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Decision Support in a Credit Environment with Fuzzy Behaviour of Cost

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Abstract

In our present study demand is assumed to be proportional to the time which is dependent on permissible trade credit. Necessary and sufficient conditions have been discussed to frame up permissible trade credit period and purchase quantity. The default risk emerging in sales revenue is incorporated in the objective of profit maximization. Both crisp and fuzzy models have been proposed to determine the optimal solution. Ordering cost, purchase cost, holding cost and selling price are considered as triangular as well as trapezoidal fuzzy numbers. Defuzzification of the seller's annual profit has been carried out by graded mean integration method and signed distance method. The eminence of fuzzy model over the crisp model in exalting profit, reducing credit period and achieving optimal solution, has been avowed through numerical examples.

Keywords and Phrases: Credit Period, Default Risk, Fuzzy Number, Defuzzification.

AMS Classification: 90B05

1 Introduction

To meet the dynamic pace of the retail sector, the seller being the decision maker, offers the credit period to settle the account which woos the buyers and enhances the market demand. This also evolves default risk for the suppliers. Most of the models discussed in the retail market situations have been studied in the crisp environment. However, in the real world, especially for new products, the relevant precise information is not possible to get due to lack of historical data. Moreover, in today's highly competitive market useful statistical data are not available. Thus fuzzy theory rather than crisp theory is well suited to this type of supply chain.

The traditional inventory model tacitly assumes that as soon as the buyers purchase the item, they have to pay for it. But in real situation it is not always practicable to make payments at the time of purchasing. Therefore, the offer of credit period has been introduced, which has changed the entire market scenario. The offer of credit period is beneficial for the sellers as well as for the buyers, as it enhances the market demand for that product without offering discount and favors the buyers to receive items with lesser price as they will pay later.

Goyal (1985) first introduced the economic order quantity model by allowing permissible delay in payment. Shah et al. (2015) developed an economic order quantity model to find out the optimal credit period and purchase quantity for the seller. Jaggi et al. (2008) introduced retailer's optimal replenishment decisions with credit linked demand under permissible delay in payments. Tripathy and Pradhan (2011) elaborated on an integrated partial backlogging inventory model having weibull demand and variable deterioration rate with the effect of trade credit. Shah et al. (2014) developed an optimal pricing and ordering policies for inventory system under trended demand when the supplier offers a credit period to the retailer and the retailer also gives a credit period to his customers. Chung and Haung (2003) developed an optimal cycle time for economic order quantity model under permissible delay in payments. Chang (2004) carried out an economic order quantity model with deteriorating items under inflation when supplier credits are linked to order quantity.

In the above cases, it has been assumed that the inventory parameters are crisp or precise or probabilistic but in reality they may deviate a little from their actual value without following any probability distribution. To deal with such type of uncertainty in inventory parameters, the notion of fuzziness has been initialized by several authors (Zimmermann (2001) and Lee (2005)). This model has also been maneuvered by various researchers. Tripathy et al. (2011) established a fuzzy economic order quantity model with reliability where the unit cost depends on demand. Dutta and Kumar (2012) developed a fuzzy inventory model without shortage where holding cost and ordering cost were taken as trapezoidal fuzzy numbers. Tripathy and Pattnaik (2009) focused on optimal disposal mechanism by considering the system cost as fuzzy under flexibility and reliability criteria. Kundu and Goswami (2003) introduced an EPQ inventory model involving fuzzy demand rate and deterioration rate. Yao et al. (2000) established a fuzzy inventory model without backorder where order quantity and total demand were treated as fuzzy numbers. Tripathy and Behera (2016) formulated a fuzzy inventory model for time deteriorating items using penalty cost under the condition of infinite production rate. Dutta and Kumar (2013) explored an optimal ordering policy for an inventory model for deteriorating items without shortage where demand rate, ordering cost and holding cost were taken as fuzzy in nature. Jaggi et al. (2012) introduced a fuzzy inventory model for deteriorating items with time varying demand and shortages. Mahata and Goswami (2007) elaborated on an economic order quantity model under the condition of permissible delay in payments in fuzzy sense. Shah et al. (2012) proposed an economic order quantity model under the condition of permissible delay in payments in fuzzy sense.

In the study developed by Shah et al. (2015), demand is assumed to be proportional to the time and is dependent on permissible trade credit. Necessary and sufficient conditions have been discussed to frame up permissible trade credit period and purchase quantity. The default risk emerging in sales revenue is incorporated in the objective of profit maximization. Here the optimal solution is obtained only in crisp environment. The present study elaborates the above model in fuzzy environment and compares the result obtained in crisp and fuzzy environment to prove the superiority of the proposed model over the crisp model in achieving the optimal solution. Here ordering cost, purchase cost, holding cost and selling price are considered as triangular as well as trapezoidal fuzzy numbers. Defuzzification of the Seller's annual Profit has been accomplished by graded mean integration method and signed distance method. Numerical examples have been cited as a proof of the validation of the model. Sensitivity analysis has also been carried out to prove the credibility of the proposed model over crisp model.

2 Notations and Assumptions

The following notations and assumptions are used to develop the model.

2.1 Notations

(i) A and $\tilde{A}$-ordering cost and fuzzy ordering cost per order

(ii) C and $\tilde{C}$-purchase cost and fuzzy purchase cost per unit

(iii) P and $\tilde{P}$-selling price and fuzzy selling price per unit

(iv) h and $\tilde{h}$-holding cost and fuzzy holding cost per unit per annum

(v) M- credit period offered by the seller to his buyers (a decision variable)

(vi) $R(M,T)$-time and credit period dependent annual demand rate

(vii) $I(t)$-inventory level at any instant of time t, $0 \leq t \leq T$

(viii) T-cycle time (a decision variable)

(ix) Q-seller's purchase quantity

(x) π(M, T) and $\tilde{\pi}\,(M, T)$-the total average profit of the seller per unit time in crisp and fuzzy environment.

(xi) $\tilde{\pi}(M, T)_{SD}$ and $\tilde{\pi}(M, T)_{GM}$ -defuzzified profit by using signed distance and graded mean integration method

2.2 Assumptions

(i) The seller deals with single item, for which replenishment rate is infinite.

(ii) Shortages are not permitted. Lead time is zero or negligible.

(iii) In global market, seller keeps selling price constant to bind his retailers.

(iv) Trade credit is similar to price discount. Demand rate is considered to be function of time and credit period

$$R\,(M, t) = a\,(1 + bt)\,M^{\beta} \tag{1}$$

where a>0 is scale demand, $0 \leq b < 1$ denotes rate of change of demand with respect to time and β>0 is constant.

(v) For seller, default risk increases when longer credit period offered to the buyer. Here, the rate of default risk giving the credit period M is assumed to be

$$F\,(M) = 1 - M^{-\gamma} \tag{2}$$

where γ>0 is a constant.

3 Mathematical Model

3.1 Crisp Model

The seller's inventory is depleting due to increasing demand and offer of credit period. The rate of change of inventory is governed by the differential equation

$$\frac{dI(t)}{dt} = -R(M, t) \tag{3}$$

with $I\,(T) = 0$. The solution of differential equation (3) is

$$I\,(t) = aM^{\beta}(T - t + \frac{1}{2}b(T^2 - t^2)) \tag{4}$$

Initially, the seller has Q units in inventory system i.e.

$$Q = I\,(0) = aM^{\beta}(T + \frac{1}{2}bT^2) \tag{5}$$

The relevant costs per cycle for the seller are

- Net revenue after default risk: SR=$P \int_o^T R\,(M,t)\,dt(1-F(M))$
- Purchase cost ; PC= CQ=$CaM^{\beta}(T+\frac{1}{2}bT^2)$
- Ordering cost ; OC=A
- Holding cost ; HC= $haM^{\beta}(\frac{T^2}{2}+\frac{1}{3}bT^3)$

Hence, the seller's annual profit per unit time is

$$\pi(M,T)=\frac{1}{T}(SR-PC-OC-HC)$$

$$=\frac{1}{T}\left(aPM^{-\gamma+\beta}(T+\frac{1}{2}bT^2)-CaM^{\beta}(T+\frac{1}{2}bT^2)-ha\beta(\frac{T^2}{2}+\frac{1}{3}bT^3)-A\right) \quad (6)$$

For maximizing annual profit per unit time with respect to credit period and cycle time, the necessary and sufficient conditions are

$\frac{\partial\pi(M,T)}{\partial M}=0,\ \ \frac{\partial\pi(M,T)}{\partial T}=0$ and

$$\begin{vmatrix} \frac{\partial^2\pi(M,T)}{\partial M^2} & \frac{\partial^2\pi(M,T)}{\partial M\,\partial T} \\ \frac{\partial\pi(M,T)}{\partial T\,\partial M} & \frac{\partial\pi(M,T)}{\partial T^2} \end{vmatrix} > 0 \quad (7)$$

3.2 Fuzzy Model

Due to uncertainty in the environment, we assume some of the parameters of the inventory system like A, C, h and P may change within certain limits. Here we have considered $\tilde{h}$, $\tilde{A}$, $\tilde{C}$ and $\tilde{P}$ are trapezoidal as well as triangular fuzzy number.

The fuzzy total profit is given by

$$\tilde{\pi}(M,T)=\frac{1}{T}\left(a\tilde{P}M^{-\gamma+\beta}(T+\frac{1}{2}bT^2)-\tilde{C}aM^{\beta}(T+\frac{1}{2}bT^2)-\tilde{h}a\beta(\frac{T^2}{2}+\frac{1}{3}bT^3)A\right) \quad (8)$$

The total profit $\tilde{\pi}(M,T)$ has been defuzzified by employing graded mean integration method and signed distance method.

When $\tilde{P}=(P_1,P_2,P_3,P_4,)$, $\tilde{C}=(C_1,C_2,C_3,C_4)$, $\tilde{h}=(h_1,h_2,h_3,h_4\ )$ and $\tilde{A}$=(A_1, A_2, A_3, A_4) are trapezoidal fuzzy numbers, by using signed distance method for defuzzification, we have

$$\tilde{\pi}_{SD}(M,T)=\frac{1}{4}\left[\tilde{\pi}_{SD_1}(M,T)+\tilde{\pi}_{SD_2}(M,T)+\tilde{\pi}_{SD_3}(M,T)+\tilde{\pi}_{SD_4}(M,T)\right] \quad (9)$$

where

$$\tilde{\pi}_{SD_1}(M,T) = \frac{1}{T}\left(aP_1M^{-\gamma+\beta}(T+\frac{1}{2}bT^2) - aC_1M^{\beta}(T+\frac{1}{2}bT^2) - h_1a\beta(\frac{T^2}{2}+\frac{1}{3}bT^3) - A_1\right)$$

$$\tilde{\pi}_{SD_2}(M,T) = \frac{1}{T}\left(aP_2M^{-\gamma+\beta}(T+\frac{1}{2}bT^2) - aC_2M^{\beta}(T+\frac{1}{2}bT^2) - h_2a\beta(\frac{T^2}{2}+\frac{1}{3}bT^3) - A_2\right)$$

$$\tilde{\pi}_{SD_3}(M,T) = \frac{1}{T}\left(aP_3M^{-\gamma+\beta}(T+\frac{1}{2}bT^2) - aC_3M^{\beta}(T+\frac{1}{2}bT^2) - h_3a\beta(\frac{T^2}{2}+\frac{1}{3}bT^3) - A_1\right)$$

$$\tilde{\pi}_{SD_4}(M,T) = \frac{1}{T}\left(aP_4M^{-\gamma+\beta}(T+\frac{1}{2}bT^2) - aC_4M^{\beta}(T+\frac{1}{2}bT^2) - h_4a\beta(\frac{T^2}{2}+\frac{1}{3}bT^3) - A_4\right)$$

Using graded mean integration method, the total profit is given by

$$\tilde{\pi}_{GM}(M,T) = \frac{1}{6}\left[\tilde{\pi}_{GM_1}(M,T) + 2\tilde{\pi}_{GM_2}(M,T) + 2\tilde{\pi}_{GM_3}(M,T) + \tilde{\pi}_{GM_4}(M,T)\right] \quad (10)$$

where

$$\tilde{\pi}_{GM_1}(M,T) = \frac{1}{T}\left(aP_1M^{-\gamma+\beta}(T+\frac{1}{2}bT^2) - aC_1M^{\beta}(T+\frac{1}{2}bT^2) - h_1a\beta(\frac{T^2}{2}+\frac{1}{3}bT^3) - A_1\right)$$

$$\tilde{\pi}_{GM_2}(M,T) = \frac{1}{T}\left(aP_2M^{-\gamma+\beta}(T+\frac{1}{2}bT^2) - aC_2M^{\beta}(T+\frac{1}{2}bT^2) - h_2a\beta(\frac{T^2}{2}+\frac{1}{3}bT^3) - A_2\right)$$

$$\tilde{\pi}_{GM_3}(M,T) = \frac{1}{T}\left(aP_3M^{-\gamma+\beta}(T+\frac{1}{2}bT^2) - aC_3M^{\beta}(T+\frac{1}{2}bT^2) - h_3a\beta(\frac{T^2}{2}+\frac{1}{3}bT^3) - A_1\right)$$

$$\tilde{\pi}_{GM_4}(M,T) = \frac{1}{T}\left(aP_4M^{-\gamma+\beta}(T+\frac{1}{2}bT^2) - aC_4M^{\beta}(T+\frac{1}{2}bT^2) - h_4a\beta(\frac{T^2}{2}+\frac{1}{3}bT^3) - A_4\right)$$

When $\tilde{P} = (P_1\ ,P_2\ ,P_3)$, $\tilde{C} = (C_1\ ,C_2\ ,C_3\ )$, $\tilde{h} = (h_1\ ,h_2\ ,h_3\ )$ and $\tilde{A}$=(A_1, A_2, A_3) are triangular fuzzy numbers, by using signed distance method for defuzzification, we have

$$\tilde{\pi}_{SD}(M,T) = \frac{1}{4}\left[\tilde{\pi}_{SD_1}(M,T) + 2\tilde{\pi}_{SD_2}(M,T) + \tilde{\pi}_{SD_3}(M,T)\right] \quad (11)$$

where

$$\tilde{\pi}_{SD_1}(M,T) = \frac{1}{T}\left(aP_1M^{-\gamma+\beta}(T+\frac{1}{2}bT^2) - aC_1M^{\beta}(T+\frac{1}{2}bT^2) - h_1a\beta(\frac{T^2}{2}+\frac{1}{3}bT^3) - A_1\right)$$

$$\tilde{\pi}_{SD_2}(M,T) = \frac{1}{T}\left(aP_2M^{-\gamma+\beta}(T+\frac{1}{2}bT^2) - aC_2M^{\beta}(T+\frac{1}{2}bT^2) - h_2a\beta(\frac{T^2}{2}+\frac{1}{3}bT^3) - A_2\right)$$

$$\tilde{\pi}_{SD_3}(M,T) = \frac{1}{T}\left(aP_3M^{-\gamma+\beta}(T+\frac{1}{2}bT^2) - aC_3M^{\beta}(T+\frac{1}{2}bT^2) - h_3a\beta(\frac{T^2}{2}+\frac{1}{3}bT^3) - A_1\right)$$

Using graded mean integration method, the total profit is given by

$$\tilde{\pi}_{GM}(M,T) = \frac{1}{6}\left[\tilde{\pi}_{GM_1}(M,T) + 4\tilde{\pi}_{GM_2}(M,T) + \tilde{\pi}_{GM_3}(M,T)\right] \tag{12}$$

where

$$\tilde{\pi}_{GM_1}(M,T) = \frac{1}{T}\left(aP_1M^{-\gamma+\beta}(T+\frac{1}{2}bT^2) - aC_1M^{\beta}(T+\frac{1}{2}bT^2) - h_1a\beta(\frac{T^2}{2}+\frac{1}{3}bT^3) - A_1\right)$$

$$\tilde{\pi}_{GM_2}(M,T) = \frac{1}{T}\left(aP_2M^{-\gamma+\beta}(T+\frac{1}{2}bT^2) - aC_2M^{\beta}(T+\frac{1}{2}bT^2) - h_2a\beta(\frac{T^2}{2}+\frac{1}{3}bT^3) - A_2\right)$$

$$\tilde{\pi}_{GM_3}(M,T) = \frac{1}{T}\left(aP_3M^{-\gamma+\beta}(T+\frac{1}{2}bT^2) - aC_3M^{\beta}(T+\frac{1}{2}bT^2) - h_3a\beta(\frac{T^2}{2}+\frac{1}{3}bT^3) - A_1\right)$$

The equations (9), (10), (11) and (12) satisfy the necessary and sufficient conditions presented in the equation (7).

4 Numerical Example

4.1 Example-1

Crisp Model

Let A=\$80 per order, a=1250 units, b=50%, h=\$6 per unit, C=\$10 per unit, P=\$16 per unit, β=6, γ=3. The solution of the crisp model is: credit period M= 0.890417 years, cycle time T = 0.430106 years, purchase quantity Q =296.757 units and total profit $\pi(M,T)$ =\$7632.70.

Fuzzy Model

Case-I

$\tilde{A}$= (76, 80, 83), $\tilde{C}$=(5, 10, 13), $\tilde{h}$=(3, 6, 8)and $\tilde{P}$=(14, 16, 19) are considered as triangular fuzzy numbers.

Scenario-1:

On applying signed distance method for defuzzification: M= 0.917546 years, T= 0.398403 years, Q =326.764 units and $\tilde{\pi}_{SD}(M,T)$ =\$8296.1.

Scenario-2:

On applying graded mean integration method for defuzzification: M= 0.908367 years, T= 0.408794 years, Q =316.404 units and $\tilde{\pi}_{GM}(M,T)$ =\$8238.04.

Case-II

$\tilde{A}$= (75, 78, 82, 84), $\tilde{C}$= (7, 9, 11, 13), $\tilde{h}$= (3, 5, 7, 9) and $\tilde{P}$= (15, 18, 20, 23) are considered as trapezoidal fuzzy numbers.

Scenario-1:
On applying signed distance method for defuzzification: M= 0.877927 years, T=0.382267 years, Q =239.699 units and $\tilde{\pi}_{SD}(M,T)$ =$7552.22.
Scenario-2:
On applying graded mean integration method for defuzzification: M=0.880744 years, T=0.387836 years, Q =248.226 units and $\tilde{\pi}_{GM}(M,T)$ =$7600.86.

4.2 Sensitivity Analysis

Table 1: Sensitivity analysis for Crisp model in example-1

Changing Parameters	Change (%)	Credit Period (M)	Cycle Time (T)	Total Profit $p(M,T)$
A	+40%	0.883920	0.508791	7564.53
	+20%	0.887023	0.471014	7597.19
	-20%	0.894200	0.385021	7671.96
	-40%	0.898535	0.333988	7716.47
C	+40%	0.737019	1.806830	5767.58
	+20%	0.819749	0.796779	6505.51
	-20%	0.968572	0.262697	9378.7
	-40%	1.071830	0.163525	12327.5
h	+40%	0.896577	0.256764	7358.21
	+20%	0.895184	0.312455	7477.92
	-20%	0.867360	0.887910	7883.64
	-40%	*	*	*
a	+40%	0.895977	0.364023	10766.4
	+20%	0.893537	0.392887	9198.12
	-20%	0.886223	0.480721	6071.03
	-40%	0.880132	0.555427	4514.66
b	+40%	0.824495	1.290600	8074.61
	+20%	0.871113	0.663983	7806.61
	-20%	0.899143	0.328371	7502.96
	-40%	0.904016	0.272541	7396.53
β	+40%	0.989885	0.198296	6693.04
	+20%	0.953084	0.255105	6931.10
	-20%	*	*	*
	-40%	*	*	*
γ	+40%	*	*	*
	+20%	*	*	*
	-20%	0.954930	0.236782	6839.47
	-40%	1.037730	0.157251	6776.64
P	+40%	1.007640	0.3083320	15168.1
	+20%	0.952572	0.3589880	11080.3
	-20%	0.818378	0.5376540	4825.40
	-40%	0.731317	0.7207440	2658.61

***indicates infeasible solution**

The following figures exhibit the effect of different system parameters on credit period, cycle time and total average profit.

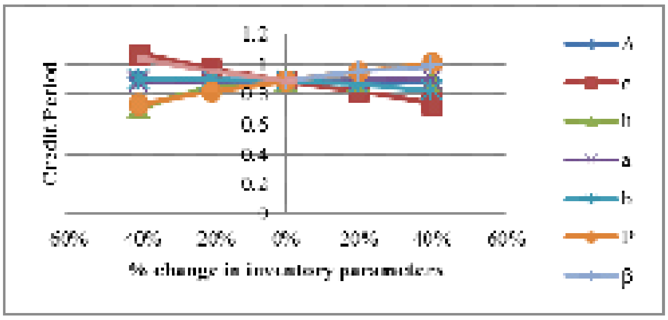

Fig.1: Sensitivity analysis for credit period

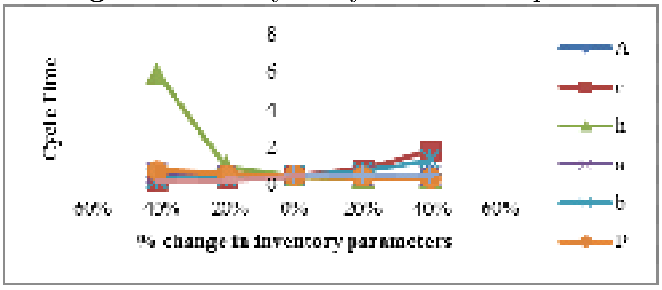

Fig.2: Sensitivity analysis for cycle Time

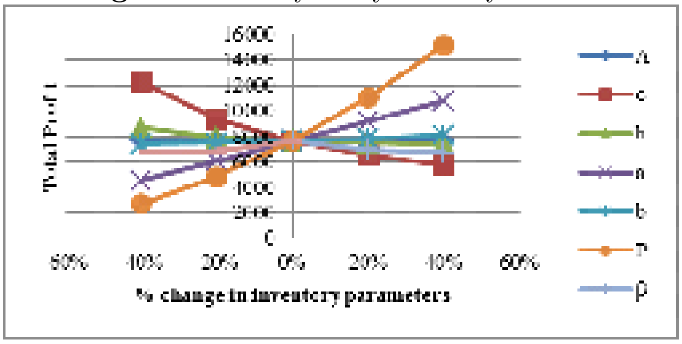

Fig.3: Sensitivity analysis for Total Profit

It is evident from the fig.1 that the credit period increases with ascent in selling price and holding cost but it decreases with rise in purchase cost and rate of change in demand. For other parameters it remains constant.

Slightly opposite behavior of fig.1 is attained by fig.2, which demonstrates that cycle time increases with incline in purchase cost and rate of change in demand and declines with increase in selling price and holding cost.

It is elucidated from fig.3 that the total profit increases when selling price and scale demand increase and declines when unit purchase cost and trade credit elasticity increase.

4.3 Comparative Analysis

Table 2: Fuzzy model when $\widetilde{\mathbf{A}}, \widetilde{\mathbf{h}}, \widetilde{\mathbf{P}}$ and $\widetilde{\mathbf{C}}$ are triangular fuzzy numbers

Method employed	M	T	Q	$\pi(M,T)$
Graded Mean Integration Method	0.908367	0.408794	316.404	8238.04
Signed Distance Method	0.917546	0.398403	326.764	8478.49

Table 3: Fuzzy model when $\widetilde{\mathbf{A}}$, $\widetilde{\mathbf{h}}$, $\widetilde{\mathbf{P}}$ and $\widetilde{\mathbf{C}}$ are trapezoidal fuzzy numbers

Method employed	M	T	Q	$\pi(M,T)$
Graded Mean Integration Method	0.948950	0.362350	360.710	10846.5
Signed Distance Method	0.948967	0.362163	360.548	10846.8

Table 4: Fuzzy model when $\widetilde{\mathbf{h}}$, $\widetilde{\mathbf{P}}$ and $\widetilde{\mathbf{C}}$ are triangular fuzzy numbers

Method employed	M	T	Q	$\pi(M,T)$
Graded Mean Integration Method	0.926269	0.170704	140.516	8294.52
Signed Distance Method	0.930152	0.204104	173.659	8492.31

Table 5: Fuzzy model when $\widetilde{\mathbf{h}}$, $\widetilde{\mathbf{P}}$ and $\widetilde{\mathbf{C}}$ are trapezoidal fuzzy numbers

Method employed	M	T	Q	$\pi(M,T)$
Graded Mean Integration Method	0.968538	0.150249	160.855	11105.6
Signed Distance Method	0.965409	0.183383	194.091	11065.6

4.4 Example-2

Crisp Model

Let A=\$100 per unit, a=1500 units, b=60%, h=\$8 per unit, C=\$12 per unit, P=\$18 per unit, β=9, γ=4. The solution of the crisp model is: credit period M= 0.932546, cycle time T =0.297535 years, purchase quantity Q =319.729 units and total profit $\pi(M,T)$ =\$8882.5.

Fuzzy Model

Case-I

$\tilde{A}$=(95, 100, 103), $\tilde{C}$=(9, 12, 16), $\tilde{h}$=(3, 8, 10)and $\tilde{P}$=(16, 18, 21) are considered as triangular fuzzy numbers.

Scenario-1:

On applying signed distance method for defuzzification: M=0.929376 years, T=0.358125 years, Q=307.702 units and $\tilde{\pi}_{SD}(M,T)$ =\$9064.34.

Scenario-2:

On applying graded mean integration method for defuzzification: M=0.93057 years, T=0.334404 years, Q=288.818 units and $\tilde{\pi}_{GM}(M,T)$=\$9001.24.

Case-II

$\tilde{A}$=(95, 97, 105, 108), $\tilde{C}$= (9, 10, 13, 15), $\tilde{h}$=(5, 7, 9, 12) and $\tilde{P}$=(16, 19, 21, 23) are considered as trapezoidal fuzzy numbers.

Scenario-1:

On applying signed distance method for defuzzification: M=0.962322 years, T=0.24744 years, Q=282.191 units and $\tilde{\pi}_{SD}(M,T)$=\$11263.6.

Scenario-2:

On applying graded mean integration method for defuzzification: M=0.965224 years, T=0.246037 years, Q=288.186 units and $\tilde{\pi}_{GM}(M,T)$=\$11484.1.

4.5 Sensitivity Analysis

Table 6: Sensitivity analysis for Crisp model in example-2

Changing Parameters	Change (%)	Credit Period (M)	Cycle Time(T)	Total Profit $\pi(M,T)$
A	+40%	0.928475	0.352772	8759.47
	+20%	0.930423	0.326250	8818.38
	-20%	0.934903	0.265909	8953.50
	-40%	0.937589	0.230156	9034.14
C	+40%	0.842970	0.709472	6054.13
	+20%	0.885271	0.452365	7189.13
	-20%	0.990263	0.196155	11589.7
	-40%	1.067410	0.124317	16443.7
h	+40%	0.933694	0.202867	8540.67
	+20%	0.933603	0.237026	8694.54
	-20%	0.928081	0.444726	9139.99
	-40%	0.902590	1.175870	9639.17
a	+40%	0.936006	0.251191	12581.3
	+20%	0.934491	0.271425	10729.3
	-20%	0.929921	0.333064	7041.76
	-40%	0.926089	0.385509	5212.33
b	+40%	0.918115	0.490526	9237.20
	+20%	0.927140	0.369024	9041.78
	-20%	0.935994	0.252412	8746.86
	-40%	0.938364	0.221605	8627.49
β	+40%	0.991949	0.169629	7813.15
	+20%	0.969049	0.208922	8102.63
	-20%	*	*	*
	-40%	*	*	*
γ	+40%	*	*	*
	+20%	0.877146	0.878704	10378.7
	-20%	0.969658	0.196845	7995.09
	-40%	1.02094	0.138028	7872.86
P	+40%	1.022070	0.203463	19389.6
	+20%	0.980375	0.242063	13575.3
	-20%	0.875959	0.383792	5260.44
	-40%	0.805643	0.535846	2649.90

***indicates infeasible solution**

The following figures exhibit the effect of different system parameters on credit period, cycle time and total average profit.

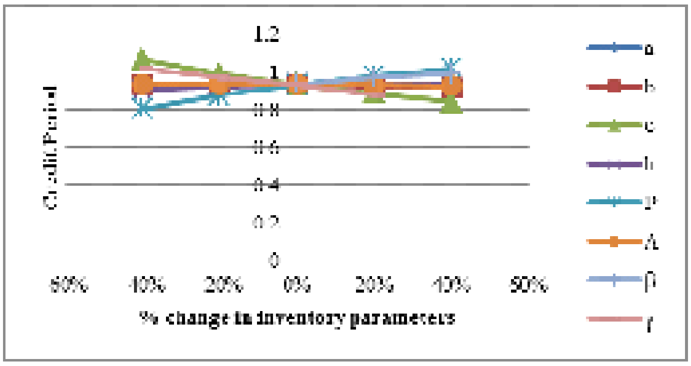

Fig.4: Sensitivity analysis for credit period

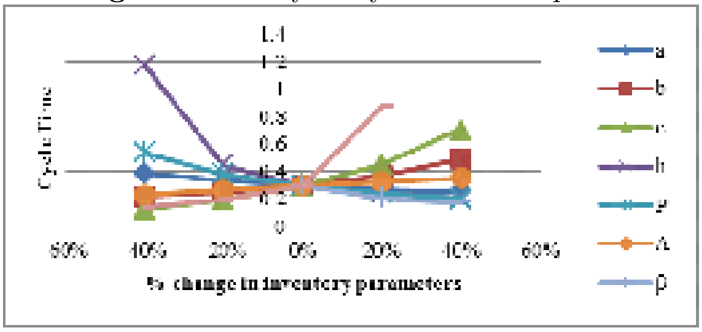

Fig.5: Sensitivity analysis for cycle Time

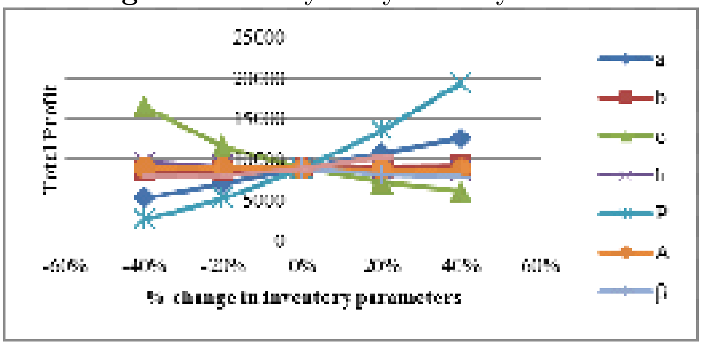

Fig.6: Sensitivity analysis for Total Profit

It is stipulated by fig.4 that the credit period increases with increase in selling price and holding cost and decreases with increase in purchase cost and rate of change in demand. For other parameters it remains constant.

Referring to fig.5, it can be observed that it behaves oppositely to fig.4. Here the cycle time increases with increase in purchase cost, scale demand and rate of change in demand and decreases with increase in selling price, holding cost.

As exhibited by fig.6 that the total profit increases when selling price and scale demand increase and declines when purchase cost and trade credit elasticity increase.

4.6 Comparative Analysis

Table 7: Fuzzy model when $\widetilde{\mathbf{A}}, \widetilde{\mathbf{h}}$, $\widetilde{\mathbf{P}}$ and $\widetilde{\mathbf{C}}$ are triangular fuzzy numbers

Method employed	M	T	Q	$\pi(M,T)$
Graded Mean Integration Method	0.930570	0.334404	288.818	9001.24
Signed Distance Method	0.929376	0.358125	307.729	9064.34

Table 8: Fuzzy model when $\widetilde{\mathbf{A}}$, $\widetilde{\mathbf{h}}$, $\widetilde{\mathbf{P}}$ and $\widetilde{\mathbf{C}}$ are trapezoidal fuzzy numbers

Method employed	M	T	Q	$\pi(M,T)$
Graded Mean Integration Method	0.965224	0.246037	288.186	11484.1
Signed Distance Method	0.962322	0.247440	282.191	11263.6

Table 9: Fuzzy model when $\widetilde{\mathbf{h}}$, $\widetilde{\mathbf{P}}$ and $\widetilde{\mathbf{C}}$ are triangular fuzzy numbers

Method employed	M	T	Q	$\pi(M,T)$
Graded Mean Integration Method	0.944255	0.139128	129.738	9351.07
Signed Distance Method	0.941054	0.183840	168.414	9338.57

Table 10: Fuzzy model when $\widetilde{\mathbf{h}}$, $\widetilde{\mathbf{P}}$ and $\widetilde{\mathbf{C}}$ are trapezoidal fuzzy numbers

Method employed	M	T	Q	$\pi(M,T)$
Graded Mean Integration Method	0.977354	0.0992367	124.730	11974.5
Signed Distance Method	0.972638	0.1221930	148.025	11676.8

Behavior of profit, credit period, cycle time and purchase quantity in both the examples has been presented in the following figures.

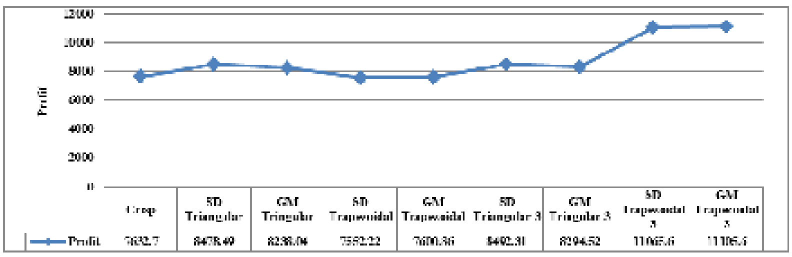

Fig.7: Behaviour of profit in example-1

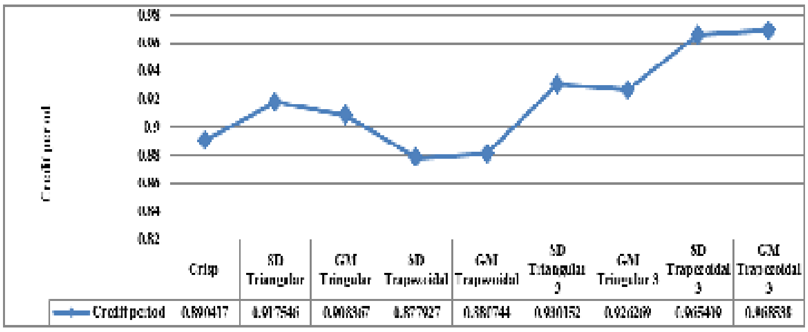

Fig.8: Behaviour of credit period in example-1

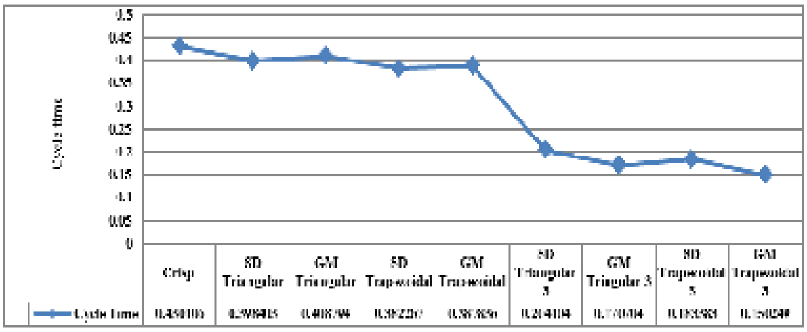

Fig.9: Behaviour of cycle time in example-1

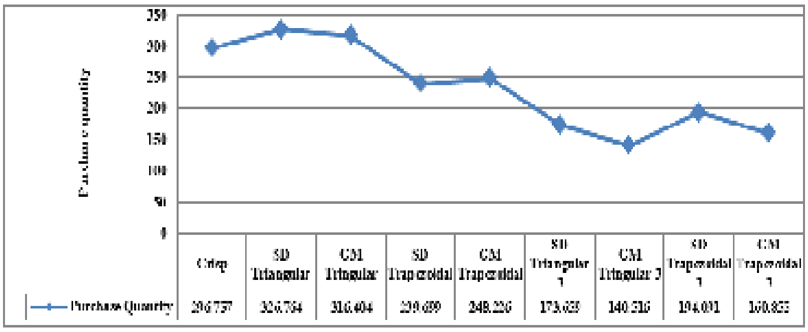

Fig.10: Behaviour of purchase quantity in example-1

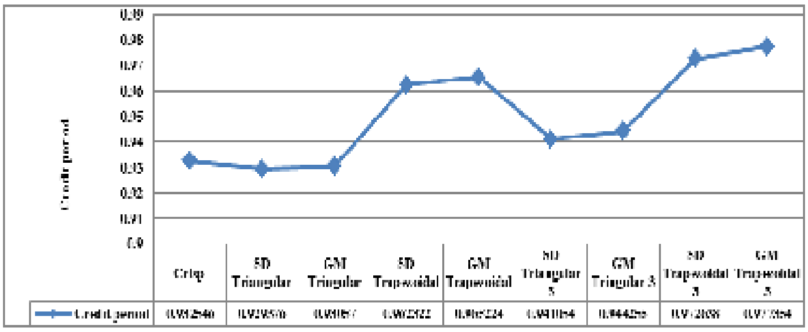

Fig.11: Behaviour of profit in example-2

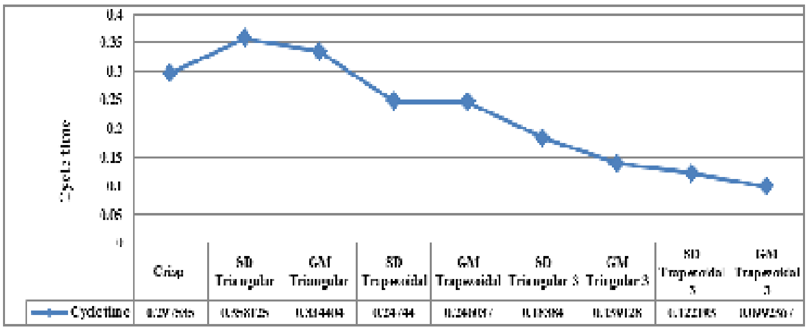

Fig.12: Behaviour of credit period in example-2

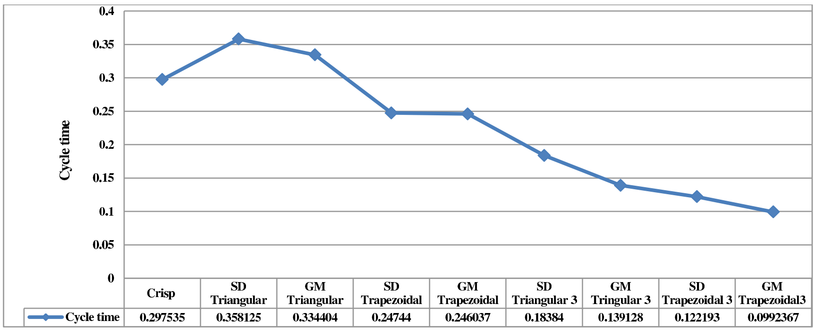

Fig.13: Behaviour of cycle time in example-2

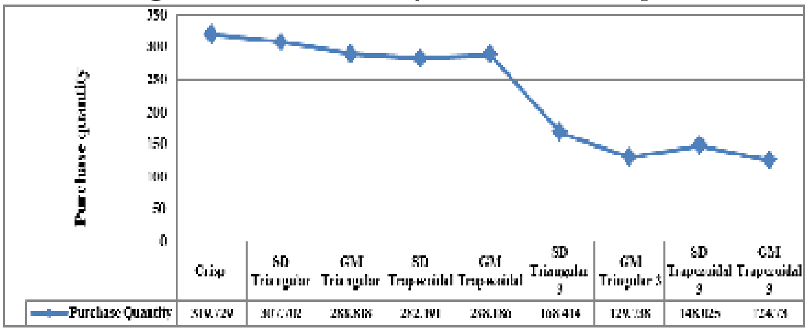

Fig.14: Behaviour of purchase quantity in example-2

5 Result & Discussion

Though utmost profit is attained by trapezoidal fuzzy number in both the examples (fig. 7&11), by treating four parameters like holding cost, unit purchase cost, ordering cost and selling price and three parameters like holding cost, unit purchase cost and selling price, as fuzzy, it cannot be adjudged as ideal one owing to possessing of lengthy credit period but shorter cycle time (fig. 8,9,12 and 13).

The next highest profit is achieved through triangular fuzzy number in both the illustrations (fig. 7&11), considering four parameters and three parameters as fuzzy, as stated earlier. It can be treated as ideal one as compared to trapezoidal fuzzy number and crisp method, as it possesses shorter credit period and lengthy cycle time. Now comparing the two methods, signed distance and graded mean integration, of defuzzification, both can be deemed as equally effective in achieving our goal as they acquire little difference. From figures 7,8,11 & 12, it is elucidated that the profit is accelerated with longer credit period.

6 Conclusion

The determination of optimal credit period and purchase quantity is inevitable for the seller. But attainability of maximum profit is not easily accessible as it is swayed by many constraints. The present paper quests on enriching the profit of the seller under the constraint of default risk. The proposed model is adorned with two different fuzzy numbers like triangular and trapezoidal. For defuzzification, signed distance and graded mean integration method have been expended. Sensitivity analysis has been accomplished both for crisp and fuzzy model. Induction of sensitivity analysis for fuzzy model, considering four parameters (holding cost, unit purchase cost, ordering cost and selling price) and three parameters (holding cost, unit purchase cost and selling price) as fuzzy, enables us to toughen the conclusion up. Result achieved elucidates the importance of fuzzy model over the crisp model in ratcheting up the profit, reducing the credit period and obtaining optimal purchase quantity. Opting for four parameters (holding cost, unit purchase cost, ordering cost and selling price) as triangular fuzzy number can be regarded as more productive in reinforcing our goal. Furthermore, both signed distance and graded mean integration methods of defuzzification can be treated as equally effective in enabling us in maximizing the profit, minimizing credit period and attaining optimal purchase quantity.

The proposed model can be extended by introducing partial credit period, shortages and deterioration of the items.

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