0.0335	0.1578	0.1578	0.1578	0.1578	0.1230	0.1230	0.1230			
8	{3,3,3,3,6,6,6,6}	0.5	0.0261	0.1552	0.1552	0.1552	0.1552	0.0948	0.0948	0.0948	0.0948		
9	{4,4,4,4,4,6,6,6,6}	0.5	0.0205	0.1240	0.1240	0.1240	0.1240	0.1240	0.0949	0.0949	0.0949	0.0949	
10	{4,4,4,4,4,7,7,7,7,7}	0.5	0.0168	0.1214	0.1214	0.1214	0.1214	0.1214	0.0786	0.0786	0.0786	0.0786	0.0786
2	{1,2}	0.1	1.2760	1.4660	-0.4660								
3	{1,1,3}	0.1	0.2156	0.5936	0.5936	-0.1871							
4	{1,1,4,4}	0.1	0.0935	0.5497	0.5497	-0.0497	-0.0497						
5	{1,1,1,5,5}	0.1	0.0391	0.3526	0.3526	0.3526	-0.0289	-0.0289					
6	{1,1,1,6,6,6}	0.1	0.0262	0.3446	0.3446	0.3446	-0.0113	-0.0113	-0.0113				
7	{1,1,1,1,7,7,7}	0.1	0.0145	0.2547	0.2547	0.2547	0.2547	-0.0062	-0.0062	-0.0062			
8	{1,1,1,1,8,8,8,8}	0.1	0.0112	0.2521	0.2521	0.2521	0.2521	-0.0021	-0.0021	-0.0021	-0.0021		
9	{1,1,1,1,1,9,9,9,9}	0.1	0.0071	0.2002	0.2002	0.2002	0.2002	0.2002	-0.0002	-0.0002	-0.0002	-0.0002	
10	{1,1,1,1,1,2,2,2,2,2}	0.1	0.0050	0.1827	0.1827	0.1827	0.1827	0.1827	0.0173	0.0173	0.0173	0.0173	0.0173
2	{1,2}	0.05	1.5125	1.5479	-0.5479								
3	{1,1,3}	0.05	0.2680	0.6209	0.6209	-0.2418							
4	{1,1,4,4}	0.05	0.1094	0.5721	0.5721	-0.0721	-0.0721						
5	{1,1,1,5,5}	0.05	0.0469	0.3658	0.3658	0.3658	-0.0487	-0.0487					
6	{1,1,1,6,6,6}	0.05	0.0301	0.3567	0.3567	0.3567	-0.0234	-0.0234	-0.0234				
7	{1,1,1,1,7,7,7}	0.05	0.0168	0.2632	0.2632	0.2632	0.2632	-0.0176	-0.0176	-0.0176			
8	{1,1,1,1,8,8,8,8}	0.05	0.0125	0.2601	0.2601	0.2601	0.2601	-0.0101	-0.0101	-0.0101	-0.0101		
9	{1,1,1,1,1,9,9,9,9}	0.05	0.0080	0.2063	0.2063	0.2063	0.2063	0.2063	-0.0079	-0.0079	-0.0079	-0.0079	
10	{1,1,1,1,1,10,10,10,10,10}	0.05	0.0064	0.2050	0.2050	0.2050	0.2050	0.2050	-0.0050	-0.0050	-0.0050	-0.0050	-0.0050

Table 3: Coefficients for computing $\hat{Q}(\xi)_{RSS}$ [In Column 4, $W = Var(\hat{Q}(\xi)_{RSS})$]

N	S_{RSS}	ξ	$\frac{W}{\sigma^2}$	1	2	3	4	5	6	7	8	9	10
2	{1,2}	0.95	9.3760	-1.3553	2.3553								
3	{1,2,3}	0.95	3.7373	-1.3556	1.0515	1.3041							
4	{1,2,3,4}	0.95	2.0495	-1.3219	0.4862	0.9932	0.8425						
5	{1,2,3,4,5}	0.95	1.2990	-1.2803	0.1940	0.6844	0.8082	0.5937					
6	{1,2,3,4,5,6}	0.95	0.8972	-1.2390	0.0245	0.4680	0.6510	0.6527	0.4428				
7	{1,2,3,4,5,6,7}	0.95	0.6565	-1.2005	-0.0818	0.3180	0.5087	0.5778	0.5339	0.3439			
8	{1,2,3,4,5,6,7,8}	0.95	0.5008	-1.1653	-0.1523	0.2116	0.3957	0.4868	0.5046	0.4436	0.2754		
9	{1,2,3,4,5,6,7,8,9}	0.95	0.3944	-1.1334	-0.2009	0.1337	0.3078	0.4048	0.4479	0.4404	0.3740	0.2258	
10	{1,2,3,4,5,6,7,8,9,10}	0.95	0.3184	-1.1046	-0.2356	0.0753	0.2391	0.3359	0.3893	0.4064	0.3859	0.3196	0.1887
2	{1,2}	0.9	5.0970	-0.7695	1.7695								
3	{1,2,3}	0.9	2.1141	-0.8365	0.8474	0.9892							
4	{1,2,3,4}	0.9	1.1817	-0.8481	0.4289	0.7767	0.6426						
5	{1,2,3,4,5}	0.9	0.7577	-0.8401	0.2064	0.5509	0.6284	0.4545					
6	{1,2,3,4,5,6}	0.9	0.5275	-0.8250	0.0745	0.3890	0.5149	0.5067	0.3399				
7	{1,2,3,4,5,6,7}	0.9	0.3882	-0.8076	-0.0097	0.2752	0.4093	0.4540	0.4143	0.2645			
8	{1,2,3,4,5,6,7,8}	0.9	0.2975	-0.7900	-0.0665	0.1935	0.3242	0.3870	0.3953	0.3443	0.2121		
9	{1,2,3,4,5,6,7,8,9}	0.9	0.2351	-0.7729	-0.1063	0.1332	0.2574	0.3256	0.3540	0.3444	0.2904	0.1742	
10	{1,2,3,4,5,6,7,8,9,10}	0.9	0.1904	-0.7568	-0.1352	0.0876	0.2049	0.2735	0.3104	0.3201	0.3016	0.2482	0.1457
2	{1,2}	0.5	0.3451	0.7344	0.2656								
3	{1,2,3}	0.5	0.1653	0.4958	0.3234	0.1808							
4	{1,2,3,4}	0.5	0.0993	0.3679	0.2818	0.2209	0.1294						
5	{1,2,3,4,5}	0.5	0.0666	0.2897	0.2382	0.2082	0.1667	0.0971					
6	{1,2,3,4,5,6}	0.5	0.0479	0.2377	0.2030	0.1863	0.1655	0.1318	0.0757				
7	{1,2,3,4,5,6,7}	0.5	0.0361	0.2008	0.1754	0.1654	0.1541	0.1363	0.1073	0.0607			
8	{1,2,3,4,5,6,7,8}	0.5	0.0282	0.1735	0.1538	0.1472	0.1408	0.1308	0.1147	0.0893	0.0499		
9	{1,2,3,4,5,6,7,8,9}	0.5	0.0227	0.1525	0.1365	0.1320	0.1282	0.1224	0.1129	0.0981	0.0757	0.0417	
10	{1,2,3,4,5,6,7,8,9,10}	0.5	0.0186	0.1360	0.1225	0.1192	0.1169	0.1135	0.1078	0.0987	0.0851	0.0650	0.0355
2	{1,2}	0.1	1.2760	1.4660	-0.4660								
3	{1,2,3}	0.1	0.3713	1.1440	0.0685	-0.2125							
4	{1,2,3,4}	0.1	0.1685	0.9594	0.2103	-0.0495	-0.1202						
5	{1,2,3,4,5}	0.1	0.0939	0.8394	0.2537	0.0414	-0.0578	-0.0767					
6	{1,2,3,4,5,6}	0.1	0.0589	0.7546	0.2655	0.0878	-0.0045	-0.0505	-0.0528				
7	{1,2,3,4,5,6,7}	0.1	0.0400	0.6914	0.2655	0.1119	0.0299	-0.0183	-0.0420	-0.0384			
8	{1,2,3,4,5,6,7,8}	0.1	0.0287	0.6422	0.2609	0.1247	0.0516	0.0062	-0.0218	-0.0347	-0.0291		
9	{1,2,3,4,5,6,7,8,9}	0.1	0.0215	0.6027	0.2546	0.1313	0.0653	0.0235	-0.0043	-0.0217	-0.0288	-0.0227	
10	{1,2,3,4,5,6,7,8,9,10}	0.1	0.0167	0.5702	0.2478	0.1345	0.0741	0.0357	0.0092	-0.0090	-0.0203	-0.0241	-0.0182
2	{1,2}	0.05	1.5125	1.5479	-0.5479								
3	{1,2,3}	0.05	0.4415	1.2166	0.0400	-0.2566							
4	{1,2,3,4}	0.05	0.2006	1.0257	0.2023	-0.0798	-0.1482						
5	{1,2,3,4,5}	0.05	0.1117	0.9009	0.2555	0.0228	-0.0830	-0.0962					
6	{1,2,3,4,5,6}	0.05	0.0701	0.8125	0.2725	0.0767	-0.0235	-0.0710	-0.0672				
7	{1,2,3,4,5,6,7}	0.05	0.0476	0.7463	0.2756	0.1059	0.0160	-0.0356	-0.0587	-0.0495			
8	{1,2,3,4,5,6,7,8}	0.05	0.0342	0.6947	0.2729	0.1222	0.0416	-0.0078	-0.0371	-0.0485	-0.0379		
9	{1,2,3,4,5,6,7,8,9}	0.05	0.0256	0.6531	0.2678	0.1313	0.0583	0.0124	-0.0174	-0.0351	-0.0405	-0.0299	
10	{1,2,3,4,5,6,7,8,9,10}	0.05	0.0198	0.6189	0.2619	0.1362	0.0693	0.0269	-0.0018	-0.0211	-0.0320	-0.0341	-0.0242

Table 4: Variances and Relative Efficiencies [In this table, $U = Var(\hat{Q}(\xi)_{GR-MVUE})$; $V = Var(\hat{Q}(\xi)_{SE})$; and $W = Var(\hat{Q}(\xi)_{RSS})$]

N	ξ	$\frac{V}{U}$	$\frac{W}{U}$	$\frac{Var(\hat{Q}(\xi)_{BLUE})}{U}$	$\frac{W}{V}$	$\frac{Var(\hat{Q}(\xi)_{BLUE})}{V}$	$\frac{Var(\hat{Q}(\xi)_{BLUE})}{W}$
2	0.95	1.0000	1.0000	0.7435	1.0000	0.7435	0.7435
3	0.95	1.9317	1.5588	1.6243	0.8069	0.8409	1.0421
4	0.95	1.4757	1.7796	2.3716	1.2059	1.6070	1.3327
5	0.95	1.9783	1.8490	3.0017	0.9346	1.5173	1.6234
6	0.95	1.6575	1.8488	3.5427	1.1154	2.1373	1.9162
7	0.95	1.9069	1.8217	4.0285	0.9553	2.1126	2.2114
8	0.95	1.6422	1.8562	4.6572	1.1303	2.8359	2.5090
9	0.95	1.9005	1.9173	5.3851	1.0088	2.8336	2.8088
10	0.95	1.7314	1.9494	6.0639	1.1259	3.5024	3.1106
2	0.9	1.0000	1.0000	0.7988	1.0000	0.7988	0.7988
3	0.9	1.9773	1.5369	1.7186	0.7773	0.8692	1.1183
4	0.9	1.4968	1.7317	2.4703	1.1569	1.6504	1.4265
5	0.9	1.9998	1.7814	3.0891	0.8908	1.5447	1.7340
6	0.9	1.6267	1.9928	4.0715	1.2251	2.5029	2.0431
7	0.9	1.6218	2.0229	4.7623	1.2473	2.9365	2.3542
8	0.9	1.1894	1.9815	5.2858	1.6660	4.4439	2.6675
9	0.9	1.1112	1.9379	5.7804	1.7440	5.2020	2.9828
10	0.9	1.0000	1.9372	6.3927	1.9372	6.3927	3.3000
2	0.5	1.0000	1.0000	1.4258	1.0000	1.4258	1.4258
3	0.5	1.2075	1.0000	1.9007	0.8282	1.5741	1.9007
4	0.5	1.0823	1.0586	2.4660	0.9781	2.2785	2.3294
5	0.5	1.0528	1.0707	2.9385	1.0170	2.7910	2.7443
6	0.5	1.0604	1.1030	3.4781	1.0402	3.2801	3.1534
7	0.5	1.0430	1.1245	4.0025	1.0781	3.8374	3.5595
8	0.5	1.0514	1.1396	4.5173	1.0840	4.2966	3.9638
9	0.5	1.0471	1.1576	5.0552	1.1054	4.8276	4.3671
10	0.5	1.0551	1.1676	5.5690	1.1066	5.2783	4.7697
2	0.1	1.0000	1.0000	0.5968	1.0000	0.5968	0.5968
3	0.1	1.0000	1.7222	1.3662	1.7222	1.3662	0.7932
4	0.1	1.2811	2.3096	2.2391	1.8028	1.7478	0.9695
5	0.1	1.1951	2.8677	3.2373	2.3995	2.7088	1.1289
6	0.1	1.5211	3.4209	4.3603	2.2489	2.8665	1.2746
7	0.1	1.4338	3.9574	5.5766	2.7600	3.8893	1.4092
8	0.1	1.7238	4.4416	6.8153	2.5766	3.9536	1.5344
9	0.1	1.5997	4.8233	7.9679	3.0152	4.9809	1.6519
10	0.1	1.5861	5.2538	9.2617	3.3124	5.8393	1.7629
2	0.05	1.0000	1.0000	0.5776	1.0000	0.5776	0.5776
3	0.05	1.0000	1.6470	1.2394	1.6470	1.2394	0.7525
4	0.05	1.1474	2.1031	1.9068	1.8330	1.6619	0.9067
5	0.05	1.0465	2.4933	2.6029	2.3826	2.4873	1.0439
6	0.05	1.2285	2.8631	3.3430	2.3307	2.7213	1.1676
7	0.05	1.1587	3.2845	4.2045	2.8346	3.6287	1.2801
8	0.05	1.3548	3.6964	5.1134	2.7284	3.7743	1.3833
9	0.05	1.2820	4.0994	6.0621	3.1977	4.7286	1.4788
10	0.05	1.4745	4.5382	7.1137	3.0778	4.8245	1.5675

4 Conclusion

The generalized ranked-set minimum variance unbiased estimator (GR-MVUE), simple estimator (SE) and ranked-set sample estimator (RSS) are all more efficient than the best linear unbiased estimator (BLUE). The GR-MVUE is the best estimators. However, the coefficients for larger sample sizes ($N > 10$) require considerable amount of computational time. Simple estimator (SE) is more efficient than the ranked-set sample estimator (RSS) as well as the best linear unbiased estimator (BLUE).

Simple estimator (SE) is most suitable estimator for estimating the quantiles of the half logistic distribution when the actual measurements of the sample observations are difficult to measure but are easily ranked.

References

[1] Balakrishnan, N. (1985). Order statistics from the half logistic distribution, *J. Statist. Comput. Simul.*, **20**, 287-309.

[2] Balakrishnan, N. and S. Puthenpura (1986). Best linear unbiased estimators of location and scale parameters of the half logistic distribution, *J. Statist. Comput. Simul.*, **25**, 193-204.

[3] Balakrishnan, N. and K. H. T. Wong (1994). Best linear unbiased estimation of location and scale parameters of the half logistic distribution based on Type II censored samples, *American Journal of Mathematical and Management Sciences*, **14**, 53-102.

[4] Bohn, Lora L. (1996). A review of nonparametric rank-set sampling methodology, *Commun. Statist.- Theory Meth.*, **25**, 2675-2685.

[5] Dell, T. R. and J. L. Cutter (1972). Ranked-set sampling theory with order statistics background, *Biometrics*, **28**, 545-555.

[6] Halls, L. K. and T. R. Dell (1966). Trials of ranked-set sampling for forage yields, *Forest Science*, **12**, 22-26.

[7] McIntyre, G. A. (1952). A method of unbiased selective sampling, using ranked sets, *Australian Journal of Agricultural Research*, **3**, 385-390.

[8] Patil, G. P., Sinha, A. K. and C. Taillie (1993). Ranked set sampling from a finite population in the presence of a trend on a site, *Journal of Applied Statistical Science*, **1**, 51-65.

[9] Stokes, S. L. and T. W. Sager (1988). Characterization of a ranked-set sample with application to estimating distribution functions, *Journal of the American Statistical Association*, **83**, 374-381.

[10] Takahasi, K. And K. Wakimoto (1968). On unbiased estimates of the population mean based on the sample stratified by means of ordering, *Annals of the Institute of Statistical Mathematics*, **20**, 1-31.