International Journal of Statistical Sciences *ISSN 1683–5603*
Vol. 2, 2003, pp 69–82

Construction of Male Life Table from Female Widowed Information of Bangladesh

Md. Rafiqul Islam[1], Md. Nurul Islam[2], Md. Ayub Ali[2] and Md. Golam Mostofa[1]
[1]*Dept. of Population Science and Human Resource Development*, Rajshahi University, Bangladesh
[2]*Dept. of Statistics*, Rajshahi University, Bangladesh

[Received October 21, 2002; Revised December 31, 2002; Accepted November 5, 2003]

Abstract

The purpose of the present study is to estimate the male adult mortality of Bangladesh. For this, the Widowhood Method is applied on the secondary data for female marital status from the 1991 census. Then male life table has been constructed from l_x values obtained for linking male infant mortality as well as male adult mortality. Age specific death rate (ASDR) for male has been estimated from the male life table. Crude death rate (CDR) for male has also been estimated using estimated ASDR and male smoothed age structure of 1991 census data. Crude birth rate (CBR) for male has been estimated from the balancing equation. The estimated population parameters indicate that Bangladesh is still experiencing a high level of fertility as well as mortality. Age distribution, age specific death rate (ASDR) and the number of persons surviving at an exact age x (l_x) for male population of Bangladesh in 1991 follow modified negative exponential model, 4^{th} degree polynomial model and 3^{rd} degree polynomial model, respectively. Model validation technique, cross validity prediction power (CVPP), showed that the mathematical models are valid.

Keywords and Phrases: Adult mortality, Widowhood method, Life table, Modeling and Cross validity prediction power (CVPP).

1 Introduction

In a country, the success of a national plan and researches depend on the correct information about the population parameters based on national data. There are several agencies, organizations and researchers who have been providing the information about the population parameters of Bangladesh. But few of them give information on national basis. Since in a developing country like Bangladesh, vital registration system has not been started yet, so, most of the sources have been providing indirect techniques. Consequently, indirect techniques have been increasing demand to estimate population parameters from very few limited and defective data. There have been developed several indirect techniques in the last few decades. Widowhood Method is one of them. Hill (1975,1977) has devised Widowhood Method to estimate adult mortality from widowhood information. Using the marital status data of Bangladesh Census 1974, estimation of adult mortality from widowhood information have been estimated by Widowhood Method (Kabir and Mosleh, 1989). Demographic parameters of Bangladesh have also been estimated by Widowhood Method based on the marital status from 1981 Census (Ali, 1990). In this study, using this method, an attempt has been made to estimate male adult mortality of Bangladesh using 1991 Census data on female marital status (BBS, 1994). Thus, the objectives of the study is to

i) estimate male adult mortality by Widowhood Method,

ii) construct an abridged life table for male from the calculated l_x values,

iii) fit some mathematical models to age distribution, ASDR and l_x for male population of Bangladesh in 1991,

iv) calculate instantaneous force of mortality (μ_x) from calculated l_x values for male population of Bangladesh in 1991.

2 Methods and Materials

2.1 Sources of Data

The male age structure and female marital status of 1991 census have been used as raw data in this study. For linking infant mortality, child mortality and adult mortality at the time of life table construction, l_2 values have been borrowed from Vital Registration System (VRS) data (BBS, 1993).

2.2 Construction of Life Tables

The secondary data on female marital status distribution for Bangladesh Population Census 1991 shows plausible pattern except some older age groups, which are not so important here. So the observed data of female marital status have been used to construct male life table after adjusting for widow remarriage.

Widowhood method proposed by Hill (1977) has been used to estimate male adult mortality using 1991 census data on marital status composition of female. For this purpose, proportion not widowed π_x has been corrected from the proportion not widowed π_x data as $\hat{\pi}_x = \frac{\pi_x}{1+p}$, where p is the proportion of widow remarriage.

According to (BFS, 1978) widow (female) remarriage of Bangladesh in 1974 was around 3.8 percent that was negligible. So, according to the suggestion of Brass (1978) male adult mortality has been estimated without adjusting for widow (female) remarriage.

Therefore, proportion not widowed female $\pi_x$has been converted to the probability of surviving ratio using some weighting factors that are given by Hill (1977) corresponding to singulate mean age at marriage (SMAM) for female and weighted singulate mean age at marriage (WSMAM) for male. Ultimately these probabilities of surviving ratios have been converted to the l_x values for the male adult ages corresponding to the representative model life tables in the adult ages (Table 1).

The indices of infant and child mortality l_2 values have been estimated from BBS data. Linking l_2 values with the male adult mortality, the l_x values for male have been estimated. For this purpose a linear function $Y_x = a + bY_x^s$ is fitted by trial and error method according to Sivamurthy and Sitharam (1980), where Y_x is the logit of l_x and Y_x^s is the logit survival function of l_x^s of standard life table (UN, 1982). Then Y_x values have been converted to l_x values, the number of persons surviving at an exact age x. The l_x values have been used to construct male life table. This life table for male has been constructed and presented in Table 2.

Table 1: Male adult Mortality of Bangladesh by Widowhood Method Using 1991 Census Data on Female Marital Status

Age Group	Proportion not widowed $\pi(x)$	Weights $W(x)$	Corrected Weights $W'(x)$	$l^m_{x+5}/l^m_{22.5}$	l^m_x
20	0.9791	0.5875	0.5695	0.9798	...
25	0.9734	0.4429	0.4249	0.9758	0.81492
30	0.9579	0.5161	0.4981	0.9656	0.81162
35	0.9336	0.5972	0.5792	0.9477	0.80314
40	0.8799	0.6777	0.6597	0.9153	0.78821
45	0.8244	0.7430	0.7250	0.8646	0.76130
50	0.7127	0.7796	0.7616	0.7978	0.71914
55	0.6493	0.8066	0.7886	0.6993	0.66353

Note: ${}^{l_{x+5}}\!/_{l_{22.5}} = W'(x)\pi\,(x-5) + [1 - W'(x)]\pi(x)$, $l^m_{22.5} = .83172$

Table 2: An Abridged Life Table for Male of Bangladesh in 1991

Age Group	l_x	q_x	L_x	T_x	e_x
0	100000	0.0910	93632	5513200	55.13
1	90903	0.0319	89161	5419568	59.62
2	87999	0.0168	87261	5330407	60.57
3	86523	0.0104	86075	5243146	60.60
4	85626	0.0070	85326	5107271	59.65
5-9	85026	0.0154	421850	5021945	59.06
10-14	83714	0.0069	417133	4600095	54.95
15-19	83139	0.0083	413968	4182963	50.31
20-24	82448	0.0099	410193	3768995	45.71
25-29	81629	0.0123	405633	3358803	41.15
30-34	80624	0.0154	400013	2953170	36.63
35-39	79381	0.0216	392613	2553158	32.16
40-44	77664	0.0323	382053	2160545	27.82
45-49	75157	0.0495	366490	1778493	23.66
50-54	71439	0.0793	343038	1412003	19.77
55-59	65776	0.1199	309168	1068965	16.25
60-64	57891	0.1863	262488	759798	13.12
65-69	47104	0.2683	203930	497310	10.56
70-74	34468	0.3715	140330	293380	8.51
75-79	21664	0.0871	103605	153050	7.06
80+	19778	1	49445	49445	2.50

Note: $L_0 = .3l_0 + .7l_1$, $L_1 = .4l_1 + .6l_{2.}$

2.3 Smoothing of Age Data for Male

To estimate the male demographic parameters of Bangladesh, male age structure of 1991 census population adjusted for undercount has been used as raw data. To evaluate this secondary data U. N. age sex accuracy index has been calculated. It is found to be 70.41 for both sexes. Whipple's index has been calculated and it is found to be 310.79 for male. Myer's index has also been calculated and it is found to be 36.14 for male. All these indices indicate that male age distribution of 1991 census data is highly inaccurate and for this reason these data need to be adjusted. In this study male age distribution of 1991 census population adjusted for undercount has been smoothed by latest smoothing method named 4253 H, twice (Velleman, 1980). Smoothing method has been accomplished using the package Minitab Release 12.1. The adjusted age structure for undercount and smoothed age distribution for male of Bangladesh have been presented in Table 3 and depict in Fig. 1.

Table 3: Adjusted (for undercount) and Male Smoothed Age Distribution of Census Population of Bangladesh in 1991

Age Group	Adjusted Population (000)	Smoothed Population (000)	Adjusted Percent *	Smoothed Percent
0-4	9482	9482	16.54	16.78
5-9	9505	8602	16.58	15.22
10-14	7175	7083	12.52	12.54
15-19	4819	5496	8.41	9.73
20-24	4356	4543	7.6	8.04
25-29	4537	4048	7.92	7.16
30-34	3495	3605	6.1	6.38
35-39	3367	3098	5.87	5.48
40-44	2519	2537	4.39	4.49
45-49	1958	2011	3.42	3.56
50-54	1687	1601	2.94	2.83
55-59	1117	1279	1.95	2.26
60-64	1251	996	2.18	1.76
65-69	653	753	1.14	1.33
70-74	692	560	1.21	0.99
75-79	273	432	0.48	0.76
80+	430	379	0.75	0.67

Source: * [2]

2.4 Estimation of Mortality Parameters

The life expectancy at different ages for male have been presented in column 6 of Table 2. It is found that the life expectancy at birth for male is 55.13. Various continuous functions of a life table are as follows: ${}_nd_x = l_x - l_{x+n}$, $\quad l_{x+n} = l_x \; {}_nP_x$, ${}_nL_x = \int_0^n l(x+t)dt$ which can be approximated as ${}_nL_x = \frac{n(l_x+l_{x+n})}{2}$ $(x \geq 2)$, $\quad T_x = \int_0^\infty l(x+t)dt$ which can be approximated as $T_x = \sum\limits_{t=0}^{\infty} L_{x+t}$, $\quad e_x = \frac{T_x}{l_x}$, ASDR $= \frac{{}_nd_x}{{}_nL_x}$ and

$$\mu_x = -\frac{1}{l_x}\frac{d}{dx}(l_x) = -\frac{d}{dx}(\ln l_x)$$

(Biswas, 1988). ASDR have been estimated from the life table and have been presented in Table 4 and depict in Fig. 2. Male crude death rate (CDR) has been estimated using these ASDR and smoothed male population applying the formula CDR = $\sum \text{ASDR}_x \text{C}_x$, where C_x is the age structure for male. It is to be found that CDR for male is 14.64.

Table 4: Age Specific Death Rate (ASDR) for Male of Bangladesh in 1981 and 1991

Age Group	ASDR 1991	Smoothed ASDR 1991	ASDR 1981 *	Smoothed ASDR 1981*
0	0.0972	0.0933	0.11453	0.11115
1	0.0326	0.0485	0.04241	0.05928
2	0.0169	0.0208	0.02237	0.02681
3	0.0104	0.0097	0.01383	0.01308
4	0.0070	0.0056	0.0093	0.00757
5-9	0.0031	0.0032	0.00411	0.00423
10-14	0.0014	0.0018	0.00181	0.00239
15-19	0.0017	0.0016	0.00227	0.0021
20-24	0.0020	0.0018	0.00252	0.00241
25-29	0.0025	0.0023	0.00324	0.00296
30-34	0.0031	0.0031	0.00405	0.00395
35-39	0.0044	0.0043	0.00568	0.0056
40-44	0.0066	0.0065	0.00846	0.00836
45-49	0.0101	0.0104	0.01295	0.013
50-54	0.0165	0.017	0.02081	0.02036
55-59	0.0255	0.0282	0.03165	0.03199
60-64	0.0411	0.0422	0.05003	0.05165
65-69	0.062	0.0552	0.07362	0.08408
70-74	0.0912	0.0848	0.13691	0.12483
75-79	0.0182	0.1674	0.16355	0.16355
80+	0.4	0.3053	.	.

Source: * [1]

2.5 Model Fitting

Using the scattered plot of ages and population (Fig. 2(a)), it is observed that population is modified negative exponentially distributed with respect to ages. Therefore, a modified negative exponential model is considered. The model is

i) $y = c + e^{-(ax+b)} + u$,

where, x represent the age group; y represent male population; a, b, c are constants and u is the error term of the model.

From the dotted plot of ages and ASDR for male, it has been seen that ASDR can be fitted by polynomial for different ages. In this case, an n^{th} degree polynomial model is treated and the model of the n^{th} degree polynomial is

ii) $y = a_0 + \sum_{i=1}^{n} a_i x^i + u$,

where, x is age group; y is ASDR; $a_0$is the constant; a_i is the coefficient of x^i (i =1, 2, 3, ..., n) and u is the error term of the model. Here we have to choose a suitable n for which the error sum of square is minimum.

Again from the scattered plot of ages and the number of persons surviving at an exact age x (l_x), it has been observed that $l_x$can also be fitted by polynomial for different ages. Therefore, an n^{th} degree polynomial model is treated and the model of the n^{th} degree polynomial function is

iii) $y = a_0 + \sum_{i=1}^{n} a_i x^i + u$,

where, x is age group; y is number of persons surviving at an exact age x (l_x); a_0 is the constant; a_i is the coefficient of x_i ($i = 1, 2, \cdots, n$) and u is the error term of the model. Here we have also to select a suitable n for which the error sum of square is minimum.

Using the software STATISTICA, all these mathematical models have been estimated.

2.6 Model Validation

To check how much those models are stable over the population, the cross validity prediction power (CVPT), ρ^2_{cv}, is applied.

Here $\rho^2_{cv} = 1 - \frac{(n-1)(n-2)(n+1)}{n(n-k-1)(n-k-2)}(1 - R^2)$. Where, n is the sample size or number of cases, k is the number of predictors in the model and the cross validated R is the correlation between observed and predicted values of the dependent variable. Using the above statistics, it can be concluded that if the prediction equation is applied to many other samples from the same population, then $(\rho^2_{cv} \times 100)\%$ of the variance on the predicted variable would be explained by the model (Stevens 1996).

2.7 Crude Birth Rate (CBR) for Male

There are various types of measures in fertility. But, in this section, we estimate CBR only. Assuming intercensal growth rate between the intercensal period from 1981 to 1991 is equal to the growth rate (r) as in 1991. Now CBR has been estimated from the balancing equation $CBR = CDR + r$; assuming the net migration rate is zero. Intercensal growth rate has been estimated using the exponential growth rate formula $P_{t_2} = P_{t_1} \exp(r(t_2 - t_1))$. Since the intercensal growth rate during 1981 to 1991 for male is 24.33 per thousand. So, in 1991 the CBR for male has been estimated as 38.97 per thousand.

3 Results and Discussion

An abridged life table for males presented in Table 2 exhibits the life expectancy at birth for males in 1991 is 55.13. In (Kabir and Mosleh, 1989), life expectancy at birth for males was 46.2 in 1974. Again in 1981 the life expectancy at birth for males was 49.81 (Ali, 1990). From the above statistics the present estimates seems to be reasonable, as the developing country like Bangladesh is gaining life expectancy at birth at the rate 0.5 in an year (UN, 1956). Again it is also found that the life expectancy at birth for male in 1991 is 56.5 (BBS, 1993) which is also close to our estimate. Therefore the life expectancy at birth for male around 55 or 56 is quite reasonable.

Again the ASDR for male of Bangladesh in 1991 presented in Table-4 shows the traditional U shaped curve. We have compared the ASDR for male of Bangladesh in 1991 to the ASDR for male of Bangladesh in 1981 presented also in Table 4. From the table, ASDR for male of Bangladesh in 1991 is lower than that of ASDR for male of Bangladesh in 1981 in each age group. Also two curves are gradually decreasing in the age interval [0,20] but gradually increasing in the age interval [20,80+].

The CDR for male of Bangladesh in 1991 is to be found in the present study as 14.64 but that in 1981 it was 15.835 (Ali, 1990). In our estimate of CBR for male of Bangladesh is 38.97 in 1991 but it was 46.83 for1981 (Ali, 1990). Thus the present study conclude that estimates are quite reasonable and the trend of fertility as well as mortality have been indicating a declining pattern.

The fitted equations are as follows

$$y = (-489.5856) + \exp(-(0.0315719)x + (9.22631)) \tag{1}$$

$$y = 0.05881 + (-0.01222)x + (0.00068)x^2 + (-0.0000141)x^3 + (9.87e-8)x^4 \tag{2}$$

$$y = (90959.33) + (-700.679)x + (20.20866)x^2 + (-0.296326)x^3 \tag{3}$$

From the above equations (1), (2) and (3), the rate of change of y with respect to x, i. e. the velocity curves have been shown in the Fig. 2(b), Fig. 3 and Fig. 4, respectively. It should be noted that the usual model, i.e., Gompertz model was also applied but seems to be worse fitted with respect to the shrinkages. Therefore, the result of Gompertz model was not shown here.

The estimated CVPP, ρ^2_{cv}, corresponding to their R^2 is shown in Table 5. From this table we see that all the fitted models in equations (1), (2) and (3) are highly cross validated and their shrinkages are 0.001, 0.021 and 0.0051, respectively. These imply that more than 99% cases the fitted models (1), (3) will be stable and 98% cases the fitted model (2) will be stable.

The information on model fitting has been presented in Table 6. From Table 6, it is shown that all the parameters of the fitted models are highly significant with large significant proportion of variance explained.

Instantaneous force of mortality (μ_x) calculated from l_x values for male population of Bangladesh in 1991 has been presented in the graph paper shown in Fig. 5. From the figure, it has been seen that force of mortality is monotonically increasing in the whole range but approximately linear in the age range '0 to 20' and strictly increasing in the age interval '20 to 80+'.

Table 5: Estimated Cross Validity Prediction Power, ρ^2_{cv} of the Predicted Equations of Age Distribution, ASDR and Number of Persons Surviving at Exact Age x (l_x) for Male of Bangladesh in 1991

Models	n	k	R^2	ρ^2_{cv}
Equation (iv)	17	1	0.9954	0.9944
Equation (v)	21	4	0.9683	0.9474
Equation (vi)	21	3	0.9890	0.9839

Table 6: Information on Model Fitting

Models	Proportion of Variance Explained	Parameters	Significant Probability (p)
Model 1	0.9954		
		a	0.0000
		b	0.0000
		c	0.0470
Model 2	0.9683		
		a_0	0.000003
		a_1	0.000030
		a_2	0.000033
		a_3	0.000020
		a_4	0.000005
Model 3	0.9890		
		a_0	0.00000
		a_1	0.02000
		a_2	0.03451
		a_3	0.00098

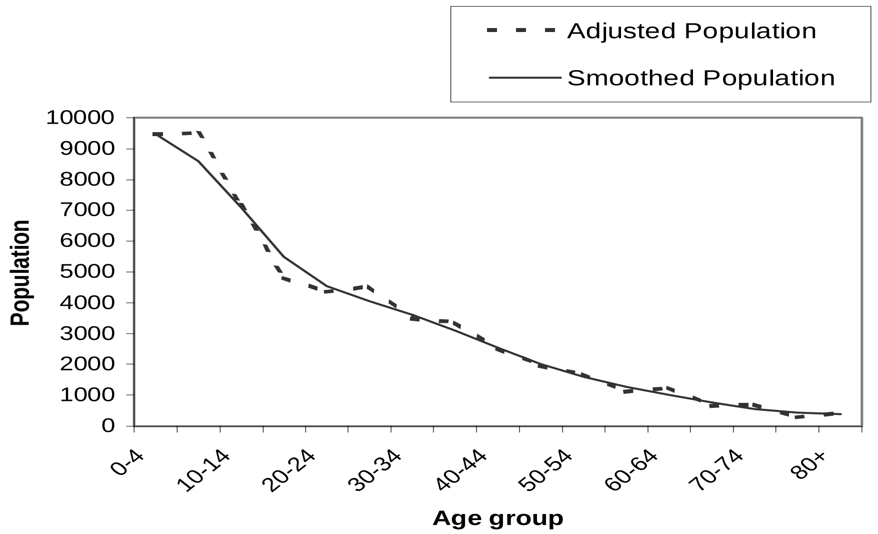

Figure 1. Adjusted (for undercount) and Smoothed Age Distribution for Male Population of Bangladesh in 1991.

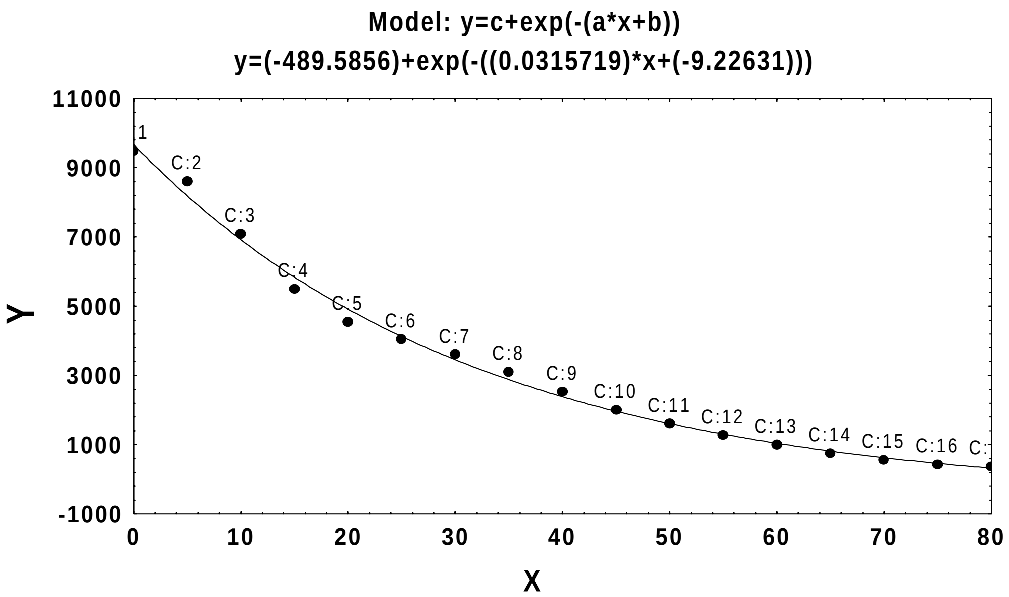

Figure 2(a). The Graph of Observed and Fitted Model for Male Age Distribution of Bangladesh in 1991.

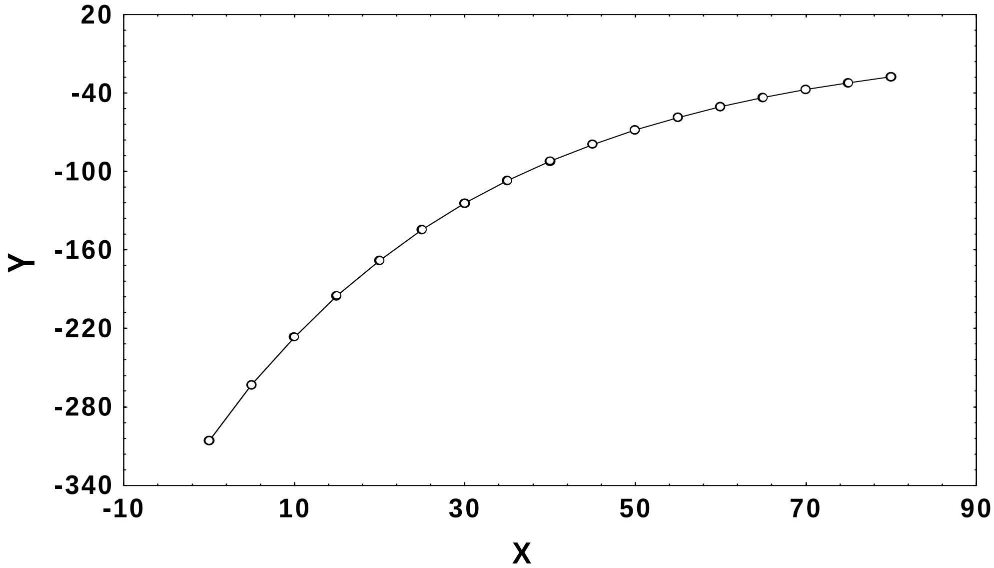

Figure 2(b). The Velocity Curve for Male Age Distribution of Bangladesh in 1991. X: Age Group, Y: Velocity.

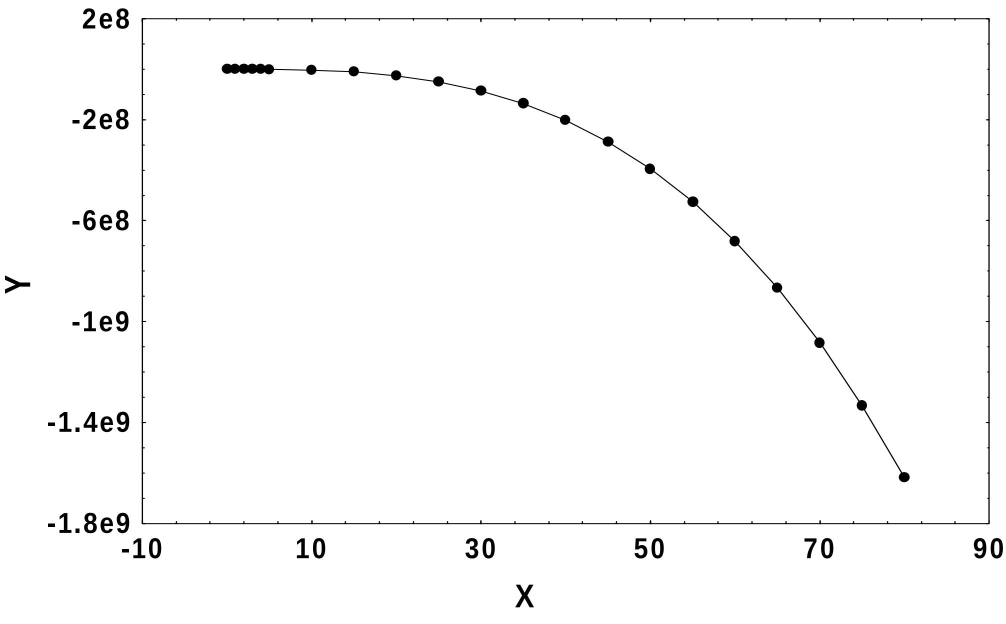

Figure 3. The Velocity Curve of Age Specific Death Rate (ASDR) for Male Population of Bangladesh in 1991.X: Age Group, Y: Velocity.

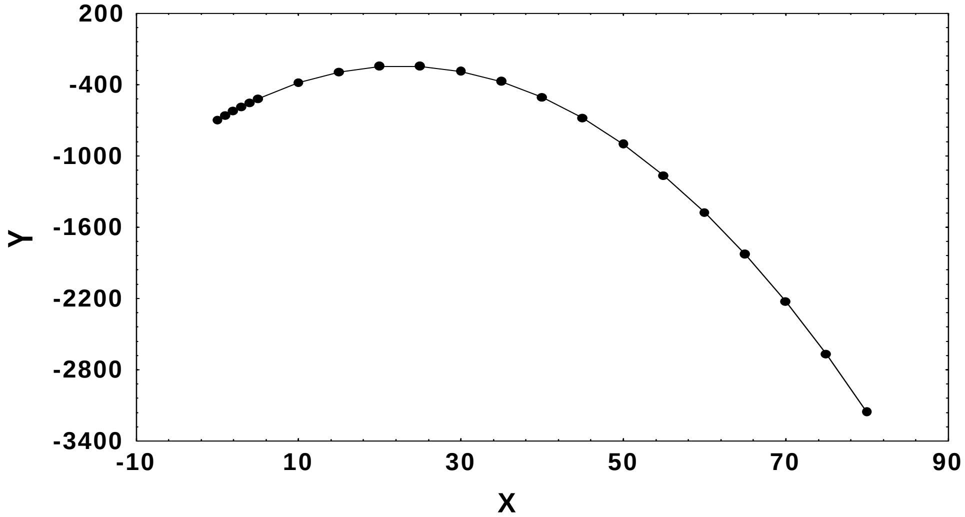

Figure 4. The Velocity Curve of Number of Persons Surviving at an Exact Age x (lx) for Male of Bangladesh in 1991. X: Age Group, Y: Velocity.

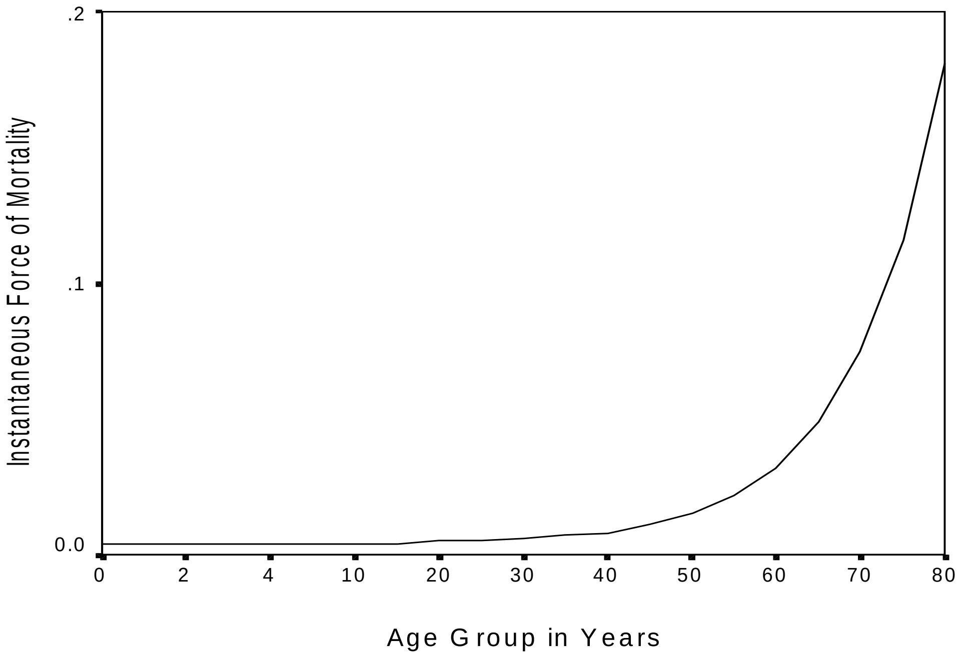

Figure 5. The Instantaneous Force of Mortality for Male Population of Bangladesh in 1991.

4 Conclusion

An abridged life table for male has been constructed using 1991 census data on female marital status by widowhood method. Then CBR, CDR and ASDR and life expectancy at birth for male have also been calculated from the same source of data. It is to be noted that the method of estimation of the population parameters ($\overset{0}{e}_0$, ASDR, CBR, CDR) from limited and defective data is likely to be fitted for developing countries like Bangladesh.

References

Ali K. (1990). Construction of Life Tables and Estimation of Demographic Parameters of Bangladesh, The Rajshahi University Studies Part-B, Vol. 18,239-250.

BBS (1994). Bangladesh Population Census 19991, Vol.1, National Series, Analytical Report, Government of the People's Republic of Bangladesh, Dhaka .

BBS (1993). Statistical Yearbook of Bangladesh of 1992; Government of the Peoples Republic of Bangladesh, Dhaka .

Brass, W. (1978). Indirect Method of Estimating Mortality: Illustrated by the Application to Middle East and North African Data, In ECWA. The population Frame Work, Data Collection, Demographic Analysis, Population Development, United Nations.

BFS (1978). First Report on Bangladesh Fertility Survey1975, Ministry of Planning. Government of Bangladesh.

Biswas, S. (1988). Stochastic Processes in Demography and Applications, Wiley Eastern Limited, New Delhi.

Hill, K. (1975). Estimation of Adult Mortality by Indirect Means, Unpublished Ph.D. Thesis, University of London, London.

Hill, K. (1977). Estimation Adult Mortality Levels From the Information on Widowhood, Population Studies. Vol. 31(i).

Kabir, M. and Mosleh Uddin, M. (1989). Techniques of Demographic Analysis, Society for Project Implementation Research Evaluation and Training, Dhaka.

Sivamurthy, M. and Sitharam, k. S. (1980). Handbook of Indirect Methods for Mortality Estimation, CDC Occasional Paper, Cairo Demographic Centre, Egypt.

Stevens, J. (1996). Applied Multivariate Statistics for the Social Sciences, Third Edition, Lawrence Erlbaum Associates, Inc., Publishers, New Jersey.

United Nations (1956) Methods for Population Projections by Sex and Age, Department of Economic and Social Affairs, Manual-iii, New York.

United Nations(1982). Model Life Tables for Developing Countries, Department of International Economic and Social Affairs, Population Studies, No.77.

Velleman, P. F. (1980). Definition and Comparison of Robust Nonlinear Data Smoothing Algorithms, Journal of the American Statistical Association, Volume 75. Number 371, 609-615.