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Groundwater Table Volatility Forecasting using Hybrid Wavelet - GARCH Model in the Northwest Bangladesh

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Abstract

The groundwater table (GwT) volatility is important for both users and policy makers in developing countries. The data on monthly groundwater table was compiled from Bangladesh Water Development Board (BWDB) for the period January, 1991 to May, 2016. In this paper, groundwater table (GwT) volatility modeling was done by using generalized autoregressive conditional heteroscedasticity (GARCH) and wavelet-GARCH models in northwest Bangladesh. The stationary of the groundwater table was examined using unit root test and to make the series stationary, it was transformed to returns. By using Box-Jenkins method, the appropriate ARIMA(p, d, q)(P, D, Q)$_m$-GARCH(r, s) model was obtained and proposed a newly hybrid wavelet-ARIMA(p, d, q)(P, D, Q)$_m$-GARCH(r, s) models to capturing groundwater table volatilities. Based on the goodness of fit criteria such as RMSE, MAE and TIC, the best model was wavelet-ARIMA(1, 0, 1)(0, 1, 1)$_{12}$ - GARCH(1, 2). The proposed model captured volatility and its forecasting performance is better than any other.

Keywords: Groundwater Table, Northwest Bangladesh, Volatility Modeling, ARIMA-GARCH Models, wavelet-ARIMA-GARCH Models, Forecasting Performance.

AMS Classification: 62M10.

1. Introduction

Water shortage is a global problem. The United Nations (UN) estimates that approximately 1.8 billion people will live in countries or regions with absolute water scarcity by 2025 (http:// www.un.org/waterforlifedecade/scarcity.shtml). Groundwater becomes a primary source of fresh water in many parts of the world, and some regions are consuming groundwater faster than it is naturally replenished, which causes groundwater table to decline unremittingly (Rodell et al., 2009). The main objective of time series modeling is to study techniques and measures for drawing inferences from past data. This procedure was used in order to study the sustainability of groundwater resources (Ali, Sarkar and Rahman, 2012). The study showed a similar output; rising trend in water table depth or declining trend in groundwater table in the North-Eastern region of Bangladesh. Also, the study revealed that if the similar trend continues, water table depth will increase significantly and will be double in most cases by 2060. Parametric regression approach has also been done in Northwest Bangladesh which displayed a drop in groundwater level in Barind area (Jahan et. al., 2010).

Groundwater table volatility is important for both users and policy makers, particularly in the developing countries. The user is concerned about groundwater table volatility because it affects hydrological asset and risk, whereas the policy maker attempts to restraint excessive volatility to ensure hydrological and environmental stability. In the both cases, an efficient quantitative tool for modeling groundwater table volatility is needed to minimize the risk of inaccurate measurement. In this regard, researchers continue to search for the best volatility model that is able to capture various stylized facts associated with well volatilities. A lot of research is being done in order to decrease the forecasting error but there is still scope to develop methods for both short and long term forecasting.

Over the last decades, ARIMA model has been widely used in predicting of geophysical as well as hydrological time series (Momani, 2009; Ye et al., 2013). However, it assumes that data are stationary and has a limited ability to capture non-stationarities and non-linearities in hydro-climatic data (Nourani et al., 2008). Recently, wavelet transformation has shown excellent performance in hydrological modeling (Okkan, 2012; Nourani et al., 2008) as well as in multiple atmospheric and environmental applications (Pal and Devara, 2012; Pal et al., 2014). Wavelet transformation decomposes the main time series into subcomponents such that the decomposed data improve the performance of geophysical and hydrological prediction models by capturing useful information at various resolution levels (Karim, 2013 and Nourani et al., 2011).

Wavelets are a mathematical expression which decomposes the original time series into various components. The wavelet components thus obtained are very helpful for improving the forecasting capability of a model by capturing useful information at various levels. Wavelet transforms proved to perform better compared to the traditional Fourier transforms (Khalek & Ali, 2016; Adamowski & Chan, 2011 and Daliakopoulos et al., 2005). The hybrid model which combines an ARIMA model with GARCH error components is applied to analyze the univariate series and to forecast the values of approximation series (Sang et al., 2013; Chen at al., 2011; Adamowski & Sun, 2010; Zhou et al., 2008 and Bollerslev, 1986). This study has been conducted by comparing the forecasting results using the Wavelet-GARCH (w-GARCH) with the ARIMA hybrid technique to verify the effectiveness of the proposed hybrid method. Results of the proposed hybrid model show significant improvements in the forecasting error. A hybrid w-GARCH model for monthly groundwater table forecasting has been proposed. Theoretical as well as empirical findings suggest that hybrid methods can be effective and efficient to improving forecasts (Kisi & Cimen, 2011).

In the present work, w-GARCH method coupled with the wavelet techniques is used to increase the efficiency of the model. Wavelet techniques were used to translate the groundwater table into various components. The decomposed components are thus used as inputs for the w-GARCH model. The purpose of the study was to examine the performance of the w-GARCH model in forecasting the groundwater table and to compare this with the performance of other existing models like Autoregressive Integrated Moving Average (ARIMA) and Generalized Autoregressive Conditional Heteroscedasticity (GARCH).

2. Materials and Methodology

2.1. Data Source

We used secondary data as per requirements of modeling and forecasting of groundwater table for northwest (NW) Bangladesh. The data involved monthly groundwater table time series was collected from Bangladesh Water Development Board (BWDB) for the period January, 19991 to May, 2016. The first subset (January 1991 to December 2012) is called in-sample data set used to build up a model for underlying data and the second subset (January, 2013 to May, 2016) is called out-sample data set used to investigate the performance of volatility forecasting.

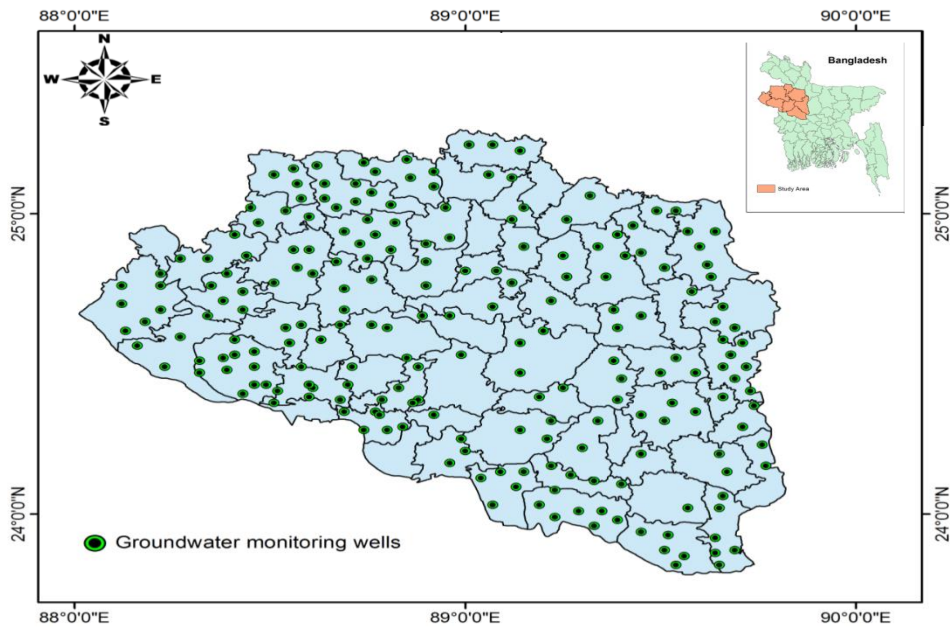


Figure 1: Study area with locations of groundwater observation wells

2.2. Modeling Groundwater Table

In this section, we briefly present the models specification, conditional distributions and forecasting criteria to model the volatility of groundwater table of NW Bangladesh. This article analyses the process and volatility of the groundwater table by using various models such as: SARIMA, SARIMA-GARCH and wavelet-SARIMA-GARCH. In this study, three different criteria, Root Mean Squared Error (RMSE), Mean Absolute Error (MAE) and Theil Inequality Coefficient (TIC) are used to evaluate the forecasting performance of the various models.

2.3. The Box-Jenkins for ARIMA Model

Autoregressive Integrated Moving Average (ARIMA) model is one of the time series forecasting methods which says that the existing value of a variable can be explained in terms of two factors; a combination of lagged values of the same variable and a combination of a constant term plus a moving average of past error terms. To build an ARIMA model one essentially use Box-Jenkins methodology (1976), which is an iterative process and involves four stages; Identification, Estimation, Diagnostic Checking and forecasting. As the Box-Jenkins models are based on the time series stationary, if underlying series is non-stationary, then first it is converted into a stationary series either by using differencing approach against time and taking the error terms of this regression (Tambi, 2005). The series stationary was tested by applying the Augmented Dickey-Fuller (ADF) (Dickey & Fuller, 1979) and Phillips-Perron (PP) unit root tests (Phillips, 1988). If it is needed for the time series to have one differential operation to achieve stationarity, it is a $I(1)$ series. Time series is $I(n)$ in case it is to be differentiated for n times to achieve stationarity.

Box-Jenkins ARIMA is known as ARIMA(p, d, q) model where p is the number of autoregressive (AR) terms, d is the number of difference taken and q is the number of moving average (MA) terms. ARIMA models always assume the variance of data to be constant. The ARIMA (p, d, q) model can be represented by the following equation:

$$\nabla^d y_t = \sum_{i=1}^{p} \varphi_i \nabla^d y_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} \tag{1}$$

where ε_t ~ N(0, σ_t^2), p and q are the number of autoregressive terms and the number of lagged forecast errors, respectively.

The identification of modeling the conditional mean value is based on the analysis of estimated autocorrelation function (ACF) and partial autocorrelation function (PACF). These estimations may be strongly inter-correlated, it is therefore recommended not to insist on unambiguous determination of the model order, but to try more models. Validation of ARMA (p, q) models is based on minimizing the Akaik's information criterion (AIC) and Bayesian information criterion (BIC). Given that hydrological data are very often characterized by high volatility, it is necessary to test the model for ARCH effect, i.e. presence of conditional heteroscedasticity (Tambi, 2005). Regarding heteroscedasticity it is therefore a situation where the condition of finite and constant variance of random components is violated. If ARCH test indicates that the variance of residuals is non-constant, we can use ARCH family models for capturing volatilities of model.

2.4. The ARCH family models

The major assumption behind the least square regression is homoscedasticity i.e. constancy of variance. If this condition is violated, the estimates will still be unbiased but they will not be minimum variance estimates. The standard error and confidence intervals calculated in this case become too narrow, giving a false sense of precision. ARCH and related models handle this by modeling volatility itself in the model and thereby correcting the deficiencies of least squares model (Dhamija and Bhalla, 2010).

2.4.1. The GARCH Model

Bollerslev (1986) developed the work of Engle in way that the conditional variance be a process of ARMA. Suppose the errors process to be as the following:

$$\varepsilon_t = \vartheta_t \sqrt{h_t}$$

In a way that $\sigma_\vartheta^2 = 1$ and

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \hat{\varepsilon}_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j} \tag{2}$$

In this condition, one needs to make sure that $\alpha_0 > 0$, $\alpha_i \geq 0$, $\beta_j \geq 0$ and $1 - \{\sum_{i=}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j\} > 0$ to see the conditional variance positive. Since ϑ_t is a white noise, the key point here is that the conditional variance of ε_t is as the following:

$$E_{t-1}\varepsilon_t = h_t$$

So, the ε_t conditional variance complies with an ARMA process like the process (1). Such models are called GARCH(p, q) where q is the number of moving average (MA) terms and p is the number of autoregressive (AR) terms. GARCH model is known as a model of heteroscedasticity which means not constant in variance. This model has been used widely in hydrological area since the data of these areas tend to have variability or highly volatile throughout the time.

2.5. Wavelet Analysis

Wavelet means a small wave or a pulse of short duration with finite energy that integrates to zero. The WT breaks the original signal into projections of translated and scaled versions of the original mother wavelet. The basis function of the wavelet analysis is the mother wavelet function (ψ) (Sang, 2012 and Li et al., 2008).

2.5.1. Continuous Wavelet Transform (CWT)

The continuous time wavelet transform of $f(t)$ with respect to a wavelet (t) is given by

$$W(m,n) = \int_{-\infty}^{\infty} f(t) \frac{1}{\sqrt{|m|}} \psi^* \left[\frac{t-m}{n}\right] dt \tag{3}$$

where, m is the scale variable, n is the translation variable and * denotes complex conjugate. The CWT maps the one dimensional function $f(t)$ to a function W(m, n) having continuous real variables m and n. The coefficients of W(m, n) at a particular scale and translation, measure how well the original function or signal $f(t)$ matches with the scaled or translated mother wavelet. However, to recover the function, all the coefficients of W(m, n) are not required. As a result CWT gives a redundant way to represent the signal (Khan & Shahidehpour, 2009).

2.5.2. Discrete Wavelet Transform (DWT)

DWT is used to decompose a signal into different resolution levels. Compared with CWT, DWT is sufficient in decomposing and reconstructing most groundwater level disturbances. It provides enough information and offers high reduction in the computational time. Multi-resolution analysis is to break a continuous real valued finite energy function into a hierarchy of approximations.

It is a technique that represents a function on many different scales, which are formed by scaled and translated mother wavelet.

In order to obtain a matrix W of wavelet coefficients for Discrete Wavelet Transform, x being a time series is defined as $x = [x_1, x_2, \ldots, x_N]^T$ with N an integer multiple of 2^j where j is the level of resolution. It is possible to define w, in order to obtain a matrix W of wavelet coefficients, which results in the discrete wavelet transform.

$$W = w.\, x \qquad (4)$$

This matrix w in (4) can be represented as $w = [w_1, w_2, \ldots w_J, v_J]^T$. Similarly, W can be defined as $W = [W_1, W_2, \ldots W_J, V_J]$. For the DWT the first J sub vectors contain all the wavelet coefficients for scale J. Each W_j column vector has $N/2\tau_j$ coefficients associated with changes on a scale of length $\tau_j = 2^{j-1}$, for j = 1, 2, 3, … , J. The final sub-vector ν_J contains just the scaling coefficients associated with averages on a scale of length 2^J.

The Multi Resolution Analysis (MRA) equation resulting from the reconstruction of the wavelet coefficients is

$$x = w^T.W = \sum_{j=1}^{J} \left(w_j^T.W_j\right) + \vartheta_J^T.V_J = \sum_{j=1}^{J} \left(D_j\right) + A_J \qquad (5)$$

In (5), the time series x is stated as the sum of a constant vector A_J and J other vectors, D_j (j=1,2,3...J), each of which contains a time series related to variations in x at a certain scale. The D_j refers to the j^{th} wavelet detail and the A_J as the approximation. MRA is intended to give good time resolution and poor frequency resolution at high frequencies and good frequency resolution and poor time resolution at low frequencies. This methodology has proved to give good results especially when the signal has high frequency components for short durations and low frequency components for long durations (Faria et al., 2009).

2.6. Forecasting Performance Measures

This article uses three different criteria, namely Root Mean Squared Error (RMSE), Mean Absolute Error (MAE), and Theil Inequality Coefficient (TIC) to compare the performance efficiency of the ARMA and ARMA-GARCH family models in the forecasting behavior of groundwater table. That model with a smaller amount would be the considered as a better and more appropriate model.

1. **Root Mean Squared Error (RMSE)**: Root Mean Square Error (RMSE) measures the difference between the true values and estimated values, and accumulates all these difference together as a standard for the predictive ability of a model. The criterion is the smaller value of the RMSE, the better the predicting ability of the model.

$$RMSE = \sqrt{\frac{\sum_{t=T+1}^{T+k}(y_t - \hat{y}_t)^2}{n}} \qquad (6)$$

2. **Mean Absolute Error (MAE)**: It takes into consideration the average of the absolute value of the residuals.

$$MAE = \sum_{t=T+1}^{T+k}\left|\frac{y_t - \hat{y}_t}{n}\right| \qquad (7)$$

3. **Theil Inequality Coefficient (TIC)**: The Theil inequality coefficient always lies between zero and one, where zero indicates a perfect fit.

$$TIC = \frac{\sqrt{\sum_{t=T+1}^{T+k}(y_t - \hat{y}_t)^2}}{\sqrt{\sum_{t=T+1}^{T+k}\hat{y}_t^2} + \sqrt{\sum_{t=T+1}^{T+k}y_t^2}} \qquad (8)$$

Where y_t is observed values, $\hat{y}_t$ is the forecasted values at time t and n is the number of forecasts.

3. Result and Discussions

3.1. Data and Stationary Examination of Variable

Monthly groundwater table of NW Bangladesh for period of January, 1991 to May, 2016 have been derived from Bangladesh Water Development Board (BWDB). Figure 1 shows the changes of actual and wavelet denoised water table for study period.

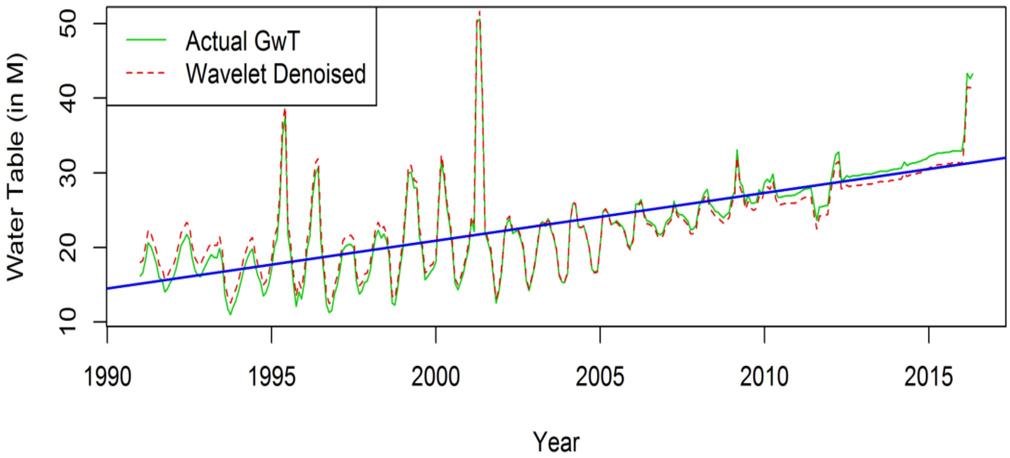


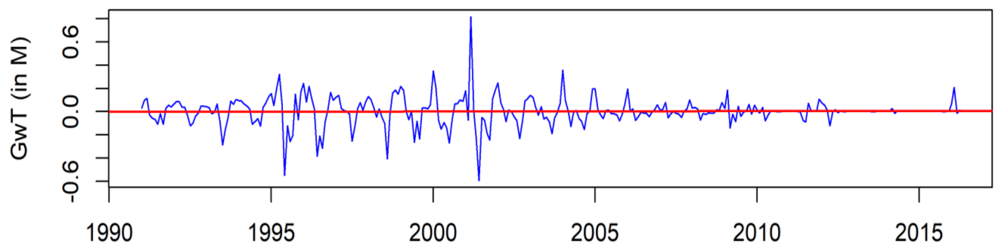
Figure 2. Actual and wavelet denoised monthly groundwater table for the period of January, 1991 to May, 2016 in the NW Bangladesh.

Since the basis of Box-Jenkins models' forecasting is the stationary of the groundwater table in question, so we use of ADF test and PP test on groundwater table. Table 1 summarized the unit root tests for groundwater table. The ADF and PP tests were used to test the null hypothesis of a unit root against the alternative hypothesis of stationarity.

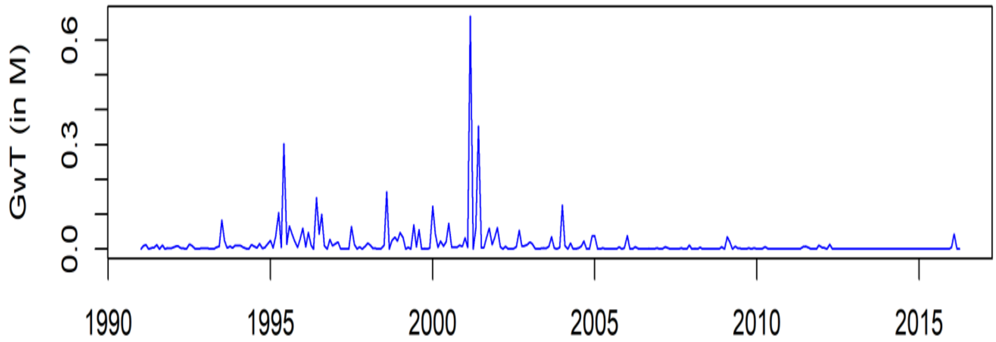
Table 1. Results of ADF and PP test of actual groundwater table.

Groundwater Table	Test	Actual		First Differenced	
		t-Statistic	p-value	t-Statistic	p-value
Actual	ADF	-0.098	0.947	-11.815	0.000
	PP	-2.494	0.086	-19.159	0.000
Wavelet-denoised	ADF	-0.396	0.907	-11.818	0.000
	PP	-2.014	0.063	-18.003	0.000

According to the Table 1 the results of ADF and PP tests show that the actual water table is non-stationary, because the p-value for both ADF and PP tests are greater than 5% level of significance and their corresponding critical values. The p-value = 0.000 < 0.001 indicates the ADF t-statistic is significant, means the first differenced transformed series is stationary. The graph plotting of the first differenced transformed series illustrates the stationarity of the first order differenced transformed groundwater level series since most of the data are located around mean of zero. The stationarity of the first differenced series then supported by the correlogram patterns of ACF and PACF for the series, where the values are reduced drastically to zero.



(a) groundwater table return (r_t) series



(b) groundwater table squared return (r_t^2) series

Figure 3. Time series plot for monthly groundwater table returns (r_t) and squared returns (r_t^2)

To transform the non-stationary to stationary groundwater table, we calculate the returns as:

$$r_t = log(y_t) - log(y_{t-1}) = log\left(\frac{y_t}{y_{t-1}}\right) \tag{9}$$

The time series plot of the transformed data that is named water table returns is shown in Figure 3(a). This plot shows that the mean of the series is now about constant. Hence, we can assume that the series is stationary. In the estimation stage, the values of Akaike information criterion (AIC) and Schwarz information criterion (SIC) are considered. In this context, the model with smaller AIC and SIC values are concluded to be the better estimation model. The variance is high that clearly exhibit volatility clustering in Figure 3(b), which allows us to carry on further to apply the ARCH family models.

In Table 2, the results of ADF and PP test show that the both actual and wavelet-denoised groundwater table returns series is stationary.

Table 2. Results of ADF and PP test of groundwater table return series.

Groundwater Table	Test	Actual		First Differenced	
		t-Statistic	p-value	t-Statistic	p-value
Actual	ADF	-12.406	0.000	-14.108	0.000
	PP	-18.191	0.000	-57.791	0.000
Wavelet-denoised	ADF	-12.565	0.000	-14.330	0.000
	PP	-15.457	0.000	-63.518	0.000

Based on the assumption of 1% significance level, all of the p-values are smaller than 0.001, which means the returns have ARCH effect. To assess the distributional properties of the actual and wavelet-denoised groundwater table return data, various descriptive statistics are reported in Table 3.

Table 3. Summary statistics of groundwater table monthly returns series

Groundwater Table	Mean	SD	Skewness	Kurtosis	Jarque-Bera	p-value
Actual	-0.0012	0.0534	-0.1285	12.7665	1209.0470	0.000
Wavelet-denoised	-0.0011	-0.0012	-0.1579	12.3662	1112.4052	0.000

Table 3 shows that the mean of actual and wavelet-denoised groundwater table returns is close to zero and the sample kurtosis for it is well above the normal value of 3. There is also evidence of negative skewness, with long left tail indicating that groundwater table has non symmetric returns. Jarque-Bera value shows that actual and wavelet-denoised groundwater table monthly returns distribution is leptokurtic and departs significantly from Gaussian distribution. Therefore, for capturing of volatilities in time series of returns, we will use the autoregressive conditional heteroscedasticity (ARCH) family models.

3.2. Model Estimation and Forecasting using the Box-Jenkins Method

In this section, we tried to build univariate model to forecast groundwater table using Box-Jenkins methodology of building ARIMA model. In order to find the most optimal lags, different AR and MA lags were tested. Autocorrelation and partial autocorrelation functions of residuals are also used. Information criteria of Akaike and Schwarz were also employed for identifying the best model. The most appropriate obtained model is ARIMA(1,0,1)(0,1,1)$_{12}$ that is an adequate choice for both cases actual and wavelet-denoised groundwater table time series:

Mean equation:

Actual groundwater table:

$$y_t = 0.056 + 0.611y_{t-1} + 0.293\varepsilon_{t-1} - 0.818\varepsilon_{t-12} + \varepsilon_t \tag{10}$$

Wavelet-denoised groundwater table:

$$y_t^* = 0.044 + 0.605y_{t-1} + 0.362\varepsilon_{t-1} - 0.822\varepsilon_{t-12} + \varepsilon_t$$

where y_t represents the estimated groundwater table. The p-values of the t-statistic of the estimated coefficients showed that all of them are highly significant for both cases. No evidence autocorrelation was found in this model's residuals (using the LM test) and DW for this model is 2.343 and 1.865, Akaike info criterion (AIC) is 4.416 and 3.742 and Schwarz criterion (SC) are 4.501 and 3.828, respectively. This ARIMA model is used to forecast the groundwater table monthly returns. The RMSE is 2.994 & 0.857, MAE is 1.528 & 1.046 and TIC values are 0.062 & 0.006, respectively.

Table 4. LM (Breusch-Godfrey) test on SARIMA residuals.

Groundwater table	F-statistic	p-value
Actual	6.581	0.0016
Wavelet-denoised	7.143	0.0009

The next step is to test whether the estimated errors are heteroscedastic or not. For this purpose, we test the presence of 'ARCH effect' in the residuals by using the Lagrange Multiplier (LM) test for returns series as suggested by Engle. The results of Lagrange Multiplier test are presented in Table 4. The p-value indicates that there is evidence of remaining ARCH effect. So, we reject the null hypothesis of absence of ARCH effect even at 1% level of significance. Hence, in next section for capturing volatilities in returns series we will use GARCH family of models.

Table 5. Results of SARIMA-GARCH model for actual and wavelet-denoised series

	Actual				**Wavelet-denoised**			
Variable	**Coefficient**	**SE**	**z-Stat**	**p-value**	**Coefficient**	**SE**	**z-Stat**	**p-value**
Drift	0.056	0.009	6.22	0.0000	0.044	0.009	4.89	0.0000
AR(1)	0.611	0.058	10.53	0.0000	0.605	0.057	10.61	0.0000
MA(1)	0.293	0.061	4.8	0.0000	0.362	0.058	6.24	0.0000
SMA(1)	-0.818	0.05	-16.36	0.0000	-0.822	0.0508	-16.18	0.0000
Variance Equation								
C	0.306	0.063	4.874	0.0000	15.216	0.38	40.049	0.0000
RESID(-1)^2	0.558	0.066	8.487	0.0000	-0.006	0.001	-6.441	0.0000
GARCH(-1)	0.539	0.127	4.235	0.0000	-0.234	0.001	-184.049	0.0000
GARCH(-2)	0.037	0.088	0.426	0.6704	-1.005	0.001	-785.421	0.0000
	AIC = 4.416, SC = 4.501, HQC = 4.450				AIC = 3.742, SC = 3.828, HQC = 4.773			

(AIC = Akaike info criterion, SC = Schwarz criterion and HQC = Hannan-Quinn criterion)

3.3. Estimation and forecasting Based on the GARCH model

Groundwater table time series is taken from January, 1991 to May, 2016 and GARCH models are fitted to the returns series. The joint estimation of mean and variance equations using "R version 3.6.1" software is shown below for GARCH models.

SARIMA - GARCH Model

A joint estimation of the SARIMA-GARCH model gives:

Variance equation:

Actual groundwater table:

$$h_t = 0.306 + 0.558\varepsilon_{t-1}^2 + 0.539h_{t-1} + 0.037h_{t-2} \quad (11)$$

Wavelet-denoised groundwater table:

$$h_t = 15.216 - 0.006\varepsilon_{t-1}^2 - 0.234h_{t-1} - 1.005h_{t-2}$$

In Equation (10) h_t is the conditional variance. The amount of p-value for parameters of mean equation and variance equation are 0.0000. So, all of the coefficients are highly significant. Akaike info criterion (AIC) is 4.416 and 3.742 and Schwarz criterion (SC) are 4.501 and 3.828 respectively.

3.4. Comparative Analysis

In order to assess the validity of forecasting the water table monthly returns through the models presented in this paper, the RMSE, MAE and TIC criteria of these models are compared with each other.

Table 6. Comparison of test statistics for hybrid SARMA - GARCH models

Criteria	Actual	Wavelet-denoised
RMSE	2.994	0.857
MAE	1.528	1.046
MAPE	6.928	4.294
TIC	0.062	0.006

According to the achieved results of Table 6, the hybrid SARIMA-GARCH model has the best value for RMSE is 2.994 & 0.857, MAE is 1.528 & 1.046 and TIC values are 0.062 & 0.006, respectively. So, the comparison of the forecasting performance through the RMSE, MAE and TIC criteria indicate that the best model is ARIMA$(1,0,1)(0,1,1)_{12}$ - GARCH(1,2). Therefore, SARIMA-GARCH model captures the volatility in the water table monthly returns and its forecast performance is more than any other models and this selected model has the lowest AIC and SC values of diagnostic checking.

4. Conclusion

This paper focuses on building a volatility model for the groundwater table using time series methodology. Monthly groundwater table for the period ranging from January, 1991 to May, 2016 are used for this purpose. Firstly, the stationary of the groundwater table is examined using unit root test such as ADF and PP tests which showed the series is non-stationary. Hence, to make the series stationary, the groundwater table is transformed to return series. In order to find the most optimal lags, different AR and MA lags were tested using the Box-Jenkins Method. The most appropriate obtained model among different models using AIC and BIC is the ARIMA$(1,0,1)(0,1,1)_{12}$. As the hydrological time series like return series may possess volatility, an attempt is made to model this volatility using GARCH family models. To capture the volatility, hybrid ARIMA(1,0,1)$(0,1,1)_{12}$ - GARCH(1,2) model is used. The wavelet-ARIMA(1,0,1)$(0,1,1)_{12}$ - GARCH(1,2) is found to be the best model with the lowest RMSE, MAE, and TIC. This model captures the volatility in the return series and provides a model with fairly good forecasting performance.

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