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On Record Values of Rayleigh Distribution

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Abstract

Records arise naturally in many fields of studies such as climatology, sports, science, engineering, medicine, traffic, and industry, among others. The Rayleigh distribution plays a pivotal role in the study of records because of its wide applicability in the modeling and analysis of life time data in these fields. This paper discusses the distribution of record values when the parent distribution is Rayleigh. The recurrence relations for moments and some distributional properties are presented. The estimation of the parameters based on record values from Rayleigh distribution is provided. The prediction of future records based on past records is also presented. We hope that the findings of this paper will be useful for the practitioners in various fields of studies and further enhancement of research in record value theory and its applications.

Keywords and Phrases: Entropy, estimation, moments, prediction, Rayleigh distribution, record values, recurrence relation.

AMS Classification: 60E05, 62E10, 62E15, 62G30.

1 Introduction

An observation is called a record if its value is greater than (or analogously, less than) all the preceding observations. The development of the general theory of statistical analysis of record values began with the work of Chandler (1952). Further development on record value distributions such as estimation of parameters, prediction of record values, characterizations, reconstruction of past record values, etc., continued with the contributions of many authors and researchers, among them Qasem (1996), Arnold et al. (1998), Rao and Shanbhag (1998), Awad and Raqab (2000), Gulati and Padgett (2003), Al-Hussaini and Ahmad (2003), Ahsanullah (2004), Klimczak and Rychlik (2005), Ahmadi et al. (2005), Ahsanullah and Aliev (2008), Balakrishnan et al. (2009), and Ahsanullah et al. (2010) are notable. Many researchers have studied the Rayleigh distribution, its characterizations and inferences based on it because of its wide applicability in biological sciences, economics, electronics, engineering, fisheries, geography, modeling and analysis of life time data, oceonography, pharmacology, physics, reliability, wireless communication system. For detailed treatment on Rayleigh distribution, the interested readers are referred to Johnson et al. (1994) and references therein. It appears from the literature that not much attention has been paid to such studies for record values of Rayleigh distribution, except the following references which were found in literature, viz., Balakrishnan and Chan (1994), Sultan and Balakrishnan (1999), and Soliman and Al-Aboud (2008). This paper discusses the distribution of record values when the parent distribution is Rayleigh. The organization of this paper is as follows. Section 2 contains the distribution of record values when the parent distribution is Rayleigh. In Section 3, we provide moments and recurrence relations for moments of record values from Rayleigh distribution. Section 4 contains survival, hazard and entropy functions. In Section 5, estimation of the parameters based on record values from Rayleigh distribution is provided. The prediction of future records based on past records is provided in Section 6. The concluding remarks are presented in Section 7.

2 Record Values

2.1 Distribution of Record Values: Definitions and Notations

Suppose that $(X_n)_{n\geq 1}$ is a sequence of *i.i.d.* (independent and identically distributed) $rv's$ (random variables) with *cdf* (cumulative distribution function) F. Let $Y_n = \max\{X_j|\ 1 \leq j \leq n\}$ for $n \geq 1$. We say X_j is an upper record value of $\{X_n|n \geq 1\}$, if $Y_j > Y_{j-1}$, $j > 1$. By definition X_1 is an upper record value. Many properties of the upper record value sequence can be expressed in terms of the cumulative hazard rate function$R(x) = -\ln \overline{F}(x)$, where $\overline{F}(x) = 1 - F(x)$, $0 < \overline{F}(x) < 1$. The function defined as $r(x) = \frac{d}{dx} R(x) = f(x) \left(\overline{F}(x)\right)^{-1}$, where f is *pdf* (probability density function) corresponding to F, is called the hazard rate. If we define $F_n(x)$ as the *cdf* of $X_{U(n)}$ for $n \geq 1$, then we have

$$F_n\ (x) = \int_{-\infty}^{x} \frac{(R\ (u))^{n-1}}{(n\ -1)!} dF(u), \quad -\infty < x < \infty. \tag{1}$$

Note that $F_n(x) = 1 - \overline{F}(x) \sum_{j=0}^{n-1} \frac{(R(x))^j}{j!}$. The *pdf* of $X_{U(n)}$, denoted by f_n, is

$$f_n(x) = \frac{(R(x))^{n-1}}{(n-1)!} f(x), \quad -\infty < x < \infty. \tag{2}$$

Note that $\overline{F}_n(x) - \overline{F}_{n-1}(x) = \frac{\overline{F}(x)}{f(x)} f_n(x)$. From here onward, for simplicity, the *nth* upper record value $X_{U(n)}$ will be denoted by $X(n)$.

2.2 Record Values of Rayleigh Distribution

In this section, we will derive the pdf and cdf of the sequence of record values $\{X(n)\}, n \geq 1$, from Rayleigh(λ) distribution.

Rayleigh Distribution: Let $Y_1 = Y_2 = N(0, \sigma^2)$ be any two independent and identically distributed (i.i.d.) Gaussian random variables(rv) with variance σ^2. Let $X = \sqrt{Y_1^2 + Y_2^2}$. Then the *rv* X defines a Rayleigh random variable with its pdf given by%

$$f(x) = \begin{cases} 2\lambda x exp(-\lambda x^2), & x>0,\ \lambda>0 \\ 0, & otherwise, \end{cases} \tag{3}$$

where $\lambda = \frac{1}{2\sigma^2} (> 0)$, known as the scale parameter of Rayleigh (λ) distribution. Suppose that $(X_n)_{n \geq 1}$ is a sequence of *i.i.d.* random variables which are Rayleigh (λ) in $(0, \infty)$, with pdf $f(x)$ given by the equation (3). The corresponding *cdf* F and the hazard rate r of the *rv* X with pdf as given in (3) are respectively

$$F(x) = 1 - exp(-\lambda x^2),$$

and

$$r(x) = f(x) / (1 - F(x)) = 2\lambda x,$$

where $x > 0, \lambda > 0$. Using equation (2), the pdf f_n of the *nth* record value $X(n)$ from Rayleigh (λ) distribution is given by

$$f_n(x) = \frac{2\lambda^n x^{2n-1} exp(-\lambda x^2)}{\Gamma(n)}, \quad n = 1, 2, 3, ..., \tag{4}$$

where $x > 0, \lambda > 0$.

3 Moments and Recurrence Relations

In this section, we will derive moments and recurrence relations for the moments of the sequence of record values $\{X(n)\}$, $n \geq 1$ from Rayleigh distribution.

3.1 Moments

We consider the Rayleigh random variable X with the distribution function as follows:

$$F(x) = 1 - exp(-\lambda x^2), x > 0, \lambda > 0, \tag{5}$$

or

$$F^{-1}(u) = \sqrt{-\frac{1}{\lambda}\ln(1-u)}, 0 < u < 1.$$

Using the Representation Theorem 8.4.1 of Ahsanullah (2004), page 256, it follows that the nth upper record from Rayleigh distribution can be written as

$$X_{U(n)} \overset{d}{=} \sqrt{X_1^2 + X_2^2 + ... + X_n^2} \tag{6}$$

where $X_i^\prime s$ are i.i.d. Rayleigh (λ) with cdf as given in (5). It is well known that $X_i^{2\prime} s$ are i.i.d. E(λ), which implies that the distribution function of $Y(n) = X_1^2 + X_2^2 + ... + X_n^2$ will be gamma (λ, n). Hence, the pdf of $X_{U(n)}$ is given by the equation (4), from which the kth moment of the nth record value $X(n)$ is easily obtained as

$$E\left[X^k(n)\right] = \frac{(\lambda)^{-\frac{k}{2}}\Gamma\left(n+\frac{k}{2}\right)}{\Gamma(n)} \tag{7}$$

Hence, by taking $k = 1$ and $k = 2$ in equation (7), the first and second moments are, respectively, given by

$$E[X(n)] = \frac{(\lambda)^{-\frac{1}{2}}\Gamma\left(n+\frac{1}{2}\right)}{\Gamma(n)}, \tag{8}$$

and

$$E\left[X^2(n)\right] = \frac{n}{\lambda}. \tag{9}$$

Thus, from equations (8) and (9), the variance of the nth record value $X(n)$ from Rayleigh distribution is easily given by

$$V[X(n)] = \left(\frac{1}{\lambda}\right)\left[n - \left\{\frac{\Gamma\left(n+\frac{1}{2}\right)}{\Gamma(n)}\right\}^2\right].$$

3.2 Odd and Even Moments of the Upper Records

Without loss of generality, we consider in this section the standard Rayleigh random variable X with the distribution function as follows:

$$F(x) = 1 - e^{-x^2/2}, x \geq 0, \tag{10}$$

or

$$F^{-1}(u) = \sqrt{-2\ln(1-u)}, 0 < u < 1.$$

The relation (6) can be written as

$$X^2_{U(n)} \stackrel{d}{=} X^2_1 + X^2_2 + ... + X^2_n \tag{11}$$

We can also write

$$X^2_{U(n)} \stackrel{d}{=} X^2_{U(k)} + Y(n-k) \tag{12}$$

The relations (6), (11) and (12) can be used to obtain the recurrence relations between odd and even moments of the upper records. For example, we have from (12)

$$X^2_{U(n+1)} \stackrel{d}{=} X^2_{U(n)} + Y(1) \tag{13}$$

Taking kth ($k \geq 1$) power of (13) and taking expectations, we obtain

$$mu^{2k}_{n+1} = \sum_{j=0}^{k} \binom{k}{j} \mu_n^{2(k-j)} \mu_1^{2j}, n \geq 0, \tag{14}$$

where $\mu^k_n = E(X^k_{U(n)})$. Thus, from equation (14), for $k = 1$, we have

$$\mu^2_{n+1} = \mu^2_n \quad + 2,$$

since $\mu^2_1 = E(X^2) = \int_0^\infty x^3 e^{-x^2/2} dx = 2$. Hence

$$\mu^2_{n+1} = 2(n+1), n \geq 0.$$

Again, from equation (14), for $k = 2$, we have

$$\begin{aligned} \mu^4_{n+1} &= \mu^4_n + 2\mu^2_n \ \ \mu^2_1 + \mu^4_1 \\ &= \mu^4_n + 8n \ \ + 8, \end{aligned}$$

since $\mu^{2n}_1 = E(X^{2n}) = \int_0^\infty x^{2n+1} e^{-x^2/2} dx = 2^n \Gamma(n+1)$, from which, in particular, we have

$$\mu^4_2 = 8 + 8 \ \ + 8 = 4(2)(3),$$

and, in general,

$$\mu_n^4 = 4n(n+1).$$

Similarly, the odd moments can easily be obtained using the relations (6).

3.3 Recurrence Relations Between Moments of the Upper Records

We note that $f(x) = xe^{-x^2/2}$, and $1 - F(x) = e^{-x^2/2}$. Therefore, $f(x) = x(1 - F(x))$. This relation can be used to find the recurrence relations between moments of the record values which are described below.

Theorem 3.3.1

$$\mu_n^{r+2} = \mu_{n-1}^{r+2} + (r+2)\mu_n^r, r = 0, 1, 2, ..., n = 2, ...$$

Proof: The proof follows by simple integration.

Theorem 3.3.2

For $m \geq 1$,

$$\mu_{m,,m+1}^{r,s+2} = \mu_m^{r+s+2} + (s+2)\mu_{m,,m+1}^{r,s}$$

Proof:

$$\begin{aligned} mu_{m,,m+1}^{r,s+2} &= \int_0^\infty \int_x^\infty \frac{x^r y^{s+2}}{\Gamma(m)} (R(x))^{m-1} r(x) f(y) dy dx \\ &= \int_0^\infty \frac{x^r}{\Gamma(m)} (R(x))^{m-1} r(x) I_x dx, \end{aligned}$$

where

$$\begin{aligned} I_x &= \int_x^\infty \frac{{}^r y^{s+2}}{\Gamma(m)} f(y) dy \\ &= -\frac{{}^r y^{s+2}}{\Gamma(m)} (1 - F(y))|_x^\infty + \int_x^\infty \frac{(r+2)y^{s+1}}{\Gamma(m)} (1 - F(y)) dy \\ &= \frac{x^{s+2}}{\Gamma(m)} (1 - F(x)) + \int_x^\infty \frac{(r+2)y^{s+1}}{\Gamma(m)} (1 - F(y)) dy \\ &= \frac{x^{s+2}}{\Gamma(m)} (1 - F(x)) + \int_x^\infty \frac{(r+2)y^s}{\Gamma(m)} f(y)) dy. \end{aligned}$$

Thus

$$\begin{aligned} mu_{m,,m+1}^{r,s+2} &= \int_0^\infty \frac{x^{r+s+2}}{\Gamma(m)} (R(x))^{m-1} f(x) dx + \int_0^\infty \frac{(r+2)x^r y^s}{\Gamma(m)} (R(x))^{m-1} r(x) f(y) dy \\ &= \mu_m^{r+s+2} + (r+2)\mu_{m,,m+1}^{r,s}. \end{aligned}$$

Theorem 3.3.3

For $n > m+1$, $r, s = 0, 1, 2, ...,$

$$\mu_{m,,n}^{r,s+2} = \mu_{m,,n-1}^{r,s+2} + (s+2)\mu_{m,,n}^{r,s}$$

Proof: The proof follows by simple integration similar to Theorem 3.3.2.

Note: The above moment relations can also be obtained as special cases of Raqab (2001).

4 Survival, Hazard and Entropy functions

In this section, we will derive the survival, hazard and entropy functions of the sequence of record values $\{X(n)\},\ n \geq 1$, from Rayleigh (λ) distribution.

4.1 Survival and Hazard Functions

The survival and hazard functions of the nth record value $X(n)$ from Rayleigh (λ) distribution are respectively given by

$$S_n(x) = 1 - F_n(x) = 1 - \frac{\gamma(n, \lambda x^2)}{\Gamma(n)}, \tag{15}$$

and

$$h_n(x) = \frac{f_n(x)}{1 - F_n(x)} = \frac{2\lambda^n x^{2n-1} exp(-\lambda x^2)}{\Gamma(n) - \gamma(n, \lambda x^2)}, \tag{16}$$

where $x > 0$, $\lambda > 0$, and $n = 1, 2, 3, ...,$ and $\gamma(a, x) = \int_0^x u^{a-1} e^{-u} du,\ \ a > 0$.

4.2 Entropy

An entropy provides an excellent tool to quantify the amount of information (or uncertainty) contained in a random observation regarding its parent distribution (population). A large value of entropy implies the greater uncertainty in the data. As proposed by Shannon (1948), entropy of an absolutely continuous random variable X having the probability density function $\phi_X(x)$ is defined as

$$H[X] = E[-\ln\{\phi_X(x)\}] = -\int_S \phi_X(x)\ \ln\{\phi_X(x)\}\, dx, \tag{17}$$

A generalization of Shannon entropy, as proposed by Renyi (1961), is defined as

$$\Im_R(\gamma) = \left[\frac{1}{1-\gamma}\right] \ln \left\{ \int_S [\phi_X(x)]^\gamma \, dx \right\}, \gamma > 0, \gamma \neq 1, \tag{18}$$

where $S = \{x\colon \phi_X(x) > 0\}$. In the limit, when $\gamma \to 1$, Shannon entropy defined by the equation (17) becomes a particular case of Renyi entropy (18).

Reyni Entropy: Hence, in view of the equation (18) and using the definition of gamma function, that is, $\int_0^\infty t^{\,a\,-\,1}\, e^{-b\,t}\, dt = \frac{1}{b^a}\Gamma(\alpha)$, $b > 0$, the expression for the Renyi entropy of the nth record value $X(n)$ from Rayleigh (λ) distribution is easily obtained as

$$\Im_{R(n)}(\gamma) = \left[\frac{1}{1-\gamma}\right] \ln \left\{ \frac{2^{\gamma-1}\Gamma\left(\frac{2\gamma n-\gamma+1}{2}\right)}{[\Gamma(n)]^\gamma \lambda^{\left(\frac{1-\gamma}{2}\right)} \gamma^{\left(\frac{2\gamma n-\gamma+1}{2}\right)}} \right\}, \gamma > 0, \gamma \neq 1. \tag{19}$$

Shannon Entropy: Thus, taking the limit in equation (19), when $\gamma \to 1$, and using the properties of digamma function (for details, see Abramowitz & Stegun (1970), and Gradshteyn & Ryzhik (2000), among others), Shannon entropy of the nth record value $X(n)$ from Rayleigh (λ) distribution is easily easily obtained as

$$H_{(n)} = \ln\left\{\frac{\Gamma(n)}{2\sqrt{\lambda}}\right\} + \left(\frac{1}{2} - n\right)\psi(n) + n, \tag{20}$$

where $\psi(z)$ denotes digamma function.

Remark: From equation (20), it is easily seen that

(i) $H_{(n+1)} - H_{(n)} = \ln(n) - \psi(n) - \frac{1}{2n}$, where $H_{(n)}$ and $H_{(n+1)}$ denote the Shannon entropies of the nth record value $X(n)$ and $(n+1)\,st$ record value $X(n+1)$ respectively from Rayleigh (λ) distribution;

(ii) the sequence $\{H_{(n)}\}$ of the Shannon entropies of the record values from a Rayleigh (λ) distribution is monotonic increasing in n, $\forall\ \lambda > 0$, which follows from the inequality $\frac{1}{2z} < \ln(z) - \psi(z) < \frac{1}{z}$, $(z > 0)$, see Alzer (1997);

(iii) the sequence $\{H_{(n)}\}$ of the Shannon entropies of the record values from a Rayleigh (λ) distribution is a monotonic decreasing convex function of λ, $\forall$ integer $n \geq 1$, which follows from (20) by differentiating it twice w.r.t. λ.

Song's Measure: Following Song (2001), it is defined as

$$\Im_{R(n)}^{/}(\gamma) = \frac{d}{d\gamma}\left[\Im_{R(n)}(\gamma)\right], \gamma > 0, \gamma \neq 1, \tag{21}$$

which is known as the gradient of the Renyi entropy. Also, for the pdf f_n of the nth record value $X(n)$, we have

$$-2\Im_{R(n)}^{/}(1) = Var\left[\ln\left\{f_n(x)\right\}\right]. \tag{22}$$

The equation (22) defines a measure of the shape of a distribution of a random variable $X(n)$ with the pdf $f_n\ (x)$ and remains invariant under location and scale transformations. Moreover, similar to the kurtosis of a distribution, the quantity $-2\Im_{R(n)}^{/}(1)$ plays an important role in comparing the shapes of various densities and measuring the heaviness of tails. By direct differentiation of the expression for Renyi entropy (19) and taking the limit, when $\gamma \to 1$, and using the properties of polygamma functions (for details, see, for example, Abramowitz & Stegun (1970), and Gradshteyn & Ryzhik (2000), among others), it is easily seen that

$$-2\Im_{R(n)}^{/}(1) = \left(n - \frac{1}{2}\right)^2 \psi^{/}(n) - n + 1, \tag{23}$$

where $n = 1, 2, 3, ...,$ and $\psi^{/}(.)$ denotes trigamma function.

5 Estimation of Parameters

This section consider the estimation of the parameters based on record values from the two parameter Rayleigh distribution.

5.1 Maximum Likelihood Estimates

A rv X is said to have a two parameter Rayleigh distribution if its pdf is given by

$$f(x, \mu, \sigma) = \frac{x-\mu}{\sigma^2} e^{-\frac{(x-\mu)^2}{2\sigma^2}},\ \mu < x < \infty,\ \sigma > 0.$$

The maximum likelihhod estimates of μ and σ based on the observed records $\mathrm{r}_i, i = 1, ..., n$ are the solutions of the following two equations, see Arnold et al., (1998).

$$n - \sum_{i=1}^{n-1} F(r_i^*) - 2F(r_n^*) = 0$$

$$n\sigma + \sum_{i=1}^{n} r_i^* - \sum_{i=1}^{n-1} r_i^* F(r_1^*) - 2r_n F(r_n^*) = 0,$$

where $r_i^* = \frac{r_i - \mu}{\sigma}, i = 1, ..., n,\ F(x) = e^{-\frac{x^2}{2}}$. These two equations need to be solved by iterative methods.

5.2 Minimum Varaince Linear Unbiased Estimates

The minimum varaince linear unbiased estimates, $\hat{\mu}$ and $\hat{\sigma}$ of μ and σ respectively are given by (see Ahsanullah (2004))

$$\hat{\mu} = \sum_{j=1}^{n} c_i r_i \text{ and } \hat{\sigma} = \sum_{j=1}^{n} d_i r_i,$$

where

$$c_1 = \frac{1}{2}\frac{a_m b_m}{D}, c_i = \frac{3c_1}{i}, i = 1, ..., n-1, c_m = 1 - \sum_{i=1}^{n-1} c_i,$$
$$d_i = -\frac{c_i}{a_m}, i = 1, ..., n-1, d_m = \sum_{i=1}^{n-1} d_i,$$
$$a_i = \sqrt{2}\frac{\Gamma(i+\frac{1}{2})}{\Gamma(i)}, b_i = \sqrt{2}[\frac{\Gamma(i+1)}{\Gamma(i+\frac{1}{2})} - \frac{\Gamma(i+\frac{1}{2})}{\Gamma(i)}], i = 1, ..., n,$$

and

$$D = a_m b_m T - 1,\ T = \frac{3}{2} + \sum_{i=1}^{n-1} \frac{1}{2i} + (2n-1)(\frac{b_{n-1}}{b_n - 1} - 1).$$

The variances and the covariances of the estimates are

$$Var(\hat{\mu}) = \sigma^2 \frac{a_n b_n}{D}, Var(\hat{\sigma}) = \sigma^2 \frac{b_n^2 T}{D}, \text{ and } Cov(\hat{\mu}, \hat{\sigma}) = -\sigma^2 \frac{b_n}{D}.$$

If μ is known then the minimum varaince linear unbiased estimates σ^* of σ is

$$\sigma^* = \frac{\Gamma(n)(x_n - \mu)}{\sqrt{2}\Gamma(n+\frac{1}{2})}.$$

6 Prediction of Record Values

Here we consider the prediction of future records based on past records. Since the variances and covariances as well as the inverse of the variance-covariance matrix are well-known, using these we will obtain the prediction of upper record values which are described below.

6.1 Prediction of the sth Upper Record Value

We will predict the sth upper record based on the observed first m $(m < s)$ records. Suppose we consider location and scale parameter of the Rayleigh distribution. The corresponding cdf is given by

$$1 - F(x) = e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}, -\infty < \mu < \infty, \sigma > 0.$$

Let $W^{/} = (W_1, W_2, ..., W_m)$, where $\sigma^2 W_i = Cov(X_{U(i)}, X_{U(s)}) = \sigma^2 \frac{\delta_i v_{n,n}}{\delta_s}$, $i = 1, 2, ..., m$. (Note $\delta_i = a_i$). Then, the best linear unbiased predictor of $X_{U(s)}$ is $\tilde{X}_{U(s}$, where

$$\tilde{X}_{U(s)} = \hat{\mu} + \hat{\sigma}\alpha^* + W'V^{-1}(X - \hat{\mu}L + \hat{\sigma}\delta),$$

or

$$\tilde{X}_{U(s)} = \hat{\mu} + \hat{\sigma}\delta_s + W'V^{-1}(X - \hat{\mu}L + \hat{\sigma}\delta),$$

where $\hat{\mu}$ and $\hat{\sigma}$ are respectively MVLUE of μ and σ, as given in 5.2 above, and

$$E(X) = E(X_{U(1)}, X_{U(2)}, ..., X_{U(m)})' = (L, \delta)'(\mu, \sigma),$$

$$\alpha^* = \sigma^{-1} E(X_{U(s)} - \mu) = \sqrt{2}\frac{\Gamma(s+\frac{1}{2})}{\Gamma(s)} = \delta_s,$$

$$E(X_{U(i)}) = \mu + \delta_i \sigma, \delta_i = \sqrt{2}\frac{\Gamma(i+\frac{1}{2})}{\Gamma(i)}, \ i = 1, 2, ..., m,$$

$$L' = (1, 1, ..., 1),$$

and

$$\delta' = (\delta_1, \delta_2, ..., \delta_m).$$

Now, we have

$\hat{\mu} = \sum_{i=1}^m c_i x_i, x_i =$ the observed value of $X_{U(i)}$, and

$\hat{\sigma} = \sum_{i=1}^m d_i x_i$. We have to simplify $W'V^{-1}(X - \hat{\mu}L + \hat{\sigma}\delta)$. For this, we note from 4.2 that

$$\begin{aligned} &W'V^{-1}(X - \hat{\mu}L - \hat{\sigma}a_n) \\ &= (0, 0, .., \tfrac{a_s}{b_s})(X - \hat{\mu}L - \hat{\sigma}a_n) \\ &= \tfrac{a_s}{b_s}(x_m - \hat{\mu} - \sigma\hat{a}_n) \\ &= 0. \end{aligned}$$

Hence, we have

$$\tilde{X}_{U(s)} = \hat{\mu} + \hat{\sigma}a_s,$$

and

$$Var(\tilde{X}_{U(n)}) = \frac{\sigma^2 b_n}{D} \ [a_n + b_n T - 2a_n].$$

7 Concluding Remarks

In this paper, we have discussed the distribution of record values when the parent distribution is Rayleigh. The recurrence relations for moments and some distributional properties are presented. The prediction of future records based on past records are presented. We have also provided the estimation of the parameters of record values from Rayleigh distribution. We hope that the findings of this paper will be a useful for the practitioners in various fields of studies and further enhancement of research in record value theory and its application.

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