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# A Comparative Study on Two Risks Using A Class of Life-Time Distributions

**Chandan Kumer Podder**

Department of Statistics
University of Chittagong
Chittagong – 4331, Bangladesh
E-mail: podder_ck@yahoo.com

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**Abstract**

In this paper, a comparative study of two risk functions based on Modified Linear Exponential (MLINEX) and Squared Error (SE) loss functions for a class of suitably chosen life-time distributions has been made.

**Keywords:** Class of life-time distributions, Jeffrey's prior, Modified linear-exponential and Squared-error loss functions, Risk function, Bayes' estimator, Minimax estimator and Admissible estimator.

**AMS Classification:** 62C25, 62F10, 62H12.

## 1. Introduction

The class of life-time distributions introduced by Prakash and Singh [5] is important in survival analysis. Suppose a random variable $X$ follows a distribution presented by a class of probability density functions in (1) with unknown parameter $\theta$ and two known positive constants $b$ and $c$:

$$f(x;\theta)=\frac{c}{\Gamma b}\frac{1}{\theta^{b}}x^{bc-1}e^{-\frac{1}{\theta}x^{c}} \;;\; x\geq 0,\; \theta>0,\, b>0,\, c>0. \qquad (1)$$

It can be seen that for different values of $b$ and $c$ the model (1) reduces to negative exponential distribution, two-parameter gamma distribution, Erlang distribution, two-parameter Weibull distribution, Rayleigh distribution and Maxwell distribution.

Properties of different life-time distributions (such as exponential distribution, Weibull distribution, two-parameter gamma distribution and Maxwell's velocity distribution) have been studied by Abu-Talebet.et. al. [1], Ahmed et. al. [2], Son and Oh [6] and Tyagi and Bhattacharya [7] etc. Prakash and Singh [5] discussed the technique of Bayesian shrinkage estimation in a class of life testing distribution.

The purpose of this paper is to study and compare the risk functions for the parameter of the class of life-time distributions (1) using MLINEX and SE loss functions.

## 2. Preliminary Theory

Let $X$ be a random variable whose distribution depends on the parameter $\theta$ and let $\Omega$ denotes the parameter space of possible values of $\theta$. Now consider the general problem of estimating the unknown parameter $\theta$, from the results of a random sample of $n$ observations. Denoting the sample observations $x_1, x_2, \cdots, x_n$ by $x$, let $\hat{\theta}$ be an estimate of $\theta$ and also let $L(\hat{\theta},\theta)$ be the loss incurred by taking the value of the parameter $\theta$ to be $\hat{\theta}$. The risk function $R(\hat{\theta},\theta)$ is the expected value of the loss function with respect to the joint distribution of sample observations.

If $l(\theta \mid x)$ is the likelihood function of $\theta$ given the sample $x$ and $\pi(\theta)$ is the prior density of $\theta$, then combining $l(\theta \mid x)$ and $\pi(\theta)$, the Bayes' estimator $\hat{\theta}$ of $\theta$ will be a solution of the equation

$$\int_{\Omega} \frac{\partial L}{\partial \hat{\theta}} l(\theta \mid x) \pi(\theta) d\theta = 0, \tag{2}$$

where $L$ stands for loss function. It is assumed that necessary regularity conditions prevail to permit differentiation under the integral sign.

Here, the following loss functions are considered:

$$\text{i)} \quad L_1(\hat{\theta}, \theta) = \varpi \left[ \left( \frac{\hat{\theta}}{\theta} \right)^{\gamma} - \gamma \ln \left( \frac{\hat{\theta}}{\theta} \right) - 1 \right]; \; \gamma \neq 0, \; \varpi > 0. \tag{3}$$

$$\text{ii)} \quad L_2(\hat{\theta}, \theta) = c (\hat{\theta} - \theta)^2; \; c > 0, \tag{4}$$

where $c$ and $\varpi$ are scale characteristics and $\gamma$ is a shape characteristic of the above loss functions.

The loss functions $L_1$ is a modified linear exponential (MLINEX) loss function which an asymmetric one and $L_2$ is the usual squared-error loss function.

## 3. Main Results

Let us consider the case of estimating the single parameter $\theta$ of the class of life-time distributions in the model (1). The likelihood function of (1) is given by

$$l(\theta \mid x) = \left\{ \left( \frac{c}{\Gamma b} \right)^n \prod_{i=1}^{n} x_i^{\,bc-1} \right\} \frac{1}{\theta^{nb}} e^{-\frac{1}{\theta}\sum_{i=1}^{n} x_i^{\,c}}$$

$$= \kappa \frac{1}{\theta^{nb}} e^{-\frac{1}{\theta}T}, \tag{5}$$

where $\kappa = \left( \frac{c}{\Gamma b} \right)^n \prod_{i=1}^{n} x_i^{\,bc-1}$ and $T = \sum_{i=1}^{n} x_i^{\,c}$.

The maximum likelihood estimator of $\theta$ is $\frac{T}{nb}$, where $T$ is defined above. $T$ is also a complete sufficient statistic for $\theta$. It is noted that the part of the likelihood function which is relevant to Bayesian inference on the unknown parameter $\theta$ is $\frac{1}{\theta^{nb}} e^{-\frac{1}{\theta}T}$.

Since the parametric range for the class of distributions (1) is $0$ to $\propto$, therefore according to the Jeffreys' [5] rule of thumb, the Jeffreys' prior becomes

$$\pi(\theta) \propto \frac{1}{\theta}; \quad \theta \geq 0, \tag{6}$$

By combining (5) and (6), we obtain the posterior distribution of $\theta$ as

$$\pi(\theta \mid x) = \frac{T^{nb}}{\Gamma(nb)} \frac{1}{\theta^{nb+1}} e^{-\frac{1}{\theta}T}; \quad \theta \geq 0, T > 0. \tag{7}$$

The mean and variance of the posterior distribution (7) are $\frac{T}{(nb-1)}$ and $\frac{T^2}{(nb-1)^2 (nb-2)}$ respectively.

Substitution from (6) and (5) in (2) yields the Bayes' estimator $\hat{\theta}$ of $\theta$ as a solution of

$$\int_0^\infty \frac{\partial L}{\partial \hat{\theta}} \frac{1}{\theta^{nb+1}} \exp\left(-\frac{1}{\theta}T\right) d\theta = 0, \tag{8}$$

For the loss function given by (4), it follows from (8),

$$\int_0^\infty \left\{\theta^{-\gamma}\hat{\theta}^{(\gamma-1)} - \hat{\theta}^{-1}\right\} \frac{1}{\theta^{nb+1}} \exp\left(-\frac{1}{\theta}T\right) d\theta = 0$$

$$\Rightarrow \hat{\theta}^{(\gamma-1)} \int_0^\infty \frac{1}{\theta^{nb+\gamma+1}} \exp\left(-\frac{1}{\theta}T\right) d\theta - \hat{\theta}^{-1} \int_0^\infty \frac{1}{\theta^{nb+1}} \exp\left(-\frac{1}{\theta}T\right) d\theta = 0$$

$$\Rightarrow \hat{\theta}^{\gamma} = \frac{\int_0^\infty \frac{1}{\theta^{nb+1}} \exp\left(-\frac{1}{\theta}T\right) d\theta}{\int_0^\infty \frac{1}{\theta^{nb+\gamma+1}} \exp\left(-\frac{1}{\theta}T\right) d\theta},$$

from which it follows that

$$\hat{\theta} = \left[\frac{\Gamma(nb)}{\Gamma(nb+\gamma)}\right]^{\frac{1}{\gamma}} T .$$

Hence the Bayes' estimator $\hat{\theta}$ for the loss function (3) is given by

$$\hat{\theta}_B = KT, \tag{9}$$

where $K = \left[\frac{\Gamma(nb)}{\Gamma(nb+\gamma)}\right]^{\frac{1}{\gamma}}$ and $T = \sum_{i=1}^{n} x_i^{\,c}$ .

Again, for the loss function given by (4), it follows from (8) that the Bayes' estimator $\hat{\theta}$ is given by

$$\Rightarrow \hat{\theta}_S = \frac{\int_0^\infty \frac{1}{\theta^{nb}} \exp\left(-\frac{1}{\theta}T\right) d\theta}{\int_0^\infty \frac{1}{\theta^{nb+1}} \exp\left(-\frac{1}{\theta}T\right) d\theta},$$

from which it follows that

$$\hat{\theta}_S = \frac{T}{(nb-1)}, \tag{10}$$

which is same as the mean of the posterior distribution (7). Since $x$ is generated from a class of life-time distributions in (1) with parameter $\theta$, then $T = \sum_{i=1}^{n} x_i^{\,c}$ is distributed as a gamma distribution with parameters $nb$ and $\frac{1}{\theta}$, i.e., $T \sim G\left(nb, \frac{1}{\theta}\right)$.

The probability density function of $T$ is

$$p(T;\theta) = \frac{1}{\Gamma(nb)} \frac{1}{\theta^{nb}} e^{-\frac{1}{\theta}T} T^{nb-1}; \; T \geq 0, \; \theta > 0. \tag{11}$$

The mean and variance of the distribution (11) are $nb\theta$ and $nb\theta^2$ respectively.

We are interested in finding the risk functions for the estimators $\hat{\theta}_B$ and $\hat{\theta}_S$ with respect to MLINEX and SE loss functions considered in (3) and (4).

The risk function of the estimator $\hat{\theta}_B$ with respect to MLINEX is given by

$$R_{ML}\left(\hat{\theta}_B,\theta\right)=E_\theta\left[L\left(\hat{\theta}_B,\theta\right)\right]$$

$$=\varpi\left[\frac{1}{\theta^\gamma}E\left(\hat{\theta}_B{}^\gamma\right)-\gamma E\left(\ln\hat{\theta}_B\right)+\gamma\ln\theta-1\right] \qquad (12)$$

Here

$$E\left(\hat{\theta}_B{}^\gamma\right)=\theta^\gamma,$$

and $E\left(\ln\hat{\theta}_B\right)=E\left[\ln\left(KT\right)\right]$

$$=\ln K+E\left(\ln T\right).$$

For simplicity,

$$E\left(\ln T\right)=\frac{1}{\Gamma\left(nb\right)}\frac{1}{\theta^{nb}}\int_0^\infty \ln T\exp\left(-\frac{1}{\theta}T\right)T^{nb-1}dT\,.$$

Using a transformation $y=\frac{1}{\theta}T$, we obtain

$$E\left(\ln T\right)=\ln\theta+\frac{1}{\Gamma\left(nb\right)}\int_0^\infty \ln y\ \exp\left(-y\right)y^{nb-1}dy$$

$$=\ln\theta+\frac{\Gamma'\left(nb\right)}{\Gamma\left(nb\right)},$$

where $\Gamma'\left(nb\right)=\int_0^\infty \ln y\exp\left(-y\right)y^{nb-1}dy$ is the first differentiation of $\Gamma\left(nb\right)$ with respect to $n$.

Thus $E\left(\ln\hat{\theta}_B\right)=\ln K+\ln\theta+\frac{\Gamma'\left(nb\right)}{\Gamma\left(nb\right)}$.

By using the above results, (12) yields

$$R_{ML}\left(\hat{\theta}_B, \theta\right) = \varpi\left[\ln \frac{\Gamma(nb+\gamma)}{\Gamma(nb)} - \gamma \frac{\Gamma'(nb)}{\Gamma(nb)}\right], \tag{13}$$

which is independent of the parameter $\theta$ and hence $\hat{\theta}_B$ is a minimax estimator of $\theta$; vide Lehmann [4].

Similarly, the risk function of the estimator $\hat{\theta}_S$ with respect to MLINEX loss function is

$$R_{ML}\left(\hat{\theta}_S, \theta\right) = \varpi\left[\frac{1}{(nb-1)^{\gamma}} \frac{\Gamma(nb+\gamma)}{\Gamma(nb)} + \gamma \ln(nb-1) - \gamma \frac{\Gamma'(nb)}{\Gamma(nb)} - 1\right] \tag{14}$$

The risk functions $R_{ML}\left(\hat{\theta}_B, \theta\right)$ and $R_{ML}\left(\hat{\theta}_S, \theta\right)$ involving the expression of gamma $(\Gamma)$ and di-gamma $(\Gamma')$ functions are complicated.

The risk functions of the estimators $\hat{\theta}_B$ and $\hat{\theta}_S$ with respect to SE loss function are as follows:

$$\begin{aligned} R_S\left(\hat{\theta}_B, \theta\right) &= E(KT-\theta)^2 \\ &= K^2 V(T) + \left[E(KT-\theta)\right]^2 \\ &= K^2\left[V(T) + \left\{E(T) - \frac{\theta}{K}\right\}^2\right] \\ &= K^2\left[nb + \left\{nb - \frac{1}{K}\right\}^2\right]\theta^2 . \end{aligned} \tag{15}$$

$$R_S\left(\hat{\theta}_S, \theta\right) = E\left(\hat{\theta}_S - \theta\right)^2 = \frac{(nb+1)}{(nb-1)^2}\theta^2 . \tag{16}$$

Both the risk functions (15) and (16) under squared error loss function are quadratic in $\theta$ and it is concluded that when $\gamma = -1$, the two estimators and hence their risks coincide.

The above risk functions of the estimators $\hat{\theta}_B$ and $\hat{\theta}_S$ with respect to MLINEX and SE loss functions are free from $c$. For different life-time distributions, such as negative exponential, Weibull and Rayleigh, it has been found that when $b = 1$, the Bayes' estimators of the parameter $\theta$ under MLINEX and SE loss functions are different but the underlying risk functions of the two estimators are identical. Again when $b = 1$, $b = \frac{3}{2}$ and $b =$ integer value, the above risk functions are the Bayes' estimators of the parameter $\theta$ of gamma distribution, Erlang distribution and Maxwell distribution respectively.

MLINEX and SE risk functions have been computed for different values of the parameter and the results are presented in the following tables.

**Table 1:** MLINEX and SE risks for $\varpi = 1$, $b = 1$, $\gamma = -3$ and $n = 10$

| $\theta$ | $R_{ML}\left(\hat{\theta}_B, \theta\right)$ | $R_{ML}\left(\hat{\theta}_S, \theta\right)$ | $R_S\left(\hat{\theta}_B, \theta\right)$ | $R_S\left(\hat{\theta}_S, \theta\right)$ |
|---|---|---|---|---|
| 0.5 | 0.5358 | 0.6131 | 0.0559 | 0.0340 |
| 1.0 | 0.5358 | 0.6131 | 0.2237 | 0.1358 |
| 1.5 | 0.5358 | 0.6131 | 0.5034 | 0.3056 |
| 2.0 | 0.5358 | 0.6131 | 0.8949 | 0.5432 |
| 2.5 | 0.5358 | 0.6131 | 1.3983 | 0.8488 |
| 3.0 | 0.5358 | 0.6131 | 2.0136 | 1.2222 |
| 3.5 | 0.5358 | 0.6131 | 2.7407 | 1.6636 |
| 4.0 | 0.5358 | 0.6131 | 3.5797 | 2.1728 |
| 4.5 | 0.5358 | 0.6131 | 4.5306 | 2.7500 |
| 5.0 | 0.5358 | 0.6131 | 5.5933 | 3.3951 |
| 5.5 | 0.5358 | 0.6131 | 6.7679 | 4.1080 |
| 6.0 | 0.5358 | 0.6131 | 8.0543 | 4.8889 |

**Table 2:** MLINEX and SE risks for $\varpi = 2, b = \frac{3}{2}, \gamma = -2$ and $n = 20$

| $\theta$ | $R_{ML}(\hat{\theta}_B, \theta)$ | $R_{ML}(\hat{\theta}_S, \theta)$ | $R_S(\hat{\theta}_B, \theta)$ | $R_S(\hat{\theta}_S, \theta)$ |
|---|---|---|---|---|
| 0.5 | 0.1391 | 0.1404 | 0.0099 | 0.0092 |
| 1.0 | 0.1391 | 0.1404 | 0.0397 | 0.0369 |
| 1.5 | 0.1391 | 0.1404 | 0.0894 | 0.0829 |
| 2.0 | 0.1391 | 0.1404 | 0.1589 | 0.1474 |
| 2.5 | 0.1391 | 0.1404 | 0.2483 | 0.2304 |
| 3.0 | 0.1391 | 0.1404 | 0.3576 | 0.3317 |
| 3.5 | 0.1391 | 0.1404 | 0.4867 | 0.4515 |
| 4.0 | 0.1391 | 0.1404 | 0.6357 | 0.5898 |
| 4.5 | 0.1391 | 0.1404 | 0.8046 | 0.7464 |
| 5.0 | 0.1391 | 0.1404 | 0.9933 | 0.9215 |
| 5.5 | 0.1391 | 0.1404 | 1.2019 | 1.1150 |
| 6.0 | 0.1391 | 0.1404 | 1.4304 | 1.3270 |

**Table 3:** MLINEX and SE risks for $\varpi = 3, b = 2, \gamma = -1$ and $n = 30$

| $\theta$ | $R_{ML}(\hat{\theta}_B, \theta)$ | $R_{ML}(\hat{\theta}_S, \theta)$ | $R_S(\hat{\theta}_B, \theta)$ | $R_S(\hat{\theta}_S, \theta)$ |
|---|---|---|---|---|
| 0.5 | 0.0254 | 0.0254 | 0.0044 | 0.0044 |
| 1.0 | 0.0254 | 0.0254 | 0.0175 | 0.0175 |
| 1.5 | 0.0254 | 0.0254 | 0.0394 | 0.0394 |
| 2.0 | 0.0254 | 0.0254 | 0.0701 | 0.0701 |
| 2.5 | 0.0254 | 0.0254 | 0.1095 | 0.1095 |
| 3.0 | 0.0254 | 0.0254 | 0.1577 | 0.1577 |
| 3.5 | 0.0254 | 0.0254 | 0.2147 | 0.2147 |
| 4.0 | 0.0254 | 0.0254 | 0.2804 | 0.2804 |
| 4.5 | 0.0254 | 0.0254 | 0.3549 | 0.3549 |
| 5.0 | 0.0254 | 0.0254 | 0.4381 | 0.4381 |
| 5.5 | 0.0254 | 0.0254 | 0.5301 | 0.5301 |
| 6.0 | 0.0254 | 0.0254 | 0.6309 | 0.6309 |

**Table 4:** MLINEX and SE risks for $\varpi = 1, b = 1, \gamma = 1$ and $n = 10$

| $\theta$ | $R_{ML}\left(\hat{\theta}_B, \theta\right)$ | $R_{ML}\left(\hat{\theta}_S, \theta\right)$ | $R_S\left(\hat{\theta}_B, \theta\right)$ | $R_S\left(\hat{\theta}_S, \theta\right)$ |
|---|---|---|---|---|
| 0.5 | 0.0498 | 0.0556 | 0.0250 | 0.0340 |
| 1.0 | 0.0498 | 0.0556 | 0.1000 | 0.1358 |
| 1.5 | 0.0498 | 0.0556 | 0.2250 | 0.3056 |
| 2.0 | 0.0498 | 0.0556 | 0.4000 | 0.5432 |
| 2.5 | 0.0498 | 0.0556 | 0.6250 | 0.8488 |
| 3.0 | 0.0498 | 0.0556 | 0.9000 | 1.2222 |
| 3.5 | 0.0498 | 0.0556 | 1.2250 | 1.6636 |
| 4.0 | 0.0498 | 0.0556 | 1.6000 | 2.1728 |
| 4.5 | 0.0498 | 0.0556 | 2.0250 | 2.7500 |
| 5.0 | 0.0498 | 0.0556 | 2.5000 | 3.3951 |
| 5.5 | 0.0498 | 0.0556 | 3.0250 | 4.1080 |
| 6.0 | 0.0498 | 0.0556 | 3.6000 | 4.8889 |

**Table 5:** MLINEX and SE risks for $\varpi = 2, b = \frac{3}{2}, \gamma = 2$ and $n = 20$

| $\theta$ | $R_{ML}\left(\hat{\theta}_B, \theta\right)$ | $R_{ML}\left(\hat{\theta}_S, \theta\right)$ | $R_S\left(\hat{\theta}_B, \theta\right)$ | $R_S\left(\hat{\theta}_S, \theta\right)$ |
|---|---|---|---|---|
| 0.5 | 0.1322 | 0.1427 | 0.0081 | 0.0092 |
| 1.0 | 0.1322 | 0.1427 | 0.0325 | 0.0369 |
| 1.5 | 0.1322 | 0.1427 | 0.0732 | 0.0829 |
| 2.0 | 0.1322 | 0.1427 | 0.1301 | 0.1474 |
| 2.5 | 0.1322 | 0.1427 | 0.2033 | 0.2304 |
| 3.0 | 0.1322 | 0.1427 | 0.2927 | 0.3317 |
| 3.5 | 0.1322 | 0.1427 | 0.3984 | 0.4515 |
| 4.0 | 0.1322 | 0.1427 | 0.5204 | 0.5898 |
| 4.5 | 0.1322 | 0.1427 | 0.6586 | 0.7464 |
| 5.0 | 0.1322 | 0.1427 | 0.8131 | 0.9215 |
| 5.5 | 0.1322 | 0.1427 | 0.9838 | 1.1150 |
| 6.0 | 0.1322 | 0.1427 | 1.1709 | 1.3270 |

**Table 6:** MLINEX and SE risk functions for $\varpi = 3, b = 2$, $\gamma = 3$ and $n = 30$

| $\theta$ | $R_{ML}\left(\hat{\theta}_B, \theta\right)$ | $R_{ML}\left(\hat{\theta}_S, \theta\right)$ | $R_S\left(\hat{\theta}_B, \theta\right)$ | $R_S\left(\hat{\theta}_S, \theta\right)$ |
|---|---|---|---|---|
| 0.5 | 0.2230 | 0.2384 | 0.0041 | 0.0044 |
| 1.0 | 0.2230 | 0.2384 | 0.0164 | 0.0175 |
| 1.5 | 0.2230 | 0.2384 | 0.0369 | 0.0394 |
| 2.0 | 0.2230 | 0.2384 | 0.0656 | 0.0701 |
| 2.5 | 0.2230 | 0.2384 | 0.1025 | 0.1095 |
| 3.0 | 0.2230 | 0.2384 | 0.1475 | 0.1577 |
| 3.5 | 0.2230 | 0.2384 | 0.2008 | 0.2147 |
| 4.0 | 0.2230 | 0.2384 | 0.2623 | 0.2804 |
| 4.5 | 0.2230 | 0.2384 | 0.3320 | 0.3549 |
| 5.0 | 0.2230 | 0.2384 | 0.4098 | 0.4381 |
| 5.5 | 0.2230 | 0.2384 | 0.4959 | 0.5301 |
| 6.0 | 0.2230 | 0.2384 | 0.5902 | 0.6309 |

## 4. Discussion

It is evident from the tables that, in every case considered, except for $\gamma = -1$, the MLINEX risk $R_{ML}\left(\hat{\theta}_B, \theta\right)$ is uniformly smaller than $R_{ML}\left(\hat{\theta}_S, \theta\right)$, when $\varpi > 0$ and $b > 0$. This implies that in case of the MLINEX loss function, the MLINEX estimator $\hat{\theta}_B$ is better compared to the SE estimator $\hat{\theta}_S$. When $\gamma = -1$, the two estimators and hence their risk functions are identical.

Again, when $\gamma < -1$, the risk of $\hat{\theta}_B$ with respect to the SE loss function is always greater than that of $\hat{\theta}_S$.

Therefore in this case, $\hat{\theta}_S$ is better compared to the estimator $\hat{\theta}_B$ when SE loss function is considered. When

$\gamma = -1$, then the two risks are equal and either estimator is admissible but when $\gamma > -1$, then $R_S\left(\hat{\theta}_B, \theta\right) < R_S\left(\hat{\theta}_S, \theta\right)$, implying $\hat{\theta}_B$ is admissible with respect to SE loss function.

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