International Journal of Statistical Sciences *ISSN 1683–5603*
Vol. 12, 2012, pp 29-46

Multi-State Markov Chain Modelling System for Environmental Impact of Climate Change

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[Received May 26, 2008; Revised March 13, 2012; Accepted March 30, 2012]

Abstract

Environmental sciences are inherently affected by uncertainty in many of the involved processes. Statistics plays, therefore, an essential role in this study. This paper discussed advance statistical tools which will have a reflective impact in these fields. The stochastic Markov chain method was applied to a data set from a rainfall network of the high Barind region, where rainfall is highly seasonal and data availability at a daily time scale or even higher temporal resolution is very limited. A detailed analysis was carried out to study the seasonal and spatial variability of many properties of the daily rainfall in order to incorporate the selected statistics. The aim of this paper is to show how it is possible to formulate in the frames of mathematical statistics the problem concerning determination of a time series data in terms of the theory of hypothesis testing. The study was to investigate prediction transition probabilities of weekly rainfall for environmental impact analysis. The rainfall events are asymptotically normally distributed. The techniques are based on Maximum likelihood estimation of Markov chain modeling system. It is important to be able to model estimate extremes of drought or flood. This robust method enabled us to identify outliers of the data. Therefore, as an application, policy makers could monitor for an increase in the rate of global warming. This study would play a vital role to mitigate future environmental impacts of climate change. It has also implications for prediction of the transition times, that is, the system spends in a given time, since this would be equivalent to predicting the duration of droughts or wet conditions. The impacts of climate change on agricultural food production are global concerns, and they are very important for Bangladesh. Global warming such as floods and droughts has major impacts in terms of human life, economic and environmental losses, and social disorder. Effective management system can mitigate the impact of global warming. The study results may be used for important environmental policy issues in Bangladesh.

Keywords and Phrases: Markov chain modeling system, Environment statistics, Conditional probability, Maximum likelihood estimate, Global warming.

AMS Classification: 62M05, 60-xx.

1 Introduction

Modelling studies that relate to the potential for substantial climate change and address questions regarding important "feedbacks" in the climate system are an exciting new direction in research. Aspects of climate change research are equally exciting areas where substantial progress is being made. In a sense, many modeling studies of the past have laid the foundations for understanding the role in climate change (Gates *et al*, 1999). A class of seasonal space-time models for general lattice systems is proposed. Covariance properties of spatial first-order models are studied. Estimation approaches in time series analysis are adopted and forecasting techniques using the seasonal space-time models are discussed. Authors found that space-time models with instantaneous spatial components give the best fit compared to other models in terms of maximizing the conditional likelihood function. The models are potentially useful for assessing the consistency of outputs from laboratory-based numerical models with field observations. Forecasting ability of the seasonal space-time models is also investigated (Niu *et al,* 2003).

Markov chain Monte Carlo techniques are used to generate samples from the posterior distributions of the parameters. Finally, this model is applied to the spatial prediction of weekly rainfall (Kim and Mallick, 2004). Markov chain method was applied to a data set from a rainfall network of the central plains of Venezuela, where rainfall is highly seasonal and data availability at a daily time scale or even higher temporal resolution is very limited. A detailed analysis was carried out to study the seasonal and spatial variability of many properties of the daily rainfall as scaling properties and autocorrelation function in order to incorporate the selected statistics and their annual cycle into an objective function to be minimized in the simulation procedure (Guenni and Bardossy, 2002). Barkotulla and Rahman (2007) indicated that the Barind regions were found to be prone to severe to moderate drought proneness in the kharif season. Chronic drought proneness was also found in the rabi season. As a result, failure of rains and the occurrence of drought during any particular growing season lead to severe food shortages.

Dynamical systems theory allows us to reconstruct some properties of a phenomenon based only on past behavior without any mechanistic assumptions or deterministic models. A near-term prediction of temperature, including a mean estimate and confidence interval, is made for 800 years into the future. The prediction suggests that the present short-time global warming trend will continue for at least 200 years and be followed by a reverse in the temperature trend (Kotov, 2003).

Environmental and ecological statistics is hovering for dramatic growth both for reasons of societal challenge and information technology. It is becoming clear that environmental and ecological statistics is demanding more and more of non-traditional statistical approaches. This is partly because environmental and ecological studies involve space, time and relationships between many variables, and require innovative and cost-effective monitoring, sampling and assessment. Also, environmental and ecological statistics methodology must satisfy environmental policy needs in addition to disciplinary and interdisciplinary environmental and ecological research imperatives (Patil *et al*, 2000). In the soil moisture balance equation, stochastic fluctuations lead to separate preferred statistical stable states with transitions between these stable states induced by environmental fluctuations (Lee and Yoo, 2001).

How much does the far future matter? This question lies at the heart of many important environmental policy issues, such as global climate change, biodiversity loss, and the disposal of radioactive waste. Although philosophers, experts, and others offer their viewpoints on this deep question, the solution of too many environmental problems lies in the willingness of the public to bear significant costs now to make the far future a better place (Layton and Levine, 2003). Characterizing the complex atmospheric variability at all pertinent temporal and spatial scales remains one of the most important challenges to scientific research today. The main issues are to quantify, within reasonably narrow limits, the potential extent of global warming, and to downscale the global results in order to describe and quantify the regional implications of global change (Bunde and Havlin, 2003).

Hydrology and water resources management are inherently affected by uncertainty in many of their involved processes, including inflows, rainfall, water demand, evaporation, etc. Statistics plays, therefore, an essential role in their study (Rios *et al*, 2002). Extreme hydrological events are inevitable and stochastic in nature. Characterized by multiple properties, the multivariate distribution is a better approach to represent this complex phenomenon than the univariate frequency analysis. However, it requires considerably more data and more sophisticated mathematical analysis (Shiau, 2003). The forecasting skill of meteorologists is determined largely by their ability to interpret output from deterministic numerical models in the light of local conditions. A statistical method for correction and interpretation of model output is the model output statistics technique. Markov chain methods are used for computation by using daily maximum temperature data from a Sydney area (Nott *et al*, 2001).

Drought is a temporary feature in the sense that, considered in the context of variability, it is experienced only when precipitation falls appreciably below normal. The drought identification and evaluation procedures slowly evolved during the first half of the twentieth century from simplistic approaches that considered the phenomenon to be a rainfall deficiency, to problem-specific models of limited applicability. Impacts also differ from one location to the next depending on the societal context in which drought is occurring (Reddy, 2001). Drought occurs when various combinations of the physical factors of the environment produce internal water stress in crop plants sufficient to reduce their productivity. Almost all the year, drought occurs in one part of the country/globe or the other with varying severities. Also, because drought affects natural habitats, ecosystems, and many economic and social sectors, from the foundation of civilization agriculture—to transportation, urban water supply, and the modern complex industries. The wide variety of sectors affected by drought, its diverse geographical and temporal distribution, and the demand placed on water supply by human-use systems make it difficult to develop a single definition of drought (Richard, 2002).

Guttman (1999) determined that the Pearson Type III distribution is the "best" universal model for computing the probability distribution. The SPI methodology allows expression of droughts (and wet spells) in terms of precipitation deficit, percent of normal, and probability of non-exceeded as well as the SPI. Whether a drought warning should be issued or not is determined by comparing the amount of predicted water shortage with the probability distribution of water deficit amounts generated by past drought events. The lead time of this drought warning system will be limited by the skill of regional climate prediction (Sui, *et al*, 2002). The use of appropriate statistical methods is essential when working with environmental data. Yet, many environmental professionals are not statisticians. A ready reference guide to the most common methods used in environmental applications, Statistics for Environmental Science and Management introduces the statistical methods most frequently used by environmental scientists, managers, and students (Manly, 2001). Estimation and prediction of the amount of rainfall in time and space is a problem of fundamental importance in many applications in agriculture, hydrology, and ecology.

Stochastic simulation of rainfall data is also an important step in the development of stochastic downscaling methods where large-scale climate information is considered as an additional explanatory variable of rainfall behavior at the local scale. Simulated rainfall has also been used as input data for many agricultural, hydrological, and ecological models, especially when rainfall measurements are not available for locations of interest or when historical records are not of sufficient length to evaluate important rainfall characteristics as extreme values (Sanso and Guenni, 2000). Climate change occurs due to natural and anthropogenic disturbances in our environment. These anthropogenic factors may contribute to the observed climatic changes and variations. The degree of climatic changes is very important in assessing environmental impacts, such as, the increase of bacteria, virus and related diseases that have been reported. The global climate change is the sum of both local and regional departures in climate elements and variables (Munn, 1998).

Extreme weather events, such as, droughts, floods, heat waves and heavy rainfall are expected to increase over the next 100 years, according to a team of scientists from the U. S. National Climatic Data Center, Ashville, North Carolina. These changes will continue to increase with the rise of "ever greater amounts of GHGs in the atmosphere." The report found that used data and climate models are necessary to examine past and future changes in climate extremes. The extreme events will cause sharply increased financial losses and are likely to lead to the extinction of more plant and animal species (Easterling. *et al*, 2000a).

2 Methodology

The secondary data has been used for the 14 stations of the Barind region. The daily rainfall data were available for 8 years during the period from 1994-2001 and were considered for this study. The daily rainfall data were reduced in the weekly form to calculate the transition probability using the Markov chain model. The probability estimate by using maximum likelihood method is considered for annual, seasonal like pre-kharif (March to May), kharif (June to October) and rabi (November to February). The week with rainfall greater than the threshold value (a minimum amount, say 2.5mm) is considered to be a wet week (Banik *et al*, 2002). The high Barind region (Brammer *et al*, 1988) is a distinct agro-ecological unit located in the northwestern part of the country. The Barind Tract is a distinct agro-ecological unit, located in the northwestern part (Rajshahi, Naogaon and Nawabganj districts) of the country, which has already been identified as the most drought prone area. The Barind Tract experiences frequent drought and it has started showing signs of desertification. This area is considered to be an ecologically fragile zone with extremely low vegetative cover (NEMAP, 1995).

2.1 Multi-State Markov Chain Modelling System

Statistical inference is an essential tool for concluding any research. The probability theory of Markov chain has been extensively developed (Medhi, 1981).

Let $n_i(t\text{-}1) = \sum_{j=1}^{m} n_{ij}(t)$. Then the conditional distribution of $n_{ij}(t)$, $j=1,2,\ldots,m$ given $n_i(t\text{-}1)$ (or given $n_k(s)$, $k=1,2,\ldots,m;\ s=0,\ldots,t\text{-}1$) is

$$\frac{n_i(t\ -1)!}{\prod_{j=1}^{m} n_{ij}(t)!}\prod_{j=‘}^{m} p_{ij}(t)^{n_{ij}(t)} \tag{1}$$

This is the same distribution, as one would obtain if one had $n_i(t\text{-}1)$ observations on a multinomial distribution with probability $p_{ij}(t)$ and with resulting numbers $n_{ij}(t)$. The distribution of the $n_{ij}(t)$ (conditional on the $n_i(0)$) is

$$\prod_{t=1}^{T}\left\{\prod_{i=1}^{m}\left[\frac{n_i(t-1)!}{\prod_{j=1}^{m} n_{ij}(t)!}\prod_{j=1}^{m} p_{ij}(t)^{n_{ij}(t)}\right]\right\}. \tag{2}$$

For a Markov chain with stationary transitional probabilities can be written in the form

$$\prod_{t=1}^{T}\prod_{g,j} p_{gj}^{n_{gj}(t)} = \prod p_{ij}^{n_{ij}} \tag{3}$$

For not necessarily stationary transitional probabilities $p_{ij}(t)$, the $n_{ij}(t)$ are a minimal set of sufficient statistics.

The stationary transitional probabilities p_{ij} can be estimated by maximizing the probability (3) with respect to the p_{ij}, subjected of course, to the restrictions $p_{ij} \geq 0$ and

$$\sum_{j=1}^{m} P_{ij} = 1, i\text{=}1,2,\ldots\ldots.,m, \tag{4}$$

when the n_{ij} are the actual observations. This probability is precisely of the same form, except for a factor that does not depend on p_{ij}, as that obtained for m independent samples, where the ith sample (i=1,2,......,m) consists of $n_i{*}{=}\sum_j n_{ij}$ multinomial trials with probabilities p_{ij} (i,j=1,2,......,m). For such samples, it is well-known and easily verified that the maximum likelihood estimates for p_{ij} are

$$\begin{aligned}\underset{ij}{\hat{p}} &= \frac{n_{ij}}{n_i^*} = \sum_{t=1}^{T} n_{ij}(t) \Big/ \sum_{k=1}^{m} \sum_{t=1}^{T} n_{ik}(t) \\ &= \sum_{t=1}^{T} n_{ij}(t) \Big/ \sum_{t=0}^{T-1} n_i(t),\end{aligned} \tag{5}$$

and hence this is also true for any other distribution in which the elementary probability is of the same form except for parameter-free factors, and the restrictions on the p_{ij} are the same. In particular, it applies to the estimation of the parameters p_{ij} in (3).

When the transitional probabilities are not necessarily stationary, the general approach can still be applied, and the maximum likelihood estimates for the $p_{ij}(t)$ are found to be

$$\hat{p}_{ij}(t) = n_{ij}(t)/n_i(t-1) = n_{ij}(t) \Bigg/ \sum_{k=1}^{m} n_{ik}(t). \tag{6}$$

The same maximum likelihood estimates for the $p_{ij}(t)$ are obtained when we consider the conditional distribution of $n_{ij}(t)$ given $n_i(t\text{-}1)$ as when the joint distribution of the $n_{ij}(1)$, $n_{ij}(2)$,......, $n_{ij}(T)$ is used and hence are also asymptotically normally distributed (Rahman, 1999a,b).

Several authors have found that sequences in daily rainfall occurrences can be described by a simple Markov chain model. Additional evidence to indicate the feasibility of using a Markov chain model has been presented by Katz (1974), Anderson and Goodman (1957), Rahman (2000), and Rahman and Mian (2002).

Let X_0, X_1, X_2, ...,X_n, be random variables distributed identically and taking only two values, namely 0 and 1, with probability one, i.e.,

$$X_n = \left\{ \begin{array}{l} 0 \text{ if the nth week is dry} \\ 1 \text{ if the nth week is wet} \end{array} \right\}$$

Under the same set up, now assume

$$P(X_{n+1}{=}x_{n+1}|X_n{=}x_n, X_{n-1}{=}x_{n-1},\ldots,X_0{=}x_0) = P(X_{n+1}{=}x_{n+1}|X_n{=}x_n, X_{n-1}{=}x_{n-1})$$

where x_0, x_1,...,$x_{n+1} \in \{0,1\}$.

In other words, it is assume that probability of wetness of any week depends only on whether the two preceding weeks were wet or dry. Given the event on previous two weeks, the probability of wetness is independent of further preceding weeks.

Let us define $Y_0=(X_0,X_1)$, $Y_1=(X_2,X_3)$, $Y_2=(X_4,X_5),\ldots$, $Y_n=(X_{2n},X_{2n+1})$,

Also consider, $P(Y_{n+1}=y_{n+1}|Y_n=y_n, Y_{n-1}=y_{n-1},\ldots, Y_0=y_0)= P(Y_{n+1}=y_{n+1}|Y_n=y_n)$ where $y_0, y_1, \ldots, y_{n+1}\in\{(0,0), (0,1), (1,0), (1,1)\}$. Now the stochastic process $\{Y_n$, $n=0, 1, 2, \ldots\}$ is a Markov Chain (Bhat, 1972; Ochi, 1990 and Chung, 1974).

Consider the transition matrix

$$\begin{bmatrix} a_{00} & a_{01} & a_{02} & a_{03} \\ a_{10} & a_{11} & a_{12} & a_{13} \\ a_{20} & a_{21} & a_{22} & a_{23} \\ a_{30} & a_{31} & a_{32} & a_{33} \end{bmatrix} \tag{7}$$

where $a_{ij} = P\left(Y_1 = j\,|Y_0 = i\right)$

$$i \text{ or } j = \left\{ \begin{array}{l} 0 \text{ stands for the state } (0,0) \\ 1 \text{ stands for the state } (0,1) \\ 2 \text{ stands for the state } (1,0) \\ 3 \text{ stands for the state } (1,1) \end{array} \right\}$$

Let $P(0,0)=P_1$, $P(0,1)=P_2$, $P(1,0)=P_3$, $P(1,1)=P_4$

Note $\sum_{i=1}^{4} P_i = 1$ $\sum_{j=0}^{3} a_{ij} = 1$ where i=0,1,2,3.

For a stationary distribution,

$$[P_1 \quad P_2 \quad P_3 \quad P_4] \begin{bmatrix} a_{00} & a_{01} & a_{02} & a_{03} \\ a_{10} & a_{11} & a_{12} & a_{13} \\ a_{20} & a_{21} & a_{22} & a_{23} \\ a_{30} & a_{31} & a_{32} & a_{33} \end{bmatrix} = [P_1 \quad P_2 \quad P_3 \quad P_4] \tag{8}$$

Here, when we find the probability of a wet spell of length k, we actually mean, it is the probability of a wet spell of length k given that this week is wet and the previous week was dry one.

Similarly we account for dry spell also.

P(W=k)=P(Wet spell of length k|this week is wet and previous week is dry)

$$P(W = k) \quad k\ odd,\ k \geq 3$$

$$= P(W = 2m - 1) \quad m \geq 2$$

$$= a_{13}a_{33}^{m-2}\left(a_{30} + a_{31}\right) \tag{9}$$

$$P(W = k) \quad k\ even,\ k \geq 4$$

$$= P(W = 2n) \quad n \geq 2$$

$$= a_{13}a_{33}^{n-2}a_{32} \tag{10}$$

$$P(W = 2) = P[(1, 0) \mid (0, 1)] = a_{12} \tag{11}$$

$P(W \geq 2t) = \sum_{k=2t}^{\infty} P(W = k),\ t \geq 2$

$$= \left[a_{32} + a_{33}\left(a_{30} + a_{31}\right)\right] \frac{a_{13}a_{33}^{t-2}}{1 - a_{33}} \tag{12}$$

$P(W \geq\ 2t - 1) = P(W \geq\ 2t) + P(W =\ 2t - 1),\ t \geq 2$

$$= a_{13}a_{33}^{t-2} \tag{13}$$

Similarly,

$P(D = k)$=P(Dry spell of length k this week is dry and previous week was wet)
$P(D = k)$ k odd, $k \geq 3$

$$= P(D = 2m - 1) m \geq 2$$

$$= a_{20}a_{00}^{m-2}\left(a_{02} + a_{03}\right) \tag{14}$$

$P(D = k)$ k even, $k \geq 4$

$$= P(D = 2n) n \geq 2$$

$$= a_{20}a_{00}^{n-2}a_{01} \tag{15}$$

Also,

$$P(D \geq 2t) = \left\{a_{01} + a_{00}\left(a_{02} + a_{03}\right)\right\} \frac{a_{20}a_{00}^{t-2}}{1 - a_{00}},\quad t \geq 2 \tag{16}$$

$P(D \geq\ 2t - 1) = P(D \geq\ 2t) + P(D =\ 2t - 1),\ t \geq 2$

$$= a_{20}a_{00}^{t-2} \tag{17}$$

Let U be the random variable such that U =number of wet weeks among *2n* week period. Therefore, *U=f(Y_0)+ f(Y_1)+. . . . . +f(Y_{n-1})*

where

$$\begin{aligned} f(0,0) &= 0 \\ f(0,1) &= 1 \\ f(1,0) &= 1 \\ f(1,1) &= 2 \end{aligned}$$

For large *n,* U=N(nμ , nσ^2) [for large n, 2n≈2n-1] (Medhi, 1981).

$$\mu = f(0,0) \times P_1 + f(0,1) \times P_2 + f(1,0) \times P_3 + f(1,1) \times P_4 \tag{18}$$

$$\sigma^2 = F'CF \tag{19}$$

$$F = \begin{pmatrix} f(0,0) \\ f(0,1) \\ f(1,0) \\ f(1,1) \end{pmatrix}$$

and C=[(c_{ij})]

where $c_{ij} = P_i z_{ij} + P_j z_{ji} - P_i \delta_{ij} - P_i P_j$

with $\delta_{ij} = \begin{cases} 1 & if\ i = j \\ 0 & otherwise \end{cases}$

$$Z = [(z_{ij})] = \begin{pmatrix} 1 + P_1 - a_{00} & P_2 - a_{01} & P_3 - a_{02} & P_4 - a_{03} \\ P_1 - a_{10} & 1 + P_2 - a_{11} & P_3 - a_{12} & P_4 - a_{13} \\ P_1 - a_{20} & P_2 - a_{21} & 1 + P_3 - a_{22} & P_4 - a_{23} \\ P_1 - a_{30} & P_2 - a_{31} & P_3 - a_{32} & 1 + P_4 - a_{33} \end{pmatrix}^{-1} \tag{20}$$

3 Results and Discussion

The multi-state Markov chain model had been tested on weekly rainfall data in the Barind region for 8 years available data from the period from 1994 to 2001. The conditional probabilities were estimated by using maximum likelihood estimation techniques. The mean distribution of wet weeks estimates for large sample case, where the number of wet week follows asymptotic normal distribution. Similar results were also reported by Banik *et al* (2002). In stochastic process, the conditional probability is always greater than the stationary probability which suggests that the effect of persistence is significant. Based on this result, it could be concluded that the Markov chain model seemed to be doing a good job in the field of environmental sciences. Probability distribution of dry spell showed more seasonal variability in rabi and pre-kharif seasons in the for Barind region (Fig. 1). The results also showed that the kharif season and annual variability of dry spell were almost similar in this region (Fig. 1 and Tables 1, 2, 3 & 4). The scenario of mean distribution of wet spell showed that more variability occurred in annual and kharif season in this region. The results also indicated that the pre-kharif and rabi seasons are almost silimar (Fig. 6 and Tables 1, 2, 3 & 4).

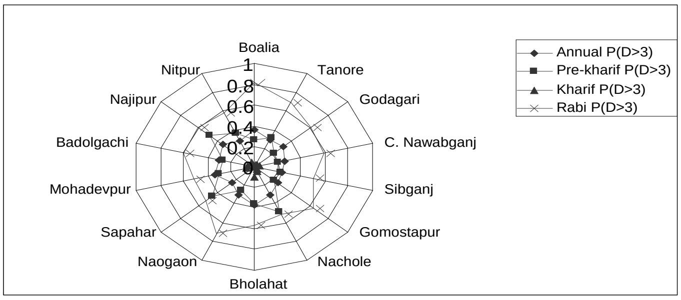

Figure 1: Seasonal variability of probability distribution of dry spell in the Barind region

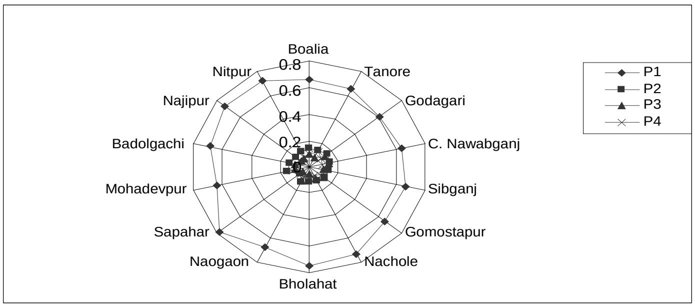

Figure 2: Annual variability of probability distribution of wet spell in the Barind region

Tables 1, 2, 3 and 4 gave the value of P_1, P_2, P_3 and P_4which discussed for the Annual, Pre-Kharif, Kharif, and Rabi seasons probabilities of wet and dry weeks for the 14 stations of Rajshahi, Chapai Nawabganj and Naogaon districts in the Barind region. Probability distribution of wet spell showed that P_1 is more variability in annually (Fig. 2), pre-kharif season (Fig.3) and rabi season (Fig. 4) and only P_4 is more variability in kharif season (Fig.5) Here the probabilities of getting at least 8, 10 and 12 wet weeks are computed under the assumption of normality using equations (18) and (19). But the corresponding probabilities given in every Tables are more reliable because of the conditions for assumptions of normality are more favorable for Markov Chain model (Rahman, 1999a; 2000; Rahman and Mian, 2002).

On the other hand, probability of a dry spell of length of at least 3 weeks shown

Table 1: Annual probability of wet and dry weeks in 14 stations in Barind region

Annual									
Station	P1	P2	P3	P4	E(U)	P(w>8)	P(w>10)	P(w>12)	P(D>3)
Boalia	0.66	0.14	0.1	0.1	11.44	0.16	0.12	0.09	0.36
Tanore	0.65	0.14	0.08	0.13	12.48	0.12	0.08	0.05	0.29
Godagari	0.61	0.16	0.13	0.1	12.74	0.11	0.08	0.06	0.3
Chapai Nawab-ganj	0.64	0.14	0.13	0.09	11.7	0.12	0.08	0.05	0.26
Sibganj	0.67	0.13	0.1	0.1	11.18	0.11	0.08	0.06	0.24
Gomostapur	0.65	0.13	0.12	0.1	11.7	0.11	0.07	0.05	0.25
Nachole	0.73	0.11	0.1	0.07	9.1	0.11	0.07	0.04	0.3
Bholahat	0.75	0.11	0.05	0.09	8.84	0.23	0.18	0.14	0.37
Naogaon	0.68	0.12	0.11	0.08	10.14	0.23	0.16	0.12	0.3
Sapahar	0.78	0.08	0.06	0.08	7.8	0.11	0.07	0.04	0.24
Mohadevpur	0.64	0.15	0.1	0.11	12.22	0.17	0.12	0.09	0.34
Badolgachi	0.69	0.13	0.1	0.08	10.14	0.19	0.13	0.09	0.3
Najipur	0.73	0.11	0.07	0.09	9.36	0.19	0.14	0.1	0.34
Nitpur	0.72	0.12	0.07	0.09	9.62	0.08	0.05	0.03	0.28

Table 2: Probability of wet and dry weeks in the pre-kharif season of 14 stations in the Barind region

Pre-kharif									
Station	P1	P2	P3	P4	E(U)	P(w>8)	P(w>10)	P(w>12)	P(D>3)
Boalia	0.51	0.18	0.18	0.13	4.03	0	0	0	0.26
Tanore	0.57	0.19	0.13	0.11	3.51	0.01	0	0	0.31
Godagari	0.47	0.18	0.24	0.11	4.16	0	0	0	0.21
Chapai Nawab-ganj	0.53	0.15	0.26	0.06	3.45	0.01	0	0	0.2
Sibganj	0.67	0.1	0.1	0.13	2.99	0.01	0	0	0.23
Gomostapur	0.42	0.17	0.25	0.17	4.94	0	0	0	0.21
Nachole	0.72	0.1	0.16	0.02	1.95	0.01	0	0	0.48
Bholahat	0.82	0.08	0.08	0.02	1.3	0.05	0.03	0.02	0.36
Naogaon	0.55	0.16	0.23	0.06	3.32	0.03	0.01	0	0.25
Sapahar	0.79	0.06	0.04	0.11	2.08	0	0	0	0.45
Mohadevpur	0.52	0.17	0.17	0.14	4.03	0.02	0.01	0	0.3
Badolgachi	0.63	0.12	0.19	0.06	2.8	0.02	0.01	0	0.27
Najipur	0.75	0.11	0.08	0.06	2.02	0.02	0.01	0	0.48
Nitpur	0.68	0.13	0.11	0.08	2.6	0	0	0	0.36

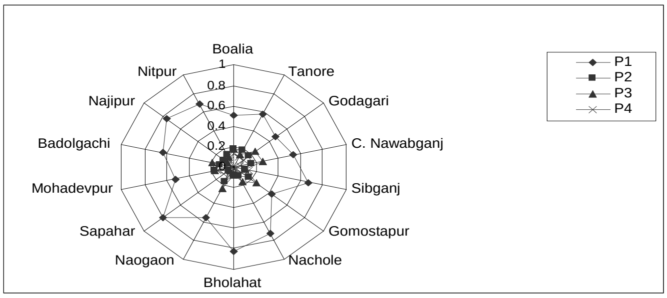

Figure 3: Annual variability of probability distribution of wet spell in the Barind region

in Tables 1 to 4, calculated using equation (16) is more satisfactory compared to that given in the method of two-state Markov Chain model because, in this case, exact probabilities are calculated without any asymptotic assumption (Banik *et al*, 2002).

Probability distribution showed that the rainfall variability was one of the most important challenges to scientific research. The results indicated that the model is used to mitigate environmental impacts of climate change. Finally the multi-state Markov chain model is applied to the spatial prediction of weekly rainfall of the Barind region.

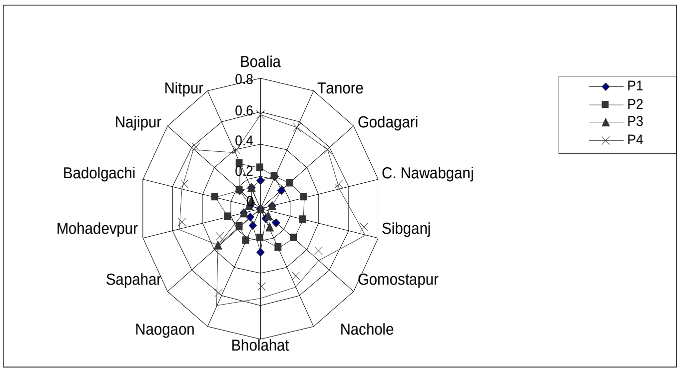

Figure 4: Kharif seasonal variability of probability distribution of wet spell in the Barind region

Table 3: Probability of wet and dry weeks in the kharif season of 14 stations in the Barind region

Pre-kharif									
Station	P1	P2	P3	P4	E(U)	P(w>8)	P(w>10)	P(w>12)	P(D>3)
Boalia	0.17	0.25	0	0.58	15.51	0.59	0.51	0.44	0
Tanore	0.22	0.22	0	0.56	14.74	0.33	0.24	0.17	0.02
Godagari	0.17	0.25	0	0.58	15.51	0.44	0.37	0.30	0.03
Chapai Nawab-ganj	0.08	0.3	0.08	0.54	16.06	0.36	0.28	0.22	0.04
Sibganj	0	0.29	0	0.71	18.81	0.33	0.26	0.21	0
Gomostapur	0.14	0.29	0.07	0.5	14.96	0.38	0.29	0.23	0.03
Nachole	0.07	0.27	0.13	0.53	16.06	0.23	0.16	0.11	0.04
Bholahat	0.27	0.18	0	0.55	14.08	0.65	0.58	0.52	0.10
Naogaon	0.11	0.22	0	0.66	16.94	0.61	0.51	0.43	0
Sapahar	0.09	0.18	0.36	0.36	13.86	0.27	0.20	0.14	0
Mohadevpur	0.11	0.22	0.11	0.56	15.95	0.56	0.48	0.41	0.06
Badolgachi	0.07	0.31	0.08	0.54	16.17	0.47	0.37	0.29	0.05
Najipur	0.17	0.17	0.08	0.58	15.51	0.49	0.41	0.34	0
Nitpur	0.15	0.31	0.15	0.39	13.64	0.19	0.14	0.10	0.04

Table 4: Probability of wet and dry weeks in the rabi season of 14 stations in the Barind region

Pre-kharif									
Station	P1	P2	P3	P4	E(U)	P(w>8)	P(w>10)	P(w>12)	P(D>3)
Boalia	0.77	0.11	0.08	0.04	2.30	0	0	0	0.81
Tanore	0.77	0.11	0.07	0.05	2.38	0.01	0	0	0.68
Godagari	0.72	0.14	0.11	0.03	2.64	0	0	0	0.60
Chapai Nawab-ganj	0.76	0.12	0.08	0.04	2.38	0	0	0	0.59
Sibganj	0.72	0.13	0.11	0.04	2.72	0	0	0	0.50
Gomostapur	0.80	0.10	0.09	0.02	1.96	0	0	0	0.63
Nachole	0.82	0.09	0.06	0.03	1.79	0	0	0	0.51
Bholahat	0.78	0.12	0.04	0.06	2.38	0	0	0	0.56
Naogaon	0.77	0.10	0.09	0.04	2.30	0.01	0	0	0.72
Sapahar	0.84	0.08	0.04	0.04	1.70	0	0	0	0.52
Mohadevpur	0.73	0.14	0.08	0.05	2.72	0	0	0	0.51
Badolgachi	0.78	0.11	0.08	0.03	2.13	0.01	0	0	0.60
Najipur	0.79	0.10	0.07	0.04	2.13	0	0	0	0.60
Nitpur	0.83	0.08	0.05	0.04	1.79	0	0	0	0.58

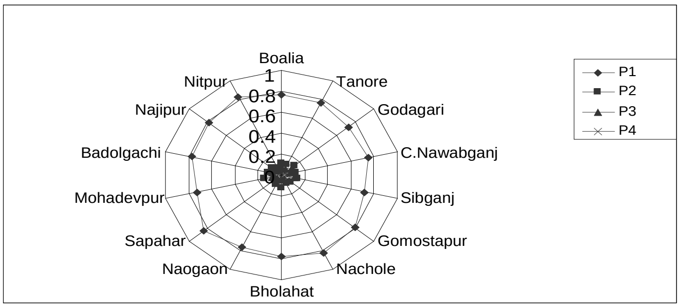

Figure 5: Rabi seasonal variability of probability distribution of wet spell in the Barind region

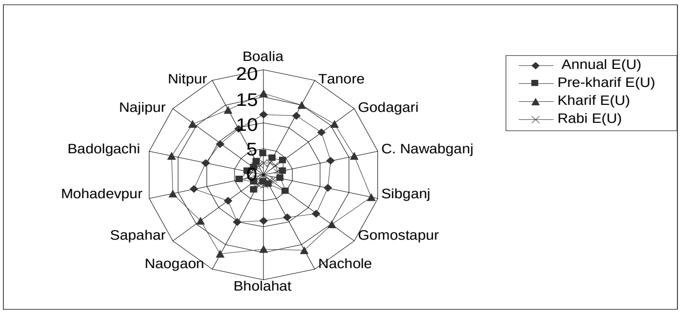

Figure 6: Seasonal varivbility of mean distribution of wet spell in the Barind region

4 Conclusions

Environmental sciences are inherently affected by uncertainty in many of the involved processes. Statistics plays, therefore, an essential role in this study. This paper discussed advance statistical tools which will have a profound impact in these fields. The stochastic Markov chain method was applied to a data set from a rainfall network of the high Barind region, where rainfall is highly seasonal and data availability at a daily time scale or even higher temporal resolution is very limited. A detailed analysis was carried out to study the seasonal and spatial variability of many properties of the daily rainfall in order to incorporate the selected statistics. The aim of this paper is to show how it is possible to formulate in the frames of mathematical statistics the problem concerning determination of a time series data in terms of the theory of hypothesis testing. The study was to investigate prediction transition probabilities of weekly rainfall for environmental impact analysis. The rainfall events are asymptotically normally distributed. The techniques are based on Maximum likelihood estimation of Markov chain modeling system. It is important to be able to model estimate extremes of drought or flood. This robust method enabled us to identify outliers of the data. As an application, policy makers could monitor for an increase in the rate of global warming.

The impacts of climate change on agricultural food production are of global concerns, and they are very important for Bangladesh. Agriculture is the single most and the largest sector of Bangladesh's economy which accounts for about 21% of the GDP and about 70% of the labor force. Agriculture in Bangladesh is already under pressure both from huge and increasing demands for food, and from problems of agricultural land and water resources depletion.

Global warming such as floods and droughts has major impacts in terms of human life, economic and environmental losses, and social disorder. Effective management system can mitigate the impact of global warming. This study would play a vital role to mitigate future environmental impacts of climate change. It has implications for prediction of the transition times, that is, the system spends in a given time, since this would be equivalent to predicting the duration of droughts or wet conditions. The study results could be used for important environmental policy issues in Bangladesh and the proposed method should be useful in real life world.

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