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# The Performance of GARCH Modeling in Forecasting Groundwater Table Fluctuation of Northwest Bangladesh

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**Abstract**

Nowadays, modelling and forecasting volatility of groundwater table have become significant interest to the practice of risk management. This paper used different volatility models; GARCH, GJR-GARCH, PGARCH, EGARCH and IGARCH to forecast groundwater table volatility of Northwest Bangladesh and find out the best model for forecast. This study uses weekly groundwater table data, collected from Bangladesh Water Development Board (BWDB) for the period January, 19991 to December, 2018. The period of January, 1991 to December, 2013 was used to build the model while remaining data were used to do out-sample forecasting and check the forecasting ability of the model. We find that the asymmetric model IGARCH following a normal error distribution yields the best forecasting performance. Our proposed model could be useful for forecasting groundwater table fluctuation of northwest Bangladesh, as well as leading to a better understanding of the groundwater table volatility, especially mitigating the problem of previous performance.

**Keyword**: Groundwater Table; Volatility modelling; Volatility foresting; GARCH family model; Northwest Bangladesh.

**AMS Classification:** 62M10.

## 1. Introduction

Forecasting volatility has attracted the interest of many academicians; hence various models ranging from simplest models such as random walk to the more complex conditional heteroscedastic models of the GARCH family have been used to forecast volatility. GARCH was used to forecast volatility for the first time by Akgiray (1989). Over the years different variations of the GARCH model has been used to forecast volatility.

Earlier, groundwater was considered to be a limitless or at least fully renewable natural resource, but in the recent past, there has been a tremendous pressure on this valuable natural resource mainly due to rapid industrialization and population growth. For an effective management of groundwater, it is important to forecast groundwater table fluctuations. Groundwater organisms possess features such as complexity, nonlinearity, being multi-scale and random, all governed by natural factors, which complicate the dynamic forecasts. Therefore, many mathematical models have been developed to simulate this complex process (Khalek& Ali, 2016 and Alvisi et al., 2006). Keeping in mind the scarcity of existing water resources in the near future and it impending threats, it has develop imperative on the part of water scientists as well as planners to quantify the available water resources for its judicial use.

Today, volatility is a vital feature of contemporary environmental time series studies, and its uses are broad and diversified. It is an essential component in the process of value at risk, portfolio management, valuation of options and financial assets, among many other uses. In this regard, the relationship between autocorrelation and variability, and predicting an inverse relationship between variability and autocorrelation in the analysis of climatic series is essential (Kim et al., 2005 and Sentana et al., 1991). According to the multivariate ARCH models, the asymmetric modeling of the conditional variance as a forecast method is important. Therefore, the modeling of the conditional variance as a univariate GJR model is also used. The moment structure of the EGARCH model was investigated (Karanasos et al., 2003).

The Box-Jenkins modeling is one of the most powerful forecasting techniques available and it can be used to analyze almost any set of data (Christodoulos et al., 2010). In hydrology, Box-Jenkins modeling is well used in groundwater table forecasting, a basis model in groundwater table forecasting, and as a benchmark forecast model for groundwater table. In many practical applications, the autoregressive integrated moving average (ARIMA) model is the most widely used Box-Jenkins models since it can handle non-stationary data. According to Shafiee and Topal (2010), the groundwater table followed a random walk and non-stationary characteristics. Therefore, ARIMA has been good potential to be a forecasting model for groundwater table. ARIMA is one of the most important time series models used in hydrological forecasting over the past three decades due to its statistical properties, accurate forecasting over a short period of time and ease of implementation (Khashei et al., 2009).

The models of the ARCH family have been extensively studied and, in particular, we can cite the works of Bollerslev et al. (1992) and Bollerslev et al. (1994). GARCH models were an extension of ARCH family models and were proposed by Bollerslev (1986) and Taylor (1986). The forecasting of volatility, as well as the comparison of the out-of-sample forecast performance of the different models, is a booming subject and several researchers have begun to work on this subject. Akgiray (1989) found that the GARCH model is superior to the EWMA (exponentially weighted moving averages) model, the ARCH model and the historical average model, predicting the monthly volatility of the US stock index. Given the great success of these models, several extensions have been developed to try to perfect this type of models and make it more and more efficient. Among these extensions, we can find the exponential GARCH or EGARCH (Nelson, 1991). For this model, conditional volatility is specified in logarithmic form, which means that there is no need to impose estimation constraints to avoid the problem of negative variance.

This property allows us to take into account the stylized fact that negative shocks imply a greater variation of volatility than positive ones. Another non-symmetric model with characteristics close to EGARCH is TGARCH, also called GJR-GARCH and developed by Zakoian (1994) and Glosten et al. (1993), respectively. The main difference between TGARCH and EGARCH is the following: TGARCH models the conditional standard deviation Instead of the conditional variance. While shocks in the volatility series tend to have long memories and, as a result, tend to impact future volatility for a long horizon, the IGARCH model (or Integrated GARCH) was proposed by Engle and Bollerslev (1986) to capture this stylized fact, as well as to make conditional volatility infinite and shocks permanent. Similarly, Ding et al. (1993) proposed the PGARCH (Power GARCH) model, which came to provide another method for modeling the long memory property in volatility. An excellent review of volatility prediction models can be found in Poon and Granger (2003).

To the best of our knowledge, this article is the first attempt to compare and study several models in order to capture and model the features of the conditional volatility of the groundwater table, producing consequently high-quality forecasts that are necessary for effective and sustainable management of groundwater table.

# 2. Methods and Materials

## 2.1 Study Area

We used secondary data as per requirements of modeling and forecasting of groundwater table for northwest (NW) Bangladesh. Rajshahi, Bogra, Pabna, Naogaon, Joypurhat, Natore, Sirajgonj and ChapaiNawabgonj is the northwest eight administrative district of Bangladesh, is located at 24.163$^{0}$N latitude and 88.40$^{0}$E longitude. The northwest part of Bangladesh is an interesting study area for its natural beauties of Barind track.

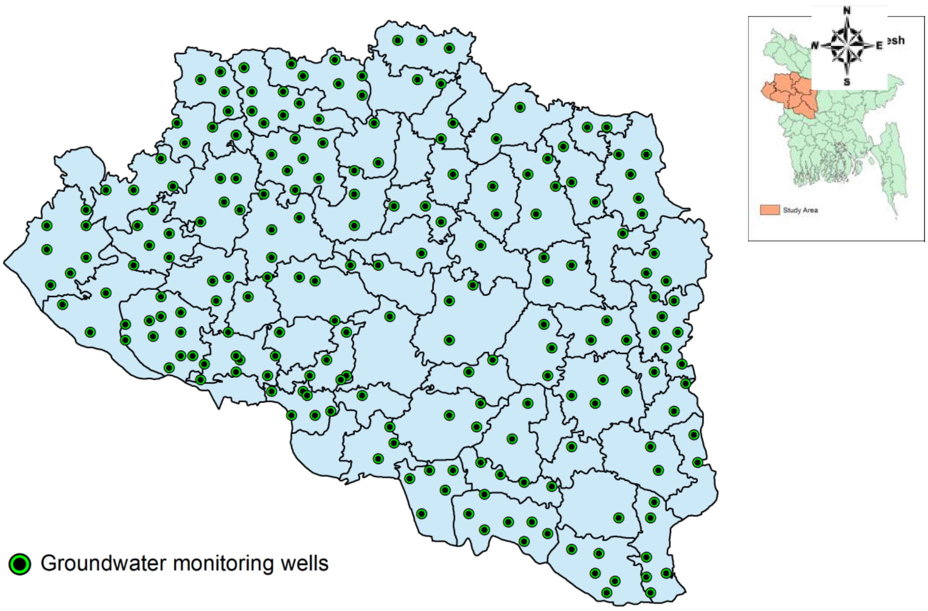



**Figure 1**: Map of Northwest Bangladesh with monitoring wells
(Source: Author)

## 2.2 Data Source

The study used monthly groundwater table for the period January, 1991 to May, 2016 of northwest Bangladesh and collected from Bangladesh Water Development Board (BWDB). Since the groundwater table time series is usually non-stationary, and hence is not appropriate for analysis, we converted the series into the rate of return on groundwater table by following logarithmic transformation. The analysis presented on this exercise involves monthly volatility forecast. Monthly return is calculated as follows:

$$r_t = log\left(\frac{y_t}{y_{t-1}}\right)$$

where $y_t$ is the groundwater level (in meter) at time $t$ and $r_t$ is the continuously compounded return at $t$. Within- standard deviation of monthly return is used as monthly realized volatility:

$$\sigma_{a,t} = \sqrt{\frac{\sum_{t=1}^{n}(r_t - \mu)^2}{n - 1}}$$

Where $\mu$ is the mean of groundwater table and n is the number of observation employed sample groundwater table in meter. There are 305 months groundwater table volatility observations in our data sample. The first one was covering the period from January, 1991 to December, 2014 (264 months) groundwater table was used to estimate our model, as well as to compute the descriptive statistics. However, the second series was covering January, 2013 and May, 2016 (41 months) and was used to evaluate the out-of-sample forecast performance of each of our models. The statistical software R 3.6.0 was used to perform the quantitative exercise.

## 2.3 GARCH (Generalized Autoregressive Conditional Heteroscedasticity)

Bollerslev (1986) and Taylor (1986) developed the GARCH($p$, $q$) model, allowing the conditional variance of the variable to be dependent on previous delays and capturing information and news contained in historical values of the variance. This model is presented as follows:

$$h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j h_{t-j} \qquad \ldots \qquad (1)$$

As the notation shows, the GARCH($p$, $q$) model contains, in addition to the term GARCH($h_{t-1}$) or delays in the conditional variance, an squared ARCH($\varepsilon_{t-1}^2$). In the literature, the GARCH(1, 1) model remains by far the most used model and hence, our choice to use this type of models. The notation of the GARCH(1, 1) model is presented below:

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}$$

This model has a non-negativity constraint for the coefficients $\alpha$ and $\beta$ so that the variance is always positive and the coefficient $\alpha_0$ must be greater than 1.

## 2.4 GJR GARCH (Glosten-Jagannathan-Runkle GARCH)

Glosten et al. (1993) have developed this model to allow conditional volatility to have different reactions to past innovations based on their signs. This model is presented as follows:

$$h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \gamma_i \varepsilon_{t-i}^2 d_{t-1} + \sum_{j=1}^{q} \beta_j h_{t-j} \qquad \dots \qquad (2)$$

where $d_{t-1}$ is a dummy variable:

$$d_{t-1} = \begin{cases} 1\,, & if\ \varepsilon_{t-1}^2 < 0\ (negative\ shocks) \\ 0\,, & if\ \varepsilon_{t-1}^2 \geq 0\ (positive\ shocks) \end{cases}$$

and $\gamma$ is the coefficient that measures the impact of news arrival. The rest of the parameters in the equation remain the same as those of the GARCH model.
In this model, the effect of good news shows its effect through $\alpha_i$, whereas the effect of negative shocks is shown by $\alpha + \gamma$. Moreover, if $\gamma \neq 0$, the impact of the arrival of news is said to be asymmetric; and when $\gamma > 0$, then volatility is marked by a leverage effect.

In order to be in line with the condition of non-negativity of the coefficients, it is necessary that $\alpha_0 > 0$, $\alpha_i > 0$, $\beta \geq 0$ and $\alpha_i + \gamma_i \geq 0$. The model could be still acceptable if $\gamma < 0$ and $\alpha_i + \gamma \geq 0$.

## 2.5 EGARCH (Exponential GARCH)

For the Exponential GARCH or EGARCH model proposed by Nelson (1991), the conditional volatility specification is given by the following formula:

$$log(h_t) = \alpha_0 + \sum_{i=1}^{p} \alpha_i \left| \frac{\varepsilon_{t-i}}{\sqrt{h_{t-i}}} \right| + \sum_{j=1}^{q} \beta_j log(h_{t-j}) + \sum_{k=1}^{r} \gamma_k \frac{\varepsilon_{t-k}}{\sqrt{h_{t-k}}} \qquad (3)$$

where $\log(h_t)$ represents the logarithm of conditional volatility, $\log(h_{t-1})$ represents the logarithm of the first lag in conditional volatility, and $\varepsilon_{t-i}$ is the term of the error at time $i$.

The use of the EGARCH model has the advantage to authorize the effects of information asymmetries to happen. In the EGARCH equation, $\gamma_k$ represents the leverage parameter used to capture the asymmetry, which is not the case for the basic GARCH model (Thomas and Mitchell, 2005).

The main contribution of this model is that it takes into account the fact that negative shocks have a greater impact on volatility than that of positive shocks.

## 2.6 Power GARCH(p, d, q)

This model was proposed by Ding et al. (1993), and has the advantage of being able to capture and model the long memory property often observed in the series of volatility. It is presented as follows:

$$h_t^d = \alpha_0 + \alpha(|\varepsilon_{t-1}| + \gamma\varepsilon_{t-1})^d + \beta h_{t-1}^d \qquad \ldots \qquad (4)$$

where $d$ is a power term, $\varepsilon_{t-1}$ represents the first lag of the error term (ARCH term), and $h_{t-1}$ is the first lag of the conditional volatility. The power term, denoted $d$, captures the standard deviation when $d$ = 1 and captures the conditional variance when $d$ = 2. The asymmetry is counted by the term $\gamma$ (Carroll and Kearney, 2009).

## 2.7 IGARCH (Integrated GARCH)

The IGARCH models, introduced by Engle and Bollerslev (1986), have the advantage of providing a statistical response to the problem of the presence of a unit root in the time series of volatility, which makes volatility shocks permanent. It is an integrated model of volatility. The formulation of this model is presented below:

$$h_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j h_{t-j}^2 \qquad \ldots \qquad (5)$$

IGARCH models are said to be volatile models, because current information remains valid for forecasting volatility across all horizons. If $\alpha_0$ = 0, we can say that the series is integrated in variance to the order $d$. And when $\alpha_0 > 0$, then the series is integrated in the order *d* with trend; where d is the number of first differences needed in order to render it stationary. As far as error distributions are concerned, GARCH model theory suggests three assumptions about the distribution of residuals. These three assumptions imply that the residuals of the GARCH may follow a normal law, a Student law or a generalized error distribution (GED).

## 2.8 Forecasting Evaluation

The approach followed in this empirical study, is to start by first estimating the conditional volatility of the groundwater table, according to the different GARCH models and according to different error distributions; and then selecting the best models in function of the significant parameters as well as Akaike (AIC) and Schwarz (BIC) information criteria and that of the maximum likelihood estimation. Once we have obtained the best GARCH models, which allow us to better express the volatility of groundwater table, we will compare the forecasting performance of these models with that of the GARCH model, using the following statistics: Root Mean Squared Error (RMSE), Mean Absolute Error (MAE) and Theil Inequality Coefficient (TIC).

$$RMSE = \sqrt{\frac{\sum_{t=1}^{n}\left(\sigma_{a,t} - \sigma_{f,t}\right)^2}{n}}$$

$$MAE = \frac{\sum_{t=1}^{n}\left|\sigma_{a,t} - \sigma_{f,t}\right|}{n}$$

$$TIC = \frac{\sqrt{\frac{1}{n}\sum_{t=1}^{n}\left(\sigma_{a,t} - \sigma_{f,t}\right)^2}}{\sqrt{\frac{1}{n}\sum_{t=1}^{n}\sigma_{a,t}^2} + \sqrt{\frac{1}{n}\sum_{t=1}^{n}\sigma_{f,t}^2}}$$

Where $\sigma_{a,t}$ is the actual volatility and $\sigma_{f,t}$ is the forecasted volatility.

If we look at MAE and MAPE, Exponential smoothing model clearly creates the best forecast, and it is followed by EGARCH. Historical mean model, once again, is the worst forecasting model. According to these two criteria's, ARCH-type models provide better forecasting than non-ARCH models.

The Theil Inequality Coefficient (TIC) is a scale invariant measure that always lies between zero and one, where zero indicates a perfect fit. Observing at this coefficient we can approximately that Random walk is the preeminent forecasting model. It is interesting to note that Exponential smoothing is no longer the best model; on the contrary, it yields the worst forecasting according to TIC. It is not easy to make a judgment if ARCH models are better than non-ARCH models; however, if we calculate the mean of the ranks of ARCH models, we can see that this mean is smaller than non-ARCH models' mean which indicates that ARCH models gives better forecasting results.

## 3. Empirical Results

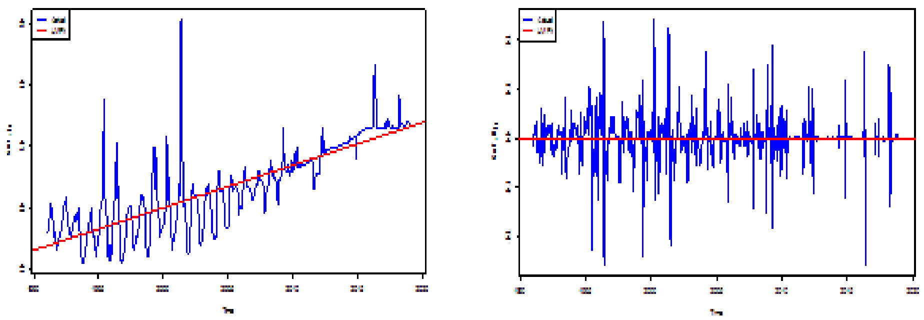
To test the stationarity of the return series we used the Augmented Dickey Fuller (ADF) unit root test (Table 1).

**Table 1:** Results of the ADF test

| Variable | ADF value | t-stat at 1% |
|---|---|---|
| Returns of groundwater table ($r_t$) | -15.425 | -3.96 |

**Table 2:** Results of the White's heteroscedasticity test

| Dependent Variable | White's statistic | Obs R^2 |
|---|---|---|
| Returns of groundwater table ($r_t$) | 308570 | 50321 |



(a) Actual series (b) log return series

**Figure 2:** Plot for groundwater table of northwest Bangladesh with simple linear regression line (a) actual series, (b) log return series

Similarly, we applied the White test for the purpose of testing the ARCH effect or the heteroskedasticity property of the errors (Table 2); this test was conducted on the residuals series taken from the following mean model regression:

$$r_t = c + \alpha r_{t-1} + \varepsilon_t$$

From the results presented in Tables 1 and 2, we conclude that the newly created return series is a stationary series. Similarly, the statistical significance of the White test led us to reject the null hypothesis of the heteroscedasticity of errors and to accept the alternative hypothesis of the heteroscedasticity of errors. So, at this stage, we can safely proceed to the estimation of our models, as the conditions for ARCH and GARCH modeling hold.

**Table 3:** Summary statistics of actual and returns series

| Descriptive statistics | Actual Series | Return Series |
|---|---|---|
| Mean | 23.1 | 0.0005 |
| Median | 23.1 | 0.0007 |
| Maximum | 50.6 | 0.241 |
| Minimum | 10.8 | -0.2584 |
| Standard deviation | 6.85 | 0.0401 |
| Asymmetry | 0.21 | -0.443 |
| Kurtosis | 2.44 | 14.461 |
| Jarque-Bera | 29.84 | 8038.439 |
| Probability | 0.000 | 0.000 |

Table 3, including the descriptive statistics for the groundwater table returns series, shows a significant difference between the maximum and minimum values, which is synonymous with high volatility in the series; in addition, the existence of a significant difference between the value of the standard deviation and the mean could only reinforce this finding. The kurtosis value, being very large compared with the value of 3, suggests the presence of a fat tail on the right side with respect to the mean and hence, the non-normality of the series. This non-normality is confirmed by the Jarque-Bera test which is significantly different from zero; so the normality hypothesis of the series cannot be accepted.

For the empirical results, they will be presented hereafter in function of the error distributions. The AIC, BIC and maximum likelihood criteria are used to find the optimal model, so that AIC and BIC are minimized and the maximum likelihood is maximized independently of the error distributions.

The first observation to be drawn from Table 4 is that the majority of the parameters are significantly different from zero, which underlines the high validity of our models. The sum of the terms α and β for the models GARCH, PGARCH and IGARCH is very close to 1, which is explained by a rather significant presence of persistence in the volatility of the groundwater table. However, the value of $\alpha$ is rather less than that of $\beta$, which means that the negative shocks on the conditional volatility of groundwater table do not have a greater impact on volatility than those of positive shocks.

For the asymmetric GARCH models, half of the parameters γ are statistically different from zero, which implies that the volatility of the groundwater table is asymmetric and, hence, the existence of leverage effects. The parameter γ of the PGARCH(1, 1, 1) model, being statistically significant and having a positive value, suggests that the impact of positive shocks on the volatility of the groundwater table is greater than that of negative shocks.

**Table 4:** Results of GARCH family model with Gaussian error distribution

| **Conditional volatility model** | **c** $\alpha_0$ | **ARCH(1)** $\alpha$ | **GARCH(1)** $\beta$ | **Leverage** $\gamma$ | **AIC** | **BIC** | **Maximum likelihood** |
|---|---|---|---|---|---|---|---|
| GARCH(1, 1) | 3.66E-06 | 0.281 | 0.682 | - | -7.289 | -7.285 | 20316.01 |
| | (0.000) | (0.000) | (0.000) | | | | |
| GJR-GARCH | 3.61E-06 | 0.033 | 0.685 | 0.262 | -7.289 | -7.284 | 20317.13 |
| | (0.000) | (0.000) | (0.000) | (0.047) | | | |
| **EGARCH** | **-1.148** | **0.410** | **0.915** | **-0.019** | **-7.495** | **-7.489** | **20733.50** |
| | **(0.000)** | **(0.000)** | **(0.000)** | **(0.005)** | | | |
| PGARCH(1,1,1) | 0.0006 | 0.245 | 0.737 | 0.037 | -7.295 | -7.289 | 20333.39 |
| | (0.000) | (0.000) | (0.000) | (0.029) | | | |
| PGARCH(1,2,1) | 0.0004 | 0.261 | 0.751 | 0.057 | -7.432 | -7.424 | 20714.06 |
| | (0.045) | (0.000) | (0.000) | (0.027) | | | |
| IGARCH | - | 0.073 | 0.927 | - | -7.229 | -7.227 | 20146.54 |
| | | (0.000) | (0.000) | | | | |

Note: Values in parentheses represent p-values.

For the Gaussian distribution, the best model of the conditional volatility of the groundwater table is EGARCH, which presents significant parameters and has the smallest AIC and BIC values while having the greater maximum-likelihood value. This model is closely followed by GARCH (1, 1), IGARCH and PGARCH(1, 1, 1). So, these are the models that will be evaluated later to test and compare their predictive performance. The other models are eliminated due to having non-significant parameters.

Results obtained when following a Student-t error distribution, are closely related to those of the normal distribution. For this error distribution, we can clearly see that the model GARCH(1, 1) is the best to capture and model conditional volatility of our data (Table 5). The parameters $\gamma$, being entirely not statistically significant, imply the non-existence of leverage effects in conditional volatility of groundwater table.

For the case of this distribution, the only models that will be kept for the final study are GARCH(1, 1) and IGARCH. The EGARCH model represents the best way to model the conditional volatility of groundwater table in the case of the generalized error is distribution (Table 6). With the exception of the EGARCH model, all the parameters $\gamma$ are not statistically significant, which implies the non-existence of leverage effects and the asymmetry of the volatility of groundwater table return series.

**Table 5:** Results of GARCH family model with Student's-t error distribution

| **Conditional volatility model** | **c** $\alpha_0$ | **ARCH(1)** $\alpha$ | **GARCH(1)** $\beta$ | **Leverage** $\gamma$ | **AIC** | **BIC** | **Maximum likelihood** |
|---|---|---|---|---|---|---|---|
| **GARCH(1, 1)** | **3.05E-06** | **0.377** | **0.661** | **-** | **-7.817** | **-7.811** | **20672.48** |
| | **(0.000)** | **(0.000)** | **(0.000)** | | | | |
| GJR-GARCH | 3.01E-06 | 0.346 | 0.664 | 0.059 | -7.417 | -7.410 | 20673.13 |
| | (0.000) | (0.000) | (0.000) | (0.082) | | | |
| EGARCH | -0.999 | 0.471 | 0.933 | -0.022 | -7.425 | -7.418 | 20696.51 |
| | (0.000) | (0.000) | (0.000) | (0.007) | | | |
| PGARCH(1,1,1) | 0.0004 | 0.287 | 0.748 | 0.040 | -7.427 | -7.421 | 20703.41 |
| | (0.000) | (0.000) | (0.000) | (0.084) | | | |
| PGARCH(1,2,1) | 3.01E-06 | 0.375 | 0.664 | 0.040 | -7.712 | -7.705 | 21714.21 |
| | (0.045) | (0.000) | (0.000) | (0.042) | | | |
| IGARCH | - | 0.116 | 0.884 | - | -7.377 | -7.374 | 20564.14 |
| | | (0.000) | (0.000) | | | | |

Note: Values in parentheses represent p values.

**Table 6:** Results of GARCH family model with Generalized Error Distribution (GED)

| Conditional volatility model | c<br>$\alpha_0$ | ARCH(-1)<br>$\alpha$ | GARCH(-1)<br>$\beta$ | Leverage<br>$\gamma$ | AIC | BIC | Maximum likelihood |
|---|---|---|---|---|---|---|---|
| GARCH(1, 1) | 2.94E-06 | 0.321 | 0.675 | - | -7.423 | -7.417 | 20683.90 |
| | (0.000) | (0.000) | (0.000) | | | | |
| GJR-GARCH | 2.90E-06 | 0.290 | 0.678 | 0.062 | -7.423 | -7.416 | 20689.32 |
| | (0.000) | (0.000) | (0.000) | (0.073) | | | |
| EGARCH | -0.996 | 0.423 | 0.931 | -0.027 | -7.430 | -7.423 | 20709.42 |
| | (0.000) | (0.000) | (0.000) | (0.027) | | | |
| PGARCH(1,1,1) | 0.0004 | 0.261 | 0.751 | 0.058 | -7.432 | -7.424 | 20713.07 |
| | (0.000) | (0.000) | (0.000) | (0.048) | | | |
| PGARCH(1,2,1) | 2.90E-06 | 0.320 | 0.678 | 0.048 | -7.423 | -7.416 | 20687.73 |
| | (0.000) | (0.000) | (0.000) | (0.047) | | | |
| IGARCH | - | 0.094 | 0.906 | - | -7.390 | -7.386 | 20594.24 |
| | | (0.000) | (0.000) | | | | |

In addition to the EGARCH model, the GARCH(1, 1) and IGARCH model will also be kept in order to compare their predictive performance in final test. At this stage and after studying and comparing the models of the GARCH family with the hope to find the best adjustments of these models, we will proceed to the last step which represents the aim and the object of this paper. In this second step, we will present a comparison of the forecasting performances for the following models: GARCH(1, 1), GJR-GARCH, EGARCH, PGARCH(1, 1, 1), PGARCH(1, 2, 1) and IGARCH.

**Table 7:** Evaluation table for forecasting performances

| Volatility model | RMSE | MAE | TIC |
|---|---|---|---|
| **Gaussian Distribution** | | | |
| GARCH(1, 1) | 0.005872 | 0.004327 | 0.954 |
| GJR-GARCH | 0.005288 | 0.003893 | 0.845 |
| EGARCH | 0.005875 | 0.004326 | 0.941 |
| PGARCH(1, 1, 1) | 0.005873 | 0.004327 | 0.951 |
| **IGARCH** | **0.005876** | **0.004326** | **0.939** |

| | | | |
|---|---|---|---|
| **Student's Distribution** | | | |
| GARCH(1, 1) | 0.005871 | 0.004329 | 0.961 |
| GJR-GARCH | 0.005577 | 0.004113 | 0.913 |
| EGARCH | 0.005523 | 0.004066 | 0.883 |
| PGARCH(1, 1, 1) | 0.005636 | 0.004156 | 0.923 |
| IGARCH | 0.005872 | 0.004328 | 0.955 |
| **Generalized Error Distribution** | | | |
| GARCH(1, 1) | 0.005870 | 0.004335 | 0.981 |
| GJR-GARCH | 0.005518 | 0.004075 | 0.922 |
| EGARCH | 0.005870 | 0.004336 | 0.983 |
| PGARCH(1, 1, 1) | 0.005577 | 0.004119 | 0.934 |
| IGARCH | 0.005870 | 0.004334 | 0.979 |

From Table 7, we observe that the ten presented models are very close to each other, but the analysis of the RMSE, MAE and TIC statistics makes possible to conclude that the IGARCH with a normal error distribution is the best model to forecast the volatility of the groundwater table. This model, compared with the others, presents the best results, by presenting the best values of the forecasting error statistics adopted in this study. Therefore, we can say that the models of conditional volatility are better than those of the IGARCH volatility for the case of the groundwater table.

## 4. Conclusion

Forecasting fluctuations of groundwater table volatility has fascinated the attention of hydrology researchers. Throughout this paper, we have tried to look for the best model to predict and forecast the volatility of the groundwater table. In order to achieve this, we have used GARCH family models, which are widely studied and analyzed, and whose performances are largely documented in the environmental and financial literature.

The IGARCH model was added to our sample models thanks to the interesting number of studies that have proved its superiority to the GARCH models, and to its main property of non-return to average. Among the results obtained at the end of this study, we found that the IGARCH models succeed in modeling and explaining, in a rather satisfactory manner, the volatility of the groundwater table fluctuation compared with the mean model, which has nevertheless succeeded in producing estimates being very close to those of the GARCH family models. The best model to forecast the volatility of groundwater table for the measurement of forecasting errors have declared the IGARCH model with Gaussian distribution of errors as a rightful winner and hence, the superiority of GARCH models in comparison to the mean model.

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