International Journal of Statistical Sciences *ISSN 1683–5603*
Vol. 15, 2015, pp 21-35

Optimisation Perspective in Managing Gender Centric Prophesied Longevity

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[Received April 13, 2017; Revised May 11, 2017; Accepted June 15, 2017; Published December 30, 2017]

Abstract

Mortality transition leads to the change in the contribution of the age group which helps predominantly in enriching the life expectancy. This contribution cannot simply be conceived from the mortality data as it may also vary gender wise. Hence, it is necessary to forecast the behaviour of the mortality transition of different age groups by gender towards the change in longevity. Attempt has been constituted in the current study to forecast the age groups by sex which will contribute more in expansion of life expectancy in future. For this purpose, the projected sex wise ASDR data of developing country like India and its states of Kerala, Maharashtra and Uttar Pradesh have been borrowed. The optimisation perspective is then employed to find out the most important age group which will contribute more in fortifying the longevity in future for both sex.

Keywords and Phrases: Mortality Transition, ASDR, Life table, Life Expectancy.

AMS Classification: 91B28

1 Introduction

Life expectancy is a measure of how long a person belonging to a particular age group is expected to survive. Improved health status throughout the developing countries led to the enhancement in life expectancy in the last few decades. India has also kept pace in the improvement in health status over the country. As a result CDR, IMR and ASDR are lessened and the life expectancy has been ratcheted up considerably.

Many states in India are forging significant improvement in reinforcing the health status, which has led to the increment in life expectancy. But the role of all the age groups in augmenting the longevity is not analogous. Game theory can be lucratively applied to have erudition on this. Game theory was pioneered by Neumann and Morgenstern [6] and later on explicated lucidly by Hillier and Liebermann[2] and Tripathy [11]. Bastian and Nair [8] proposed game theory approach to find out the age groups playing overriding role in enhancing life expectancy in India and some of its major states. Papers on life expectancy and mortality rate have been published by several authors. Tripathy and Pati [12] focused on that infant and child mortality that affect the health of the persons in general and the health of women in particular, thereby affecting the life expectancy. Dubey [10] projected the sex wise population and ASDR of India using Cohort-component and Lee-carter model. Sasson [4] adopted a multidimensional approach to life span inequality to study the trends in both life expectancy and life span variation by educational attainment in the United States. Li [5] examined the application of a Poisson common factor model for the projection of mortality. Goldstein and Cassidy [7] evinced mathematically how varying the pace of senescence influences life expectancy. Doblhammer [3] studied whether a woman's reproductive history influences her life span. Mayhew and Smith [9] presented new methods for comparing past improvements in life expectancy and also future prospects, using data from five developed and low-mortality countries. Poppel et. al. [1] investigated the role of urbanisation and plague on the changes in life expectancy amongst artists.

In the present study, a game theory prospective has been employed by using projected ASDR extracted from Dubey [10] and estimated life expectancy for both sexes in India and states of Kerala, Maharashtra and Uttar Pradesh, to find out the age group affecting mostly the life table in future. The objective of this paper is to make an initiative for finding the age group which plays major role in enriching the longevity by gender.

2 Methodology

In a two person zero sum game, if maximum of row minima $\neq$ minimum of column maxima, it does not possess a saddle point. In such a case to solve the game, the players must determine an optimal mixture of strategies to find a saddle point.

There is no pure strategy (Nash equilibrium) in this game. If we play this game, we should be unpredictable. That is, we should randomize (or mix) between strategies so that we don't get exploited.

The important observation is that if a player is using a mixed strategy at equilibrium then both players should have the same expected payoff from the strategies they are mixing. We can easily find the mixed strategy (Nash equilibrium) in 2x2 games using this observation.

The optimal strategy mixture for each player may be determined by assigning to each strategy its probability of being selected. For the following two person zero sum game,

$$\begin{array}{cc} & \textit{Player B} \\ & \begin{array}{cc} y_1 & y_2 \end{array} \\ \textit{Player A} \quad \begin{array}{c} x_1 \\ x_2 \end{array} & \left[\begin{array}{cc} a_{11} & a_{12} \\ a_{21} & a_{22} \end{array}\right] \end{array}$$

the player A plays his strategies x_1 and x_2 with probabilities p_1 and q_1 respectively, where

$$p_1 = \frac{a_{22} - a_{21}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$$

and $q_1 = 1 - p_1$ (because $p_1 + q_1 = 1$)

Likewise, the player B plays his strategies y_1 and y_2 with probabilities p_2 and q_2 respectively, where

$$p_2 = \frac{a_{22} - a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$$

and $q_2 = 1 - p_2$ (because $p_2 + q_2 = 1$)

As per principle of dominance for easiness of solution, it is convenient to deal with smaller payoff matrices. The size of the payoff matrix can be considerably reduced by using the principle of dominance, which includes, if each element in rth row is less than or equal to the corresponding element in any other row, say sth , then the player A will never choose rth strategy or in other words, rth strategy is said to be dominated by the sth strategy. If all the elements of a column, say C_r, are greater than or equal to the corresponding elements in any other column, say C_s, then the column C_r is dominated by the column C_s. If the convex linear combination of some rows dominates the ith row, then the ith row will be deleted. If the ith row dominates the convex linear combination of some other rows, then one of the rows involving in the combination may be deleted. Similar arguments follow for columns also. By using dominance property, we reduce the size of the matrix to 2x2, the aforementioned

method is used to solve the game.

In case of $2 \times n$ or $m \times 2$ game, after applying the dominance property, if a 2x2 matrix is not obtained, graphical method can be employed to have such a matrix. In which two axes of unit length apart are drawn and scale on each axis is marked. All the lines given in the matrix are drawn. Then the lowest point in the uppermost boundary, denoted by P, the interaction point of two strategies, helps in obtaining a 2x2 matrix.

3 Analysis

The methodology has been implemented to the gender wise data on projected ASDR and estimated Life Expectancy of India for 2016, 2021 & 2026.

Table 1: Sex wise Projected ASDR of India for the years 2016, 2021 & 2026

Age Group	Projected ASDR of India					
	2016 Male	2016 Female	2021 Male	2021 Female	2026 Male	2026 Female
0-4	10.31	10.17	8.05	8.49	6.34	7.08
5-9	0.85	0.83	0.67	0.68	0.53	0.55
10-14	0.70	0.69	0.60	0.60	0.51	0.53
15-19	1.07	1.20	0.96	1.08	0.85	0.97
20-24	1.56	1.50	1.42	1.33	1.28	1.18
25-29	2.04	1.44	1.91	1.28	1.79	1.13
30-34	2.59	1.46	2.43	1.28	2.27	1.13
35-39	3.45	1.71	3.26	1.51	3.08	1.34
40-44	4.48	2.22	4.17	1.98	3.87	1.77
45-49	6.45	3.22	5.99	2.90	5.54	2.62
50-54	9.43	4.86	8.67	4.35	7.94	3.90
55-59	14.02	8.26	12.90	7.53	11.82	6.87
60-64	21.90	13.81	20.09	12.56	18.37	11.42
65-69	35.60	23.24	33.41	21.51	31.28	19.90
70+	74.81	61.05	70.68	57.83	66.63	54.79

Table 2: Sex wise Estimated life expectancy of India for 2016, 2021 & 2026

Age Group	Estimated Life Expectancy of India					
	2016 Male	2016 Female	2021 Male	2021 Female	2026 Male	2026 Female
0-4	72.1	75.4	73.1	75.6	72.1	77.6
5-9	72.6	75.9	73.3	75.9	72.6	77.7
10-14	67.7	71.1	68.4	71.0	67.7	72.8
15-19	62.8	66.2	63.5	66.1	62.8	67.9
20-24	57.9	61.3	58.6	61.3	57.9	63.0
25-29	53.1	56.5	53.7	56.4	53.1	58.2
30-34	48.3	51.7	48.9	51.6	48.3	53.3
35-39	43.5	46.8	44.2	46.7	43.5	48.4
40-44	38.8	41.9	39.4	41.9	38.8	43.5
45-49	34.1	37.1	34.8	37.0	34.1	38.7
50-54	29.5	32.4	30.1	32.3	29.5	33.9
55-59	25.1	27.7	25.6	27.6	25.1	29.1
60-64	21.4	24.9	22.2	24.9	21.4	26.9
65-69	17.2	20.5	18.0	20.5	17.2	22.4
70+	13.4	16.4	14.1	16.4	13.4	18.3

Taking into account the ASDR and Life expectancy of males and females in 2016, we have the following tables.

Table 3: ASDR and Life expectancy for males of India in 2016

Age Group	Males of India 2016		Row Minimum
	Projected ASDR	Estimated Life Expectancy	
0-4	10.31	72.1	10.31
5-9	0.85	72.6	0.85
10-14	0.70	67.7	0.70
15-19	1.07	62.8	1.07
20-24	1.56	57.9	1.56
25-29	2.04	53.1	2.04
30-34	2.59	48.3	2.59
35-39	3.45	43.5	3.45
40-44	4.48	38.8	4.48
45-49	6.45	34.1	6.45
50-54	9.43	29.5	9.43
55-59	14.02	25.1	14.02
60-64	21.90	21.4	21.4
65-69	35.60	17.2	17.2
70+	74.81	13.4	13.4
Column Maximum	74.81	72.6	

Table 4: The ASDR and Life expectancy of females of India in 2016

Age Group	Females of India 2016		Row Minimum
	Projected ASDR	Estimated Life Expectancy	
0-4	10.17	75.4	10.17
5-9	0.83	75.9	0.83
10-14	0.69	71.1	0.69
15-19	1.20	66.2	1.20
20-24	1.50	61.3	1.50
25-29	1.44	56.5	1.44
30-34	1.46	51.7	1.46
35-39	1.71	46.8	1.71
40-44	2.22	41.9	2.22
45-49	3.22	37.1	3.22
50-54	4.86	32.4	4.86
55-59	8.26	27.7	8.26
60-64	13.81	24.9	13.81
65-69	23.24	20.5	20.5
70+	61.05	16.4	16.4
Column Maximum	61.05	75.9	

It is evident from table 3 & table 4 that maximum of the Row Minima $\neq$ minimum of the Column maxima i.e. 20.5$\neq$61.05. So, they don't possess a saddle point. Therefore dominance property is to be applied to reduce the size of the matrices. In table 3, the age groups 10-14 to 50-54 are dominated by the age group 0-4. In table 4 age groups 10-14 to 55-59 are dominated by age group 0-4. The table 5 & table 6 represent the ASDR and the Life expectancy after applying the dominance property.

Table 5: Reduced matrix after applying dominance property

Age Group	Males of India 2016	
	Projected ASDR	Estimated Life Expectancy
0-4	10.31	72.1
5-9	0.85	72.6
55-59	14.02	25.1
60-64	21.90	21.4
65-69	35.60	17.2
70+	74.81	13.4

Table 6: Reduced matrix after applying dominance property

Age Group	Females of India 2016	
	Projected ASDR	Estimated Life Expectancy
0-4	10.17	75.4
5-9	0.83	75.9
60-64	13.81	24.9
65-69	23.24	20.5
70+	61.05	16.4

Further 2x2 matrices have been formed by exerting graphical method.

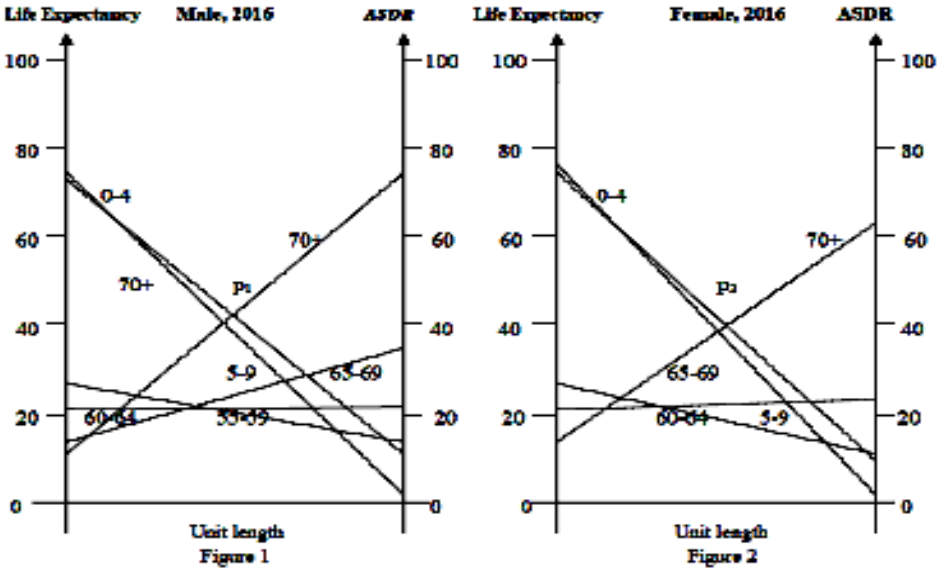

Figure 1&2: Graphical representation of projected ASDR vis-à-vis estimated life expectancy

In Figure 1 & Figure 2, the lowest points P_1 and P_2 in the upper most boundaries are presented as the interaction of two strategies 0-4 and 70+. The two strategies corresponding to the points P_1 and P_2 help in shortening the matrices into 2x2 forms.

Table 7: 2x2 matrix derived from Figure 1

Age Group	Males of India 2016	
	Projected ASDR	Estimated Life Expectancy
0-4	10.31	72.1
70+	74.81	13.4

Table 8: 2x2 matrix derived from Figure 2

Age Group	Females of India 2016	
	Projected ASDR	Estimated Life Expectancy
0-4	10.17	75.4
70+	61.05	16.4

Let p_1 and q_1 be the probabilities of males and p_2 and q_2 that of females for the age groups of 0-4 and 70+ respectively. Then

$$p_1 = \frac{13.4 - 74.81}{(10.13 + 13.4) - (72.1 + 74.81)} = 0.4977$$

$$and \quad q_1 = 1 - p_1 = 1 - 0.4977 = 0.5023, q_1 > p_1$$

$$p_2 = \frac{16.4 - 61.05}{(10.17 + 16.4) - (75.4 + 61.05)} = 0.4064$$

$$and \quad q_2 = 1 - p_2 = 1 - 0.4064 = 0.5936, \quad q_2 > p_2$$

For both the genders, the probability of the age group of 70+ is more exhibiting the importance of the age group in expanding the life expectancy.

Applying the same method for the data of males and females of India for 2021 & 2026, the results obtained are presented in table 9.

Now the effectiveness of the proposed method has been studied by applying it for the data of Kerala, Maharashtra and Uttar Pradesh.

The projected ASDR and estimated Life expectancy of males and females of Kerala for 2016, 2021 and 2026 are presented in the table 9 & table 10.

Table 9: Sex wise projected ASDR of Kerala for 2016, 2021 & 2026

Age Group	Projected ASDR of Kerala					
	2016 Male	2016 Female	2021 Male	2021 Female	2026 Male	2026 Female
0-4	1.61	1.02	1.26	0.72	1.00	0.51
5-9	0.16	0.07	0.12	0.05	0.09	0.03
10-14	0.21	0.13	0.18	0.10	0.15	0.08
15-19	0.48	0.28	0.44	0.23	0.40	0.20
20-24	0.91	0.42	0.85	0.35	0.80	0.30
25-29	1.35	0.39	1.28	0.32	1.23	0.27
30-34	1.51	0.51	1.41	0.43	1.31	0.36
35-39	2.42	0.71	2.32	0.61	2.23	0.52
40-44	3.02	0.80	4.22	0.67	2.64	0.56
45-49	5.12	1.23	5.74	1.03	4.60	0.87
50-54	8.60	2.22	7.25	1.93	7.99	1.67
55-59	12.42	4.14	11.71	3.68	11.39	3.27
60-64	21.25	6.93	18.76	6.08	20.16	5.33
65-69	29.98	12.31	29.39	10.88	27.72	9.62
70+	80.86	50.47	68.05	46.85	77.10	43.50

Table 10: Sex wise estimated Life Expectancy of Kerala for 2016, 2021 & 2026

Age Group	Estimated Life Expectancy of Kerala					
	2016 Male	2016 female	2021 Male	2021 female	2026 Male	2026 Female
0-4	72.8	80.4	73.1	81.8	73.5	83.4
5-9	71.9	79.6	72.3	80.9	72.6	82.4
10-14	67.0	74.6	67.3	75.9	67.6	77.4
15-19	62.0	69.6	62.3	70.9	62.6	72.5
20-24	57.1	64.6	57.4	66.0	57.7	67.5
25-29	52.2	59.7	52.5	61.1	52.8	62.5
30-34	47.3	54.7	47.6	56.1	47.9	57.6
35-39	42.5	49.8	42.7	51.1	43.0	52.6
40-44	37.7	44.8	37.9	46.2	38.2	47.7
45-49	32.9	39.9	33.1	41.3	33.4	42.7
50-54	28.2	34.9	28.4	36.3	28.7	37.8
55-59	23.6	30.1	23.9	31.5	24.1	32.9
60-64	20.7	28.9	20.9	30.5	21.3	32.2
65-69	16.5	24.3	16.8	25.8	17.1	27.5
70+	12.4	19.8	12.7	21.3	12.9	22.9

Applying the above methodology for the sex wise data on ASDR and Life Expectancy of Kerala for 2016, it is observed that the maximum of the row minima $\neq$ minimum of the Column maxima. Dominance property has been utilised and the table 11 & table 12 are obtained.

Table 11: Reduced matrix after applying dominance property

Age Group	Males of Kerala 2016	
	Projected ASDR	Estimated Life Expectancy
0-4	1.61	72.8
35-39	2.42	42.5
40-44	3.02	37.7
45-49	5.12	32.9
50-54	8.60	28.2
55-59	12.42	23.6
60-64	21.25	20.7
65-69	29.98	16.5
70+	80.86	12.4

Table 12: Reduced matrix after applying dominance property

Age Group	Females of Kerala 2016	
	Projected ASDR	Estimated Life Expectancy
0-4	1.02	80.4
45-49	1.23	39.9
50-54	2.22	34.9
55-59	4.14	30.1
60-64	6.93	28.9
65-69	12.31	24.3
70+	50.47	19.8

For further reduction of the above matrices to 2x2 form the graphical method has been employed.

The 2x2 matrices and the probabilities obtained from Figure 3 & Figure 4 are presented below.

Table 13: 2x2 matrix derived from Figure 3

Age Group	Males of Kerala 2016	
	Males of Kerala 2016	Projected ASDR
0-4	1.61	72.8
70+	80.86	12.4

Table 14: 2x2 matrix derived from Figure 4

Age Group	Females of Kerala 2016	
	Projected ASDR	Estimated Life Expectancy
0-4	1.02	80.4
70+	50.47	19.8

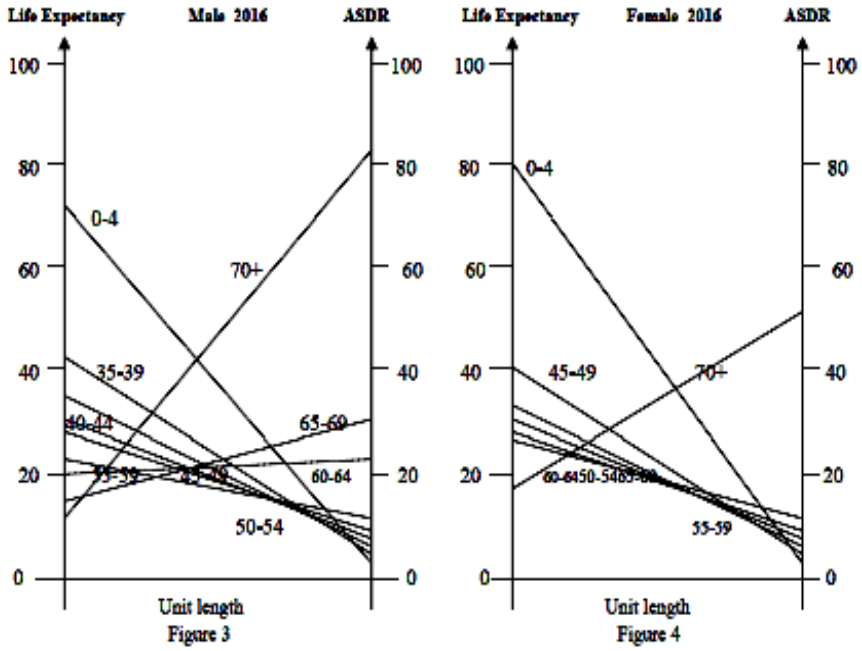

Figure 3&4: Graphical representation of projected ASDR vis-à-vis estimated life expectancy

If p_1 and q_1 are defined as the probabilities of males and p_2 and q_2 that of females for the age groups 0-4 and 70+ respectively, then

$$p_1 = \frac{12.4 - 80.86}{(1.61 + 12.4) - (80.86 + 72.8)} = 0.4902$$

$$and \quad q_1 = 1 - p_1 = 1 - 0.4902 = 0.5098,\ q_1 > p_1$$

$$p_2 = \frac{19.8 - 50.47}{(1.02 + 19.8) - (80.4 + 50.47)} = 0.2787$$

$$and \quad q_2 = 1 - p_2 = 1 - 0.2787 = 0.7213,\ q_2 > p_2$$

This implies that in Kerala for 2016, 70+ age group is important in enriching the life expectancy.

Proceeding in the same way for 2021 and 2026, the result obtained is presented in table-19.

The following table 15, table 16, table 17 & table 18 represent the gender wise projected ASDR and estimated life expectancy of Maharashtra and Uttar Pradesh for the years 2016, 2021 & 2026.

Table 15: Sex wise Projected ASDR of Maharashtra for 2016, 2021 & 2026

Age Group	Projected ASDR of Maharashtra					
	2016 Male	2016 Female	2021 Male	2021 Female	2026 Male	2026 Female
0-4	4.26	4.04	3.05	2.73	2.14	1.80
5-9	0.31	0.26	0.21	0.16	0.14	0.10
10-14	0.40	0.34	0.32	0.25	0.26	0.19
15-19	0.66	0.84	0.56	0.69	0.46	0.56
20-24	1.21	0.93	1.09	0.75	0.98	0.59
25-29	2.27	0.87	2.27	0.69	2.26	0.53
30-34	2.22	1.00	2.05	0.81	1.89	0.65
35-39	3.83	1.25	3.76	1.02	3.68	0.82
40-44	4.22	1.47	3.90	1.17	3.58	0.92
45-49	5.74	2.40	5.26	1.97	4.79	1.60
50-54	7.25	3.62	6.41	3.01	5.62	2.48
55-59	11.71	6.44	10.44	5.46	9.24	4.58
60-64	18.76	11.75	16.75	10.06	14.86	8.57
65-69	29.39	18.86	26.39	16.22	23.56	13.79
70+	68.05	55.54	62.68	50.02	57.46	44.70

Table 16: Sex wise Estimated Life expectancy of Maharashtra for 2016, 2021 & 2026

Age Group	Estimated Life Expectancy of Maharashtra					
	2016 Male	2016 Male	2021 Male	2021 Female	2026 Male	2026 Female
0-4	77.9	74.2	73.1	79.9	77.0	82.2
5-9	77.5	73.8	72.3	79.3	76.3	81.5
10-14	72.6	68.8	67.3	74.4	71.4	76.5
15-19	67.6	63.9	62.3	69.4	66.4	71.6
20-24	62.7	58.9	57.4	64.5	61.4	66.6
25-29	57.8	54.1	52.5	59.6	56.6	61.7
30-34	52.9	49.3	47.6	54.7	51.8	56.8
35-39	48.0	44.6	42.7	49.8	46.9	51.8
40-44	43.1	39.9	37.9	44.9	42.3	46.9
45-49	38.3	35.2	33.1	39.9	37.6	42.0
50-54	33.4	30.6	28.4	35.1	32.9	37.1
55-59	28.6	26.0	23.9	30.3	28.3	32.3
60-64	26.7	22.9	20.9	28.7	25.8	31.2
65-69	22.2	18.7	16.8	24.3	21.5	26.7
70+	18.0	14.7	12.7	19.9	17.4	22.4

Table 17: Sex wise Projected ASDR of Uttar Pradesh for 2016, 2021 & 2026

Age Group	Projected ASDR, Uttar Pradesh					
	Male 2016	Female 2016	Male 2021	Female 2021	Male 2026	Female 2026
0-4	14.39	11.18	10.99	7.46	8.19	4.80
5-9	1.03	0.93	0.77	0.66	0.56	0.42
10-14	0.85	0.63	0.72	0.48	0.59	0.35
15-19	1.36	1.17	1.25	0.91	1.14	0.70
20-24	2.17	1.72	2.10	1.34	2.03	1.03
25-29	2.27	2.03	2.17	1.67	2.06	1.34
30-34	3.24	1.87	3.19	1.50	3.15	1.17
35-39	4.32	2.20	4.27	1.83	4.21	1.50
40-44	4.87	2.47	4.49	2.03	4.12	1.63
45-49	6.76	3.75	6.15	3.20	5.55	2.70
50-54	9.66	5.37	8.65	4.45	7.66	3.63
55-59	14.06	8.05	12.52	6.68	11.04	5.46
60-64	24.36	13.06	22.45	10.68	20.55	8.57
65-69	38.33	22.40	35.84	19.08	33.32	16.02
70+	79.89	63.12	76.28	58.26	72.55	53.41

Table 18: Sex wise Estimated Life expectancy of Uttar Pradesh for 2016, 2021 & 2026

Age Group	Estimated Life Expectancy, Uttar Pradesh					
	Male 2016	Female 2016	Male 2021	Female 2021	Male 2026	Female 2026
0-4	70.6	74.8	71.7	76.7	72.6	78.6
5-9	71.7	75.5	72.3	76.8	72.9	78.3
10-14	66.9	70.7	67.4	71.9	67.9	73.4
15-19	61.9	65.7	62.5	66.9	63.0	68.4
20-24	57.1	60.9	57.6	62.1	58.2	63.5
25-29	52.4	56.1	52.8	57.3	53.4	58.6
30-34	47.6	51.3	48.1	52.4	48.6	53.8
35-39	42.9	46.5	43.4	47.6	43.9	48.9
40-44	38.2	41.7	38.7	42.8	39.3	44.1
45-49	33.6	36.9	34.0	37.9	34.6	39.2
50-54	29.0	32.1	29.4	33.1	29.9	34.4
55-59	24.5	27.5	24.9	28.4	25.4	29.6
60-64	20.4	24.4	21.1	25.9	21.9	27.6
65-69	16.4	20.0	16.99	21.4	17.7	23.0
70+	12.5	15.8	13.1	17.2	13.8	18.7

After applying the proposed methodology to the above data, the results are exhibited in table 19.

The following table 19 explicitly focuses on the age groups playing dominant role in enriching the life expectancy.

Table-19: Predominant age groups affecting the life expectancy

Year Country/State	2016		2021		2026	
	Male	Female	Male	Female	Male	Female
India	70+	70+	70+	70+	70+	70+
Kerala	70+	70+	70+	70+	70+	70+
Maharashtra	70+	70+	70+	70+	70+	70+
Uttar Pradesh	0-4	70+	0-4	70+	70+	70+

4 Conclusion

The propensity of life expectancy employing mortality data has been prevised effectively through game theory. Application of the method for sex wise data has evinced that age group of 70+ plays dominant role in elevating longevity for all the three years in India. Similar consequences have been achieved for Kerala and Maharashtra. But in Uttar Pradesh in 2016 & 2021, age group of 0-4 helps in augmenting life expectancy for male while for female; it is 70+ age group which elevates the longevity. In 2026, for both sexes age group of 70+ helps in enriching the longevity. Hence it can be conceded that with the passage of time the old age mortality is becoming influential in escalating the longevity. Erudition on the behaviour of mortality is essential for decision maker for framing suitable policies as it affects the longevity. Therefore it is imperative to forecast the behaviour of mortality rate and life expectancy and thus previse the age groups affecting the life expectancy for future planning. Optimisation perspective helps us in acquiring knowledge in this direction. For future research game theory concept can be employed effectively for analysing other demographic problems.

References

[1] Frans Van Poppel, Dirk J. Van de kaa & Govert E. Bijwaard (2013). *Life expectancy of artists in the low countries from the fifteenth to the twentieth century.* Population Studies, 67:3, 275-292.

[2] Frederick S. Hillier & Gerald J. Liebermann (2001). *Introduction to Operations Research*, (7th edition), Mc Graw- Hill.

[3] Gabriele Doblhammer (2000). *Productive history and mortality later in life: A comparative study of England and Wales and Austria.* Population Studies.169-176, 54.

[4] Issac Sasson (2016). *Trends in life expectancy and life span variation by educational attainment: United States*, 1990-2010. Demography, 53(2): 269-293.

[5] Jackie Li (2013). *A Poisson common factor model for projecting mortality and life expectancy jointly for females and males*. Population Studies, Vol.67, No. 1, 111-126.

[6] John Von Neumann & Oskar Morgenstern (2004). *Theory of Games and Economic Behavior*, (60th Anniversary edition), Princeton.

[7] Joshua R. Goldstein and Thomas Cassidy (2012). *How slowing senescence translates into longer life expectancy*. Population Studies, Vol. 66, No. 1, pp. 29-37.

[8] K. S. Bastian & Mohanachandran Nair (2009). *Mortality changes and life expectancy-an application of Operations Research*. Demography India, Vol. 38, No. 1, pp. 103-116.

[9] Les Mayhew & David Smith (2015). *On the decomposition of life expectancy and limits of limits*. Population Studies. Vol. 69, No. 1, 73-89.

[10] Manisha Dubey (2016). *Trends and prospects of mortality by age and sexing India: 1991-2030*. International Institute of Population Sciences (IIPS).

[11] P. K. Tripathy (2007). *Operations Research: Methods and Practice*, (2nd edition), Kalyani Publishers, New Delhi.

[12] P. K. Tripathy and A Pati (2003). *Bio-social relationship in fertility transition for promoting family planning programme in India*. Population Stabilisation and Development. Edited Book of International Conference on Population and Development, Editor-B. Mishra, Published by- Council of Cultural Relations and Cultural Growths & Supported by UNFPI & PFI, India, pp. 780-788.