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Stochastic Study of Asymptotic Behavior of Rainfall Distribution in Rajshahi

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Abstract

The weather plays a significant role in the agriculture of a country. The rainfall, which can be considered as one of the most important of the weather factors in Bangladesh, is highly variable and hence sometimes unpredictable. A study of rainfall probability is an approach to sound planning against the hardships caused by large variation in rainfall. Daily rainfall data for a period of 27 years from 1972 to 1998 of Rajshahi district were collected from Bangladesh Meteorological Department and analyzed to estimate the asymptotic behaviors of rainfall distribution in Rajshahi district. A Markov chain model is established to fit Rajshahi daily rainfall data for the various aspects of rainfall occurrence patterns and could be mathematically derived from the Markov chain by using maximum likelihood estimate and these were also established to fit the observed data. The exact mean and variance were also the same as the asymptotic mean and variance for each month. The distribution of number of success is asymptotic normal distribution. The rainfall probability was not found to be vary much during rabi (November-February) season. But much more variation of rainfall probabilities were observed during both kharif (June-October) and pre-kharif (March-May) seasons. Strictly speaking, one would presume continuous variation in these probabilities also within monthly, seasonally and annually. Based on these findings and using chi-square test, it could be concluded that the model fit is good. The probability of occurrence of rainfall is of vital importance in efficient planning and execution of water programs for agricultural development and environmental strategies in Bangladesh. This kind of information is very helpful in determining water needs for supplemental irrigation for agriculture and also for urban areas. These requirements could be translated in terms of worth for additional reservoir storage, if there is storage of water, or suitable drainage system if there is excessive rainfall.

Keywords and Phrases: Markov Chain Model, Maximum Likelihood Estimate, Rainfall Probability, Asymptotic Normal Distribution and Chi-square Test.

1 Introduction

A Markov chain is sometimes a suitable probability model for certain time series in which the observation at a given time is the category into which an individual falls. The simplest Markov chain is that in which there are a finite number of states or categories and a finite number of equidistant time points at which observations are made, the chain is of first-order, and transition probabilities are the same for each time interval. When there are many observations in each of the initial states and the same set of transition probabilities operate, the methods of statistical inference is used for this model.

Statistical inference is an essential tool for concluding any research. The probability theory of Markov chain has been extensively developed, whereas statistical inference concerning Markov chain model has not been comparatively developed yet (Billingsley, 1961). There is a tendency for rainy days and dry days to cluster and to form respective sequences. This reality of meteorological persistence can best be described by a Markov chain model of proper order corresponding to the order of conditional dependence of the physical phenomena (Rahman, 1999a,b).

Drought is temporary but complex feature of the climatic system of a given region with widespread significance (Olapido, 1985), which is usually caused by precipitation deficit (Gregory, 1986). Such natural disasters leave a long lasting effect on a social and economic fabric of a region where it strikes, sometimes-requiring relief efforts on a global scale (Ahmed, 1995). A drought can be recognized when it hits a given region, here is no universal definition of this term. Various characteristics of droughts, including definitions and their meteorological, hydrological and economic aspects were discussed by Doornkamp et al. (1980), Giambelluca et al. (1988), Dennet et al. (1985), Olapido (1985), Ahmed (1991) and reviewed extensively by Gregory (1986) and Nieuwolt (1986).

The significance of rainfall analysis has been highlighted by hydrological and climatological studies because of its influence in all human activities such as agricultural, industrial and domestic (Rahman, 2000). Amounts of rainfall vary from place to place and from month to month. It is essential to know the average amount and variability of rainfall for the purpose of agricultural, hydrological planning, industrial and water management. Variability of rainfall is especially important because it has got effects on both hydrology and agriculture of the high Barind region (Rahman and Alam, 1997). Inference problems, such as estimation and hypothesis testing, involving Markov chains were considered by several authors (Anderson and Goodman, 1957; Billingsley, 1961; Lee et al., 1970). These problems were studied not only for their theoretical interest but also for their applications in diverse areas.

A rainfall simulation model based on a first-order Markov chain has been developed to simulate the annual variation in rainfall amount that is observed in Bangladesh. The distribution of number of success is asymptotic normal distribution. When actual and simulated daily rainfall data were used to derive a crop simulation model, there is no significant difference of rice yield response (Rahman, 2000). The rainfall simulation model was tested by (Rahman, 1999a,b). An attempt has been made in this study to estimate the asymptotic behaviors of rainfall distribution in Rajshahi district.

2 Materials and Methods

2.1 Study Areas and Data Collection

The study area is a distinct agro-ecological unit located in the North- Western part of the country. This area lies in the driest part of the country. Climatic conditions are fairly uniform. However, high standard deviation figures indicate considerable year-to-year variability in all the following seasonal parameters.

- Mean annual rainfall is about 1300 - 1400 mm.
- The mean length of the pre-monsoon transition period is 50-60 days, of which about two-thirds are 'dry' days.
- The mean length of the reference rainfed kharif growing period is 185-190 days over most of the region, but it exceeds 190 days in the southeast.
- Almost the whole of the region lies in the zones with the longest cool winter period and the highest number of days with maximum temperatures above 40^0C.

Limited surface-water supplies are available in tanks. Ground water supplies generally are poor in the more hilly western part, but somewhat better in valleys and in more shallowly dissected areas towards the eastern boundary of the region. The predominant land use is transplanted aman grown as a single crop. Local varieties are used. Broadcast aus are grown before aman locally in the east, mainly in years when early rains occur. Gram, barley and mustard are grown to a limited extent after Aman, but generally produce poor yields. With irrigation, HYV boro is generally grown followed by T.Aman, but dryland rabi crops grown with hand irrigation on the banks of tanks and locally on adjoining fields.

The properties of rainfall occurrence were derived from the probability model and require two conditional probabilities as $P_1 = P_r\{\text{wet day/previous day wet}\}=P_r(w/w)$ and $P_0 = P_r\{\text{wet day/previous day dry}\}=P_r(w/d)$. The Markov chain model used the daily rainfall data of Rajshahi district for a period of 27 years from 1972–1998. The conditional probabilities in a Markov chain were estimated by using maximum likelihood estimation techniques. Rainfall data were obtained from the Bangladesh Meteorological Department (BMD).

3 Model Building

3.1 Markov Chain Model

A two-state Markov chain method involves the calculation of two conditional probabilities: (1) α, the probability of a wet day following a dry day, and (2) β, the probability of a dry day following a wet day. The two-state Markov chain for the combination of conditional probabilities is:

		Future State	
Present State		Dry	Wet
	Dry	1-α	α
	Wet	β	$1-\beta$

Let us consider the conditional probabilities which are denoted by

$$\begin{aligned} P_0 &= P_r\{W/D\} \\ P_1 &= P_r\{W/W\} \end{aligned} \tag{1}$$

This sequence is irreducible Markov chain with two ergodic states. Its stationary probability distribution has a probability of success

$$P = P_0/(1-(P_1-P_0)) \tag{2}$$

The actual distribution of $n_{ij}(t)$ is multiplied by an appropriate function of factorials. Let $n_i(t-1) = \sum_{j=1}^{m} n_{ij}(t)$. Then the conditional distribution of $n_{ij}(t)$, $j = 1, 2, \cdots, m$, given $n_i(t-1)$ (or given $n_k(s)$, $k = 1, 2, \cdots, m$; $s = 0, \cdots, t-1$) is

$$\frac{n_i(t-1)!}{\prod_{j=1}^{m} n_{ij}(t)!} \prod_{j=1}^{m} p_{ij}(t)^{n_{ij}(t)} \tag{3}$$

This is the same distribution, as one would obtain if one had $n_i(t-1)$ observations on a multinomial distribution with probability $p_{ij}(t)$ and with resulting numbers $n_{ij}(t)$. The distribution of the $n_{ij}(t)$ (conditional on the $n_i(0)$) is

$$\prod_{t=1}^{T} \left\{ \prod_{i=1}^{m} \left[\frac{n_i(t-1)!}{\prod_{j=1}^{m} n_{ij}(t)!} \prod_{j=1}^{m} p_{ij}(t)^{n_{ij}(t)} \right] \right\}. \tag{4}$$

For a Markov chain with stationary transition probabilities, a stronger result concerning sufficiency follows that the set $n_{ij} = \sum_{t=1}^{T} n_{ij}(t)$ forms a set of sufficient statistics.

This follows from the fact that, when the transition probabilities are stationary, the probability can be written in the form

$$\prod_{t=1}^{T}\prod_{i,j=1}^{m} p_{ij}(t)^{n_{ij}(t)} = \prod_{i,j=1}^{m} p_{ij}^{n_{ij}} \tag{5}$$

For not necessarily stationary transition probabilities $p_{ij}(t)$, the $n_{ij}(t)$ are a minimal set of sufficient statistics.

3.2 Maximum Likelihood Estimates

The stationary transition probabilities p_{ij} can be estimated by maximizing the probability (5) with respect to the p_{ij}, subject of course to the restrictions $p_{ij} \geq 0$ and

$$\sum_{j=1}^{m} P_{ij} = 1,\ i = 1, 2, \cdots, m \tag{6}$$

when the n_{ij} are the actual observations. This probability is precisely of the same form, except for a factor that does not depend on p_{ij}, as that obtained for m independent samples, where the ith sample ($i = 1, 2, \cdots, m$) consists of $n_i^\star = \sum_j n_{ij}$ multinomial trials with probabilities p_{ij} $(i, j = 1, 2, \cdots, m)$. For such samples, it is well-known and easily verified that the maximum likelihood estimates for p_{ij} are

$$\begin{aligned} \hat{P}_{ij} &= \frac{n_{ij}}{n_i^\star} = \sum_{t=1}^{T} n_{ij}(t) / \sum_{k=1}^{m}\sum_{t=1}^{T} n_{ik}(t) \\ &= \sum_{t=1}^{T} n_{ij}(t) / \sum_{t=0}^{T-1} n_i(t) \end{aligned} \tag{7}$$

and hence this is also true for any other distribution in which the elementary probability is of the same form except for parameter-free factors, and the restrictions on the p_{ij} are the same. In particular, it applies to the estimation of the parameters p_{ij} in (5).

When the transition probabilities are not necessarily stationary, the general approach can still be applied, and the maximum likelihood estimates for the $p_{ij}(t)$ are found to be

$$\hat{p}_{ij}(t) = n_{ij}(t)/n_i(t-1) = n_{ij}(t)/\sum_{k=1}^{m} n_{ik}(t). \tag{8}$$

The same maximum likelihood estimates for the $p_{ij}(t)$ are obtained when we consider the conditional distribution of $n_{ij}(t)$ given $n_i(t-1)$ as when the joint distribution of the $n_{ij}(1), n_{ij}(2), \cdots, n_{ij}(T)$ is used. Formally these estimates are the same as one would obtain if for each is t and one had $n_i(t-1)$ observations on a multinomial distribution with probabilities $p_{ij}(t)$ and with resulting numbers $n_{ij}(t)$. The estimates can be described in the following way: Let the entries $n_{ij}(t)$ for a given t be entered in a two-way $m \times m$ table. The estimate $p_{ij}(t)$ is the i, jth entry in the table divided by the sum of the entries in the ith row. In order to estimate p_{ij} for stationary chain, add the corresponding entries in the two-way tables for $t = 1, 2, \cdots, T$, obtaining a two-way table with entries $n_{ij} = \sum_t n_{ij}(t)$. The estimate of p_{ij} is the i, jth entry of the table of n_{ij}'s divided by the sum of the entries in the ith row (Anderson and Goodman, 1957; Medhi,1981).

3.3 Asymptotic Behavior of Estimators

Let $P = (p_{ij})$ and let $p_{ij}^{[t]}$ be the elements of the matrix p^t. Then $p_{ij}^{[t]}$ is the probability of state j at time t given state i at time 0. Let $n_{k;ij}(t)$ be the number of sequences including state k at time 0, i at time $t-1$ and j at time t. Then we seek the low order moments of

$$n_{ij}(t) = \sum_{k=1}^{m} n_{k;ij}(t). \tag{9}$$

The probability associated with $n_{k;ij}(t)$ is $p_{ki}^{[t-1]} p_{ij}$ with a sample size of $n_k(0)$. Thus

$$\varepsilon n_{k;ij}(t) = n_k(0) p_{ki}^{[t-1]} p_{ij}, \tag{10}$$

$$\text{Var}\{n_{k;ij}(t)\} = n_k(0) p_{ki}^{[t-1]} p_{ij} [1 - p_{ki}^{[t-1]} p_{ij}], \tag{11}$$

To find the asymptotic behavior of the $\hat{p}_{ij}$, first consider the $n_{ij}(t)$. Let us consider that $n_k(0)/\sum n_j(0) \to \eta_k (\eta_k > 0, \sum \eta_k = 1)$ as $\sum n_j(0) \to \infty$. For each $i(0)$, the set $n_{i(0)i(1)i(2)\cdots i(T)}$ are simply multinomial variables with sample size $n_{i(0)}(0)$ and parameters $p_{i(0)i(1)}\ p_{i(1)i(2)} \cdots p_{i(T-1)i(T)}$, and hence are asymptotically normally distributed as the sample size increases. The $n_{ij}(t)$ are linear combinations of these multinomial variables, and hence are also asymptotically normally distributed (Rahman,1999a,b).

3.4 Tests of Hypotheses

On the basis of the asymptotic distribution theory discussed in the preceding section, we can derive certain methods of statistical inference. Here we shall assume that every $p_{ij} > 0$. First we consider testing the hypothesis that certain transition probabilities p_{ij} have specified values p_{ij}^0. We make use of the fact that under the null hypothesis the $(n_i^\star)^{1/2}(\hat{p}_{ij} - p_{ij}^0)$ have a limiting normal distribution with means zero, and variances and covariances depending on p_{ij}^0 in the same way as observations for multinomial estimates. We can use standard asymptotic theory for multinomial or normal distributions to test a hypothesis about one or more p_{ij}, or determine a confidence region for one or more p_{ij}.

As a specific example consider testing the hypothesis that $p_{ij} = p_{ij}^0$, $j = 1, 2, \cdots, m$, for given i. Under the null hypothesis,

$$\sum_{j=1}^{m} n_i^{\star} \frac{(\hat{p}_{ij} - p_{ij}^0)^2}{p_{ij}^0} \tag{12}$$

has an asymptotic χ^2-distribution with $m - 1$ degrees of freedom (according to the usual asymptotic theory of multinomial variables). Thus the critical region of first test of this hypothesis at significance level α consists of the set $\hat{p}_{ij}$ for which (12) is greater than the a significance point of the χ^2-distribution with $m - 1$ degrees of freedom.

In the stationary Markov chain, p_{ij}, is the probability that an individual in state i at time $t-1$ moves to state j at t. A general alternative to this assumption is that the transition probability depends on t; let us say it is $p_{ij}(t)$. We test the null hypothesis $H : p_{ij}(t) = p_{ij}$ $(t = 1, 2, \cdots, T)$. Under the alternate hypothesis, the estimates of the transition probabilities for time t are

$$\hat{p}_{ij}(t) = \frac{n_{ij}(t)}{n_i(t-1)}. \tag{13}$$

The likelihood function maximized under the null hypothesis is

$$\prod_{t=1}^{T} \prod_{i,j} \hat{p}_{ij}^{n_{ij}(t)} \tag{14}$$

The likelihood function maximized under the alternate is

$$\prod_{t} \prod_{i,j} \hat{p}_{ij}^{n_{ij}(t)}(t) \tag{15}$$

The ratio is the likelihood ratio criterion

$$\lambda = \prod_{t} \prod_{i,j} \left[\frac{\hat{p}_{ij}}{\hat{p}_{ij}(t)} \right]^{n_{ij}(t)} \tag{16}$$

For a given i, the set $\hat{p}_{ij}(t)$ has the same asymptotic distribution as the estimates of multinomial probabilities $p_{ij}(t)$ for T independent samples. The asymptotic distribution of $-2\log\lambda_i$ is χ^2 with $(m-1)(T-1)$ degrees of freedom. The preceding remarks relating to the contingency table approach dealt with a given value i. Hence, the hypothesis can be tested separately for each value of i. Similarly, the test criterion based on (16) can be written as

$$\sum_{i=1}^{m} -2\log\lambda_i = -2\log\lambda \tag{17}$$

4 Results and Discussion

The properties of rainfall occurrence were derived from the probability model and require two conditional probabilities $P_1 = P_r\{\text{wet day/previous day wet}\}=P_r(w/w)$, $P_0 = P_r\{\text{wet day/previous day dry}\}=P_r(w/d)$. The fit of the Markov chain model had been tested on data of daily rainfall at Rajshahi district for a period of 27 years from 1972 − 1998. The conditional probabilities in a Markov chain were estimated by using maximum likelihood estimation techniques.

A seasonal trend of the probabilities was evident and showed maximum rainfall (Table 1) occurrence in May (.47) pre-kharif season, in July (.70) kharif season and in February (.30) rabi season for the period 1972 − 1980.

A seasonal trend of the probabilities is evident and showed maximum rainfall (Table 2) occurrence in May (.48) pre-kharif season, in July (.74) kharif season and in February (.31) rabi season for the period 1981 − 1989.

Table 1: The conditional probabilities (p_o, p_1) for Markov chain, the probability of success (p), exact and asymptotic mean and variance of the number of success (suggested by daily rainfall data at Rajshahi for the year-1972-80).

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Po	.03	.10	.06	.11	.22	.37	.44	.41	.36	.18	.03	.02
P_1	.28	.30	.31	.36	.47	.62	.70	.66	.61	.43	.28	.27
P	.05	.08	.07	.14	.29	.49	.59	.55	.50	.26	.11	.03
Exact mean	1.33	2.11	2.33	4.33	8.56	13.33	18.4	16.89	14.33	7.56	3.45	.89
Asy. mean	1.33	2.11	2.33	4.33	8.56	13.33	18.44	16.89	14.33	7.56	3.45	.89
Exact var.	2.06	3.19	3.35	5.77	9.45	11.27	11.34	12.15	11.97	9.45	1.54	1.39
Asy.var.	2.06	3.19	3.35	5.77	9.45	11.27	11.34	12.15	11.97	9.45	1.54	1.39

Table 2: The conditional probabilities (p_o, p_1) for Markov chain, the probability of success (p), exact and asymptotic mean and variance of the number of success (suggested by daily rainfall data at Rajshahi for the year-1981-89).

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
P_o	.03	.06	.08	.14	.23	.35	.49	.45	.42	.15	.03	.03
P_1	.28	.31	.33	.39	.48	.60	.74	.70	.64	.38	.28	.28
P	.04	.07	.10	.19	.30	.42	.65	.60	.51	.18	.04	.04
Exact mean	1.33	2.11	3.22	5.56	9.44	13.78	20.11	18.67	15.33	5.56	1.22	1.22
Asy. mean	1.33	2.11	3.22	5.56	9.44	13.78	20.11	18.67	15.33	5.56	1.22	1.22
Exact var.	2.05	3.19	4.59	7.04	10.34	11.90	11.05	12.01	11.55	7.00	1.87	1.89
Asy. var.	2.05	3.19	4.59	7.04	10.34	11.90	11.05	12.01	11.55	7.00	1.87	1.89

A seasonal trend of the probabilities was evident and showed maximum rainfall (Table 3) occurrence in May (.48) pre-kharif season, in July (.74) kharif season and in February (.33) rabi season for the period 1990 − 1998.

Table 3: The conditional probabilities (p_o, p_1) for Markov chain, the probability of success (p), exact and asymptotic mean and variance of the number of success (suggested by daily rainfall data at Rajshahi for the year-1990-98).

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
P_o	.04	.08	.08	.13	.23	.37	.49	.45	.40	.15	.04	.02
P_1	.29	.33	.28	.33	.48	.62	.74	.70	.65	.40	.29	.27
P	.05	.11	.11	.17	.30	.49	.66	.61	.55	.20	.06	.03
Exact mean	1.67	3.00	3.33	5.00	9.33	14.67	20.33	18.78	15.78	6.11	1.56	.89
Asy. mean	1.67	3.00	3.33	5.00	9.33	14.67	20.33	18.78	15.78	6.11	1.56	.89
Exact var.	2.48	4.23	4.63	5.67	10.53	11.99	11.16	11.94	11.41	7.73	2.39	1.33
Asy. var.	2.48	4.23	4.63	5.67	10.53	11.99	11.16	11.94	11.41	7.73	2.39	1.33

Table 4: The occurrence and non-occurrence data of pre-kharif (March-May), kharif (June-October) and rabi (November-February) seasons of rainfall for the 9 consecutive years from 1972-1980. A dry day is denoted by state 0 and a wet day is denoted by state 1.

		No Rain	Rain	Total
Pre-Kharif	No Rain	596	93	689
	Rain	91	48	139
	Total	687	141	828
	$-2\log\lambda$	13.55		
	χ^2 (1 d.f. at 5% level)	.00393		
		No Rain	Rain	Total
Kharif	No Rain	491	238	729
	Rain	243	405	648
	Total	734	643	1377
	$-2\log\lambda$	54.10		
	χ^2 (1 d.f. at 5% level)	.00393		
		No Rain	Rain	Total
Rabi	No Rain	1008	30	1038
	Rain	30	15	45
	Total	1038	45	1083
	$-2\log\lambda$	19.68		
	χ^2 (1 d.f. at 5% level)	.00393		

Rigorously speaking, one would presume continuous variation in these probabilities also within monthly, seasonally and annually, yet there did not seem to be much variation in the rabi season. Significant variations were found in kharif and pre-kharif seasons. The results are in consistent with the findings of Rahman and Alam (1997). Tables 1 to 3 showed that the distribution of number of success is asymptotic normal distribution. The exact mean and variance were also the same as the asymptotic mean and variance for each month. Similar results were also reported by Rahman (1999a). Based on these findings it could be concluded that the model seems to be doing a reasonable good job.

Firstly, the seasonal occurrences and non-occurrences of rainfall data of Rajshahi district for the nine consecutive years from 1972-1980 were shown in Table 4. The test of null hypothesis that the chain is of order 0 against the alternative hypothesis that is of order 1. The chi-square value of the transition matrices (Table 4) were $P_r\{\chi^2 \geq 13.55\}$ for pre-kharif, $P_r\{\chi^2 \geq 54.10\}$ for kharif and $P_r\{\chi^2 \geq 19.68\}$ for rabi season being very small indicating the null hypothesis that the chain is of order 0 is rejected.

Table 5: The occurrence and non-occurrence data of pre-kharif (March-May), kharif (June-October) and rabi (November-February) season of rainfall for the 9 consecutive years from 1981-1989. A dry day is denoted by state 0 and a wet day is denoted by state 1.

		No Rain	Rain	Total
Pre-Kharif	No Rain	551	114	665
	Rain	113	50	163
	Total	664	164	828
	$-2\log\lambda$	6.04		
	χ^2 (1 d.f. at 5% level)	.00393		
Kharif		No Rain	Rain	Total
	No Rain	494	219	713
	Rain	222	442	644
	Total	716	661	1377
	$-2\log\lambda$	78.59		
	χ^2 (1 d.f. at 5% level)	.00393		
Rabi		No Rain	Rain	Total
	No Rain	996	35	1031
	Rain	33	15	51
	Total	1029	53	1082
	$-2\log\lambda$	22.26		
	χ^2 (1 d.f. at 5% level)	.00393		

Secondly, similar results were found in the next decade 1981-1989 at Rajshahi district (Table 5). The chi-square value of the transition matrices (Table 5) were $P_r\{\chi^2 \geq 6.04\}$ for pre-kharif, $P_r\{\chi^2 \geq 78.59\}$ for kharif and $P_r\{\chi^2 \geq 22.26\}$ for rabi season being very small the null hypothesis that the chain is of order 0 is rejected.

Finally, similar results were also found in the next decade 1990-1998 at Rajshahi district (Table 6). The chi-square value of the transition matrices (Table 6) were $P_r\{\chi^2 \geq 8.42\}$ for pre-kharif, $P_r\{\chi^2 \geq 80.33\}$ for kharif and $P_r\{\chi^2 \geq 14.80\}$ for rabi season being very small the null hypothesis that the chain is of order 0 is rejected. Similar analysis also reported by Rahman (2000).

Table 6: The occurrence and non-occurrence data of pre-kharif (March-May), kharif (June-October) and rabi (November-February) season of rainfall for the 9 consecutive years from 1990-1998. A dry day is denoted by state 0 and a wet day is denoted by state 1.

		No Rain	Rain	Total
Pre-Kharif	No Rain	565	109	674
	Rain	104	50	154
	Total	669	159	828
	$-2\log\lambda$	8.42		
	χ^2 (1 d.f. at 5% level)	.00393		
Kharif		No Rain	Rain	Total
	No Rain	475	218	693
	Rain	221	463	684
	Total	696	681	1377
	$-2\log\lambda$	80.33		
	χ^2 (1 d.f. at 5% level)	.00393		
Rabi		No Rain	Rain	Total
	No Rain	970	46	1016
	Rain	48	18	66
	Total	1018	64	1082
	$-2\log\lambda$	14.80		
	χ^2 (1 d.f. at 5% level)	.00393		

5 Conclusions

Markov chain model was found to fit Rajshahi daily rainfall data for the various aspects of rainfall occurrence patterns and could be mathematically derived from the Markov chain and these were also found to fit the observed data. This was of interest in giving further evidence to the closeness of fit the model as well as showing that such properties of rainfall occurrence may depend simply on the Markov chain model and its probability. The weather plays an important role in the agriculture of our country. The rainfall, which considered as one of the most important of the weather factors in Bangladesh, is highly variable and hence sometimes unpredictable. A study of rainfall probability was an approach to sound planning against the hardships caused by large variation in rainfall.

The mean values of rainfall over a number of years and for different places were generally available. There was information on the probability of occurrence of various amounts of rainfall in relation to time and location. This information is of vital importance in efficient planning and execution of water programs for agricultural development and environmental strategies in Bangladesh. To make a cost-benefit study, it is necessary to consider probabilities of deficit quantities of rainfall that must be provided to meet the requirements of a project. This kind of information comes very handy in determining water needs for supplemental irrigation for agriculture and also for urban areas. These requirements could be translated in terms of cost for additional reservoir storage if there is storage of water, or suitable drainage system if there is excessive rainfall.

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