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### An Efficient Product-type Exponential Estimator of Finite Population Mean using Variable Transformation

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#### Abstract

This paper proposes an efficient product-type exponential estimator for estimating the finite population mean using variable transformation. The expressions of the bias and mean square error of the proposed estimator, to the first order of approximation, are derived in general form. It has been shown that the proposed estimator is more efficient than the usual unbiased estimator, product estimator, product-type exponential estimator due to Bahl and Tuteja [1] and product-type exponential estimator due to Tailor and Tailor [6]. An empirical study is carried out in support of theoretical findings.

**Keywords and Phrases:** Simple random sampling, product estimator, product-type exponential estimator, variable transformation, bias and mean square error.

AMS Classification: 62D05.

## 1 Introduction

In survey sampling, when the auxiliary variable x is negetively correlated with the study variable y and complete information on x is available, the product method of

estimation is followed to estimate the population mean  $(\overline{Y})$  or the population total (Y). The conventional product estimator is given as

$$\overline{y}_P = \overline{y} \frac{\overline{x}}{\overline{X}}.$$
(1)

With a view to improving estimates of the population mean  $(\overline{Y})$  of the study variable y, Bahl and Tuteja [1] introduced the product-type exponential estimator as

$$\overline{y}_{Pe} = \overline{y}exp\left(\frac{\overline{x} - \overline{X}}{\overline{x} + \overline{X}}\right).$$
(2)

Motivated by the works of Oyenka [3], Srivenkataramana [5] and Tailor and Sharma[7], we have used variable transformation to estimate the population mean. The obvious advantage of variable transformation is the introduction of an additional auxiliary (transformed) variable without additional cost, since new auxiliary variable is a transformation of an already observed auxiliary variable.

Tailor and Tailor [6] proposed dual to Bahl and Tuteja product-type exponential estimator as

$$\overline{y}_{Pe}^* = \overline{y} \exp\left(\frac{\overline{X} - \overline{x}^*}{\overline{X} + \overline{x}^*}\right),\tag{3}$$

where  $\overline{x}^* = \frac{N\overline{X}-n\overline{x}}{N-n}$ , which is arrived at from the variable transformation given by  $x_i^* = \frac{N\overline{X}-nx_i}{N-n}$ ,  $i = 1, 2, \dots, N$ .

## 2 Bias and mean square error of competing estimators

It is well known that mean per unit estimator  $\overline{y}$  is an unbiased estimator of population mean  $\overline{Y}$  and its variance is given by

$$V\left(\overline{y}\right) = MSE\left(\overline{y}\right) = \theta \overline{Y}^2 C_y^2,\tag{4}$$

where  $\theta = \frac{N-n}{Nn} = \frac{1-f}{n}$ ,  $f = \frac{n}{N}$  and  $C_y^2 = \frac{S_y^2}{\overline{Y}^2}$ .

We also have, to  $O(n^{-1})$ ,

$$B\left(\overline{y}_{P}\right) = \theta \overline{Y} \rho C_{y} C_{x} \tag{5}$$

$$MSE\left(\overline{y}_{P}\right) = \theta \overline{Y}^{2} \left(C_{y}^{2} + C_{x}^{2} + 2\rho C_{y}C_{x}\right)$$

$$\tag{6}$$

$$B\left(\overline{y}_{Pe}\right) = \theta \overline{Y}\left(\frac{1}{2}\rho C_y C_x - \frac{1}{8}C_x^2\right) \tag{7}$$

$$MSE\left(\overline{y}_{Pe}\right) = \theta \overline{Y}^2 \left(C_y^2 + \frac{1}{4}C_x^2 + \rho C_y C_x\right)$$
(8)

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$$B\left(\overline{y}_{Pe}^{*}\right) = \theta \overline{Y}\left(\frac{3n^{2}}{8(N-n)^{2}}C_{x}^{2} + \frac{n}{2(N-n)}\rho C_{y}C_{x}\right)$$

$$\tag{9}$$

and

$$MSE(\bar{y}_{Pe}^{*}) = \theta \overline{Y}^{2} \left( C_{y}^{2} + \frac{n^{2}}{4(N-n)^{2}} C_{x}^{2} + \frac{n}{(N-n)} \rho C_{y} C_{x} \right),$$
(10)

where  $\rho$  is the correlation coefficient between y and x assumed to be negative.

## 3 Proposed estimator

We propose the following product-type exponential estimator of population mean  $\overline{Y}$ :

$$\overline{y}_{Pe}^{**} = \overline{y} \left[ \left\{ \exp\left(\frac{\overline{X} - \overline{x}^*}{\overline{X} + \overline{x}^*}\right) \right\} \left(\frac{\overline{X}}{\overline{x}^*}\right)^a \right],\tag{11}$$

where  $\alpha$  is a scalar quantity and  $\overline{x}^*$  is as defined earlier.

To obtain the bias and mean square error of the proposed estimator  $\overline{y}_{Pe}^{**}$ , we define the following quantities:

$$\overline{y} = \overline{Y}(1+e_0), \ \overline{x} = \overline{X}(1+e_1), \ \text{such that} \ E(e_0) = E(e_1) = 0, \ E(e_0^2) = \theta C_y^2, \ E(e_1^2) = \theta C_x^2, \ E(e_0e_1) = \theta \rho C_y C_x.$$

Expressing (11) in terms of e's, we have

$$\overline{y}_{Pe}^{**} = \overline{Y} \left[ 1 + e_0 + \left(a + \frac{1}{2}\right) \frac{n}{N-n} e_1 + \left(a + \frac{1}{2}\right) \frac{n}{N-n} e_0 e_1 + \left(\frac{4a^2 + 8a + 3}{8}\right) \frac{n^2}{\left(N-n\right)^2} e_1^2 \right] \right]$$
(12)

which gives rise to

$$B\left(\overline{y}_{Pe}^{**}\right) = \theta \overline{Y}\left[\left(\frac{4a^2 + 8a + 3}{8}\right)\frac{n^2}{\left(N - n\right)^2}C_x^2 + \left(a + \frac{1}{2}\right)\frac{n}{N - n}\rho C_y C_x\right]$$
(13)

and

$$MSE\left(\overline{y}_{Pe}^{**}\right) = \theta \overline{Y}^{2} \left[ C_{y}^{2} + \left(a + \frac{1}{2}\right)^{2} \frac{n^{2}}{\left(N-n\right)^{2}} C_{x}^{2} + 2\left(a + \frac{1}{2}\right) \frac{n}{N-n} \rho C_{y} C_{x} \right].$$
(14)

The above expressions of bias and mean square error are, to the first order of approximation, i.e., to  $O(n^{-1})$ .

# 4 Efficiency comparison

In this section, we have derived the conditions under which the proposed estimator  $\overline{y}_{Pe}^{**}$  is more efficient than the estimators  $\overline{y}$ ,  $\overline{y}_P$ ,  $\overline{y}_{Pe}$  and  $\overline{y}_{Pe}^*$ . From (4), (6), (8), (10) and (14), we find  $MCE(\overline{z}^{**}) \leftarrow MCE(\overline{z})$ 

$$MSE\left(\overline{y}_{Pe}^{*}\right) < MSE\left(\overline{y}\right)$$
  
if  $\left(a + \frac{1}{2}\right) \frac{n}{N-n} \left[\left(a + \frac{1}{2}\right) \frac{n}{N-n} + 2k\right] < 0$ , where  $k = \rho \frac{C_y}{C_x}$   
*i.e., if*  $\min\left\{-\frac{1}{2}, -\left(\frac{N-n}{n}\right)2k - \frac{1}{2}\right\} < a < \max\left\{-\frac{1}{2}, -\left(\frac{N-n}{n}\right)2k - \frac{1}{2}\right\}$ .  
(15)  

$$MSE\left(\overline{y}_{Pe}^{**}\right) < MSE\left(\overline{y}_P\right)$$
  
if  $\left\{\left(a + \frac{1}{2}\right) \frac{n}{N-n} - 1\right\} \left\{\left(a + \frac{1}{2}\right) \frac{n}{N-n} + 1 + 2k\right\} < 0$   
*i.e., if*  $\min\left\{\frac{2N-3n}{2n}, (-2k-1)\left(\frac{N-n}{n}\right) - \frac{1}{2}\right\} < a < \max\left\{\frac{2N-3n}{2n}, (-2k-1)\left(\frac{N-n}{n}\right) - \frac{1}{2}\right\}$ .  
(16)

$$MSE\left(\overline{y}_{Pe}^{**}\right) < MSE\left(\overline{y}_{Pe}\right)$$
  
if  $\left\{\left(a+\frac{1}{2}\right)\frac{n}{N-n}-\frac{1}{2}\right\}\left\{\left(a+\frac{1}{2}\right)\frac{n}{N-n}+\frac{1}{2}+2k\right\} < 0$   
*i.e., if min*  $\left\{\frac{N-2n}{2n}, (-2k-\frac{1}{2})\left(\frac{N-n}{n}\right)-\frac{1}{2}\right\} < a < max\left\{\frac{N-2n}{2n}, (-2k-\frac{1}{2})\left(\frac{N-n}{n}\right)-\frac{1}{2}\right\}$   
(17)

and

$$MSE\left(\overline{y}_{Pe}^{**}\right) < MSE\left(\overline{y}_{Pe}^{*}\right)$$

$$if\{\left(a+\frac{1}{2}\right)-\frac{1}{2}\}\left[\frac{n}{N-n}\left\{\left(a+\frac{1}{2}\right)+\frac{1}{2}\right\}+2k\right]<0$$
  
*i.e.*, *if* min  $\left\{0, -2k\left(\frac{N-n}{n}\right)-1\right\}< a < max \left\{0, -2k\left(\frac{N-n}{n}\right)-1\right\}.$  (18)

# 5 Optimum choice of the scalar $\alpha$

Differentiating (14) with respect to  $\alpha$  and equating it to zero, we get the optimum value of  $\alpha$  as

$$\alpha_{opt.} = -\left(\frac{N-n}{n}\right)\rho\frac{C_y}{C_x} - \frac{1}{2} = -\left(\frac{N-n}{n}\right)k - \frac{1}{2}$$
(19)

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Substituting this value of  $\alpha$  in (14), the optimum value of the  $MSE(\overline{y}_{Pe}^{**})$  can be

$$MSE(\overline{y}_{Pe}^{**})_{opt.} = \theta \overline{Y}^2 C_y^2 (1 - \rho^2).$$
<sup>(20)</sup>

The expression in (20) is equal to the mean square error of linear regression estimator  $\overline{y}_{lr} = \overline{y} + \hat{\beta}(\overline{X} - \overline{x})$ . Thus, the proposed estimator  $\overline{y}_{Pe}^{**}$  can be used as an alternative to the usual linear regression estimator  $\overline{y}_{lr}$  when  $\alpha$  is chosen optimally. It is apt to mention here that, for large samples, i.e., when  $n \to N$ , the proposed estimator approaches the corresponding parameter implying thereby that the proposed estimator is consistent.

### 6 Empirical Study

For the purpose of numerical illustration, we refer to Weisberg [8] wherein the given sample quantities have been considered as the corresponding population quantities. We consider the sample size n = 10.

The variables are

y: Rate (1973 accident rate per million vehicle miles)

x: SLIM = Speed limit =(in 1973, before the 55 mph limits)

and N = 39, n = 10,  $\overline{Y} = 3.93$ ,  $\overline{X} = 55$ ,  $C_y = 0.51$ ,  $C_x = 0.11$ ,  $\rho = -0.68$ .

Using the conditions which we have obtained in section 4, we calculate the range of the scalar  $\alpha$  in which the proposed estimator  $\overline{y}_{Pe}^{**}$  is more efficient than the estimators  $\overline{y}, \overline{y}_{P}, \overline{y}_{Pe}$  and  $\overline{y}_{Pe}^{*}$  and present the same in Table 1.

**Table 1:** The range of  $\alpha$  in which  $\overline{\mathbf{y}}_{\mathbf{P}e}^{**}$  is more efficient than  $\overline{\mathbf{y}}, \overline{\mathbf{y}}_{\mathbf{P}}, \overline{\mathbf{y}}_{\mathbf{P}e}$  and  $\overline{\mathbf{y}}_{\mathbf{P}e}^{*}$ 

Estimators	$\overline{y}$	$\overline{y}_P$	$\overline{y}_{Pe}$	$\overline{y}_{Pe}^{*}$
Range of $\alpha$	(-0.50, 17.78)	(2.4, 16.33)	(0.95, 16.33)	(0, 17.28)

It may be noted here that the common range of the scalar  $\alpha$  in which  $\overline{y}_{Pe}^{**}$  is more efficient than  $\overline{y}$ ,  $\overline{y}_P$ ,  $\overline{y}_{Pe}$  and  $\overline{y}_{Pe}^{*}$  is (2.4, 16.33). To evaluate the performance of the  $\overline{y}_{Pe}^{**}$  over the existing estimators, we have computed percent relative efficiencies (PREs) of  $\overline{y}_{Pe}^{**}$  with respect to  $\overline{y}$ ,  $\overline{y}_P$ ,  $\overline{y}_{Pe}$  and  $\overline{y}_{Pe}^{*}$  in the common range (2.4, 16.33) of  $\alpha$  using respective formulae given by

$$PRE\left(\overline{y}_{Pe}^{**}, \ \overline{y}\right) = \frac{MSE\left(\overline{y}\right)}{MSE\left(\overline{y}_{Pe}^{**}\right)} \times 100$$
(21)

$$PRE\left(\overline{y}_{Pe}^{**}, \overline{y}_{P}\right) = \frac{MSE\left(\overline{y}_{P}\right)}{MSE\left(\overline{y}_{Pe}^{**}\right)} \times 100$$
(22)

$$PRE\left(\overline{y}_{Pe}^{**}, \overline{y}_{Pe}\right) = \frac{MSE\left(\overline{y}_{Pe}\right)}{MSE\left(\overline{y}_{Pe}^{**}\right)} \times 100$$
(23)

$$PRE\left(\overline{y}_{Pe}^{**}, \overline{y}_{Pe}^{*}\right) = \frac{MSE\left(\overline{y}_{Pe}^{*}\right)}{MSE\left(\overline{y}_{Pe}^{**}\right)} \times 100$$
(24)

**Table 2:** PREs of  $\overline{\mathbf{y}}_{\mathbf{P}e}^{**}$  with respect to the estimators  $\overline{\mathbf{y}}$ ,  $\overline{\mathbf{y}}_{\mathbf{P}}$ ,  $\overline{\mathbf{y}}_{\mathbf{P}e}$  and  $\overline{\mathbf{y}}_{\mathbf{P}e}^{*}$ 

$\alpha$	$PRE\left(\overline{y}_{Pe}^{**}, \ \overline{y}\right)$	$PRE\left(\overline{y}_{Pe}^{**}, \overline{y}_{P}\right)$	$PRE\left(\overline{y}_{Pe}^{**}, \overline{y}_{Pe}\right)$	$PRE\left(\overline{y}_{Pe}^{**}, \overline{y}_{Pe}^{*}\right)$
2.4	132.76	100.00	114.89	126.22
2.85	138.29	104.17	119.67	131.48
3.15	141.90	106.89	122.80	134.92
4.50	158.04	119.05	136.77	150.26
5.25	166.31	125.28	143.93	158.13
6.35	176.43	132.90	152.69	167.75
7.80	184.72	139.15	159.86	175.63
8.64	185.99	140.10	160.96	176.84
9.37	185.07	139.40	160.16	175.96
10.25	181.14	136.45	156.76	172.22
11.50	171.57	129.24	148.48	163.12
12.75	158.46	119.36	137.13	150.66
13.85	145.35	109.49	125.79	138.20
14.55	136.83	103.07	118.41	130.10
15.30	127.76	96.24	110.56	121.47
16.33	115.68	87.14	100.17	109.99

It is observed from Table 2 that the performance of proposed estimator  $\overline{y}_{Pe}^{**}$  is better than the estimators  $\overline{y}$ ,  $\overline{y}_P$ ,  $\overline{y}_{Pe}$  and  $\overline{y}_{Pe}^*$ , the maximum gain in efficiency being attained at the optimum value  $\alpha = 8.64$ . It may also be noticed that in the event of an absolute deviation to the extent of 70% from the optimum value of  $\alpha$ , the superiority of the proposed estimator is maintained.

## 7 Conclusion

We have considered the problem of estimating the population mean of the study variable when the population mean of auxiliary variable is known using variable transformation. The proposed estimator, to the first order of approximation, is more efficient than the existing estimators. Theoretical findings have been demonstrated through empirical investigation.

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