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Estimating and Testing Coefficient of Dispersion from a Class of Arbitrary Populations

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Abstract

In this paper we suggest some improved estimation strategies for the coefficient of dispersion (CD) parameter using shrinkage and pretest principles. These methods are useful when some uncertain prior information (UPI) or non-sample information (NSI) regarding the CD is available. Indeed, the shrinkage and pretest methods use the NSI in some optimal sense. The performance of these estimators, with respect to asymptotic mean squared error (AMSE), is examined both analytically and numerically. It is exhibited that suggested estimators outshine the the empirical estimator of CD in some part of the parameter space. The finding of this research invalidates the global minimum AMSE property of the benchmark estimator. In an effort to appreciate the relative behavior of listed estimators for finite sample scenario, a Monte carlo simulation study is planned and performed.

Keywords and Phrases: Coefficient of Dispersion, Asymptotic Test and Interval, Shrinkage and Pretest Estimation, Local Alternatives, Asymptotic Properties, Relative Performance and Monte Carlo Simulation.

1 Introduction and Preliminaries

The coefficient of dispersion is widely used in many branches of life science, for example genetics, astrophysics, geophysics, and plant breeding a few to mention. The coefficient of dispersion is used to detect a type of spatial dispersion. A CD is less than 1 is indicative of an under-spread (uniform) spatial distribution and a CD greater than 1 indicates a over-dispersed (clumped) spatial distribution. The population coefficient of dispersion (γ) is defined as

$$\gamma = \frac{\sigma^2}{\mu}, \quad \mu \neq 0,$$

where μ and σ^2 are the mean and the variance of the variable X, respectively, of a class arbitrary population. This class is subject to existence of first four moments.

This research is motivated by diverse applications of CD and its involvement in many areas of scientific research. The current paper is mainly focusing on the applications of CD. Our results build on the asymptotic normality of estimate of CD. We develop the inferential tools for CD parameter. Our asymptotic work uses the general asymptotic theory for preliminary test estimation as developed by Ahmed (2001) and others.

The sample estimate of the dispersion measure (based on sample $(X_1, \cdots, X_n)$ is

$$\hat{\gamma} = \frac{S^2}{\bar{X}}$$

where $\bar{X}$ and S^2 are sample mean and sample variance,respectively given by

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i, \quad \text{and} \quad S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$$

Further, the asymptotic normality of $\boldsymbol{e}$ is presented in the following lemma.

Lemma 1: Under assumed regularity conditions as $n \to \infty$, then

$$\sqrt{n}\,(\hat{\gamma} - \gamma) \xrightarrow[D]{} \mathcal{N}(0, \nu^2),$$

where

$$\nu^2 = \frac{1}{\mu}\gamma^3 - \gamma^2 - \frac{2\mu_3}{\mu^2}\gamma + \frac{\mu_4}{\mu^2},$$

the notation $\xrightarrow[D]{}$means *convergence in distribution.*

If the parent population is Poisson distribution with parameter λ, then we have

$$\mu = \lambda, \quad \mu_2 = \sigma^2, \quad \mu_3 = \lambda, \quad \mu_4 = 3\lambda^2 + \lambda,$$

hence,

$$\sqrt{n}\,(\hat{\gamma} - \gamma) \xrightarrow[D]{} \mathcal{N}(0, 2).$$

Interestingly, the asymptotic distribution is parameter free and is useful for inference purposes.

However, in this investigation we are not restricted to above distributions, our subsequent results hold for a class of arbitrary distributions and the remaining discussion follows, otherwise.

2 Inference regarding Coefficient of Dispersion

In the following two subsections we consider interval estimation and testing hypothesis regarding the parameter of interest λ.

2.1 Asymptotic Interval Estimation

First, consider

$$Z_1[\hat{\gamma}] = \frac{\sqrt{n}\{\hat{\gamma} - \lambda\}}{\nu} \xrightarrow[D]{} \mathcal{N}(0,1).$$

Since ν depends on the unknown quantity λ and other parameters, this result will not immediately yield confidence intervals for λ for unknown ν. Another pivotal quantity can be defined by replacing ν by its empirical estimate $\hat{\nu}$ in the denominator of $Z_1[\hat{\gamma}]$ and is given by

$$Z_2[\hat{\gamma}] = \frac{\sqrt{n}\{\hat{\gamma} - \lambda\}}{\hat{\nu}},$$

where

$$\hat{\nu}^2 = \frac{1}{\hat{\mu}}\hat{\gamma}^3 - \hat{\gamma}^2 - \frac{2\hat{\mu}_3}{\hat{\mu}^2}\hat{\gamma} + \frac{\hat{\mu}_4}{\hat{\mu}^2},$$

where $\hat{\mu}_3$ and $\hat{\mu}_4$ are consistent estimator of μ_3 and μ_4, respectively.

Further, $\hat{\nu}$ is a consistent estimator of ν, hence $Z_2[\hat{\gamma}] \xrightarrow[D]{} \mathcal{N}(0,1)$. Therefore, for large n, it is appropriate to approximate the distribution of $Z_2[\hat{\gamma}]$ by the standard normal distribution. An asymptotic $(1-\alpha)100\%$ confidence interval for γ is

$$Pr\left\{\hat{\gamma} - z_{\frac{\alpha}{2}}\left(\frac{\hat{\nu}^2}{n}\right)^{1/2} \leq \gamma \leq \hat{\gamma} + z_{\frac{\alpha}{2}}\left(\frac{\hat{\nu}^2}{n}\right)^{1/2}\right\} \rightarrow 1-\alpha \quad \text{as} \quad n \rightarrow \infty,$$

where $z_{\frac{\alpha}{2}}$ is the $\left(1-\frac{\alpha}{2}\right)100$ percentile point of the standard normal distribution.

2.2 Asymptotic Tests and Power

We propose the following three test statistics to test for $H_o : \lambda = \lambda_o$ against $H_a : \lambda \neq \lambda_o$ or $\lambda < \lambda_o$ or $\lambda > \lambda_o$. These statistics are given by

$$\mathcal{T}_i = \frac{\{\sqrt{n}(\hat{\gamma} - \lambda_o)\}^2}{\hat{\nu}_i^2}, \quad i = 1,2,3.$$

where

$$\hat{\nu}_1^2 = \frac{1}{\hat{\mu}}\gamma_o^3 - \gamma_o^2 - \frac{2\hat{\mu}_3}{\hat{\mu}^2}\gamma_o + \frac{\hat{\mu}_4}{\hat{\mu}^2},$$

$$\hat{\nu}_2^2 = \frac{1}{\hat{\mu}}\hat{\gamma}^3 - \hat{\gamma}^2 - \frac{2\hat{\mu}_3}{\hat{\mu}^2}\hat{\gamma} + \frac{\hat{\mu}_4}{\hat{\mu}^2},$$

and

$$\hat{\nu}_3^2 = \frac{1}{\hat{\mu}}(\hat{\lambda}^S)^3 - (\hat{\lambda}^S)^2 - \frac{2\hat{\mu}_3}{\hat{\mu}^2}\hat{\lambda}^S + \frac{\hat{\mu}_4}{\hat{\mu}^2}.$$

The shrinkage estimator $\hat{\lambda}^S$ will be introduced in Section 3. For large n and under the null hypothesis, $\mathcal{T}_i$ follows a χ^2-distribution with one degree of freedom, which provides the asymptotic critical values.

It is important to note that for a fixed alternative that is different from the null hypothesis the power of all three tests statistics will converge to one as $n \to \infty$. Thus, to explore the asymptotic power properties of $\mathcal{T}_i$, we consider a sequence of local alternative $\{A_n\}$. When λ is the parameter of interest, such a sequence may be specified by

$$A_n : \lambda_n = \lambda_o + \frac{\omega}{\sqrt{n}}, \tag{2.1}$$

where ω is a fixed real number. Obviously, λ approaches λ_o at a rate to $n^{-1/2}$. Stochastic convergence of $\hat{\gamma}$ to the parameter λ ensures that $\hat{\gamma} \xrightarrow[p]{} \lambda$ under local alternatives as well, where the notation $\xrightarrow[p]{}$means *convergence in probability*.

The following lemma, which we present without proof, characterizes the asymptotic powers of the three test statistics under local alternatives.

Lemma 2 Under local alternatives in (2.1) the following results hold:

1. $\sqrt{n}(\hat{\gamma} - \gamma) \xrightarrow[D]{} \mathcal{N}(0, \tau^2)$,

2. $\mathcal{T}_i$ has asymptotically a noncentral χ^2-distribution with 1 degree of freedom and noncentrality parameter $\Omega = \frac{\omega^2}{\nu^2}$.

Hence, the power calculations of the proposed test statistic can be accomplished by using noncentral χ^2-distribution. We remark that $\hat{\gamma}$ is a nonlinear function of asymptotically normal quantities; as such, the distribution of $\hat{\gamma}$ may not be well approximated by a normal distribution for small sample sizes.

Now, we consider two improved estimation procedures based on shrinkage and the preliminary test techniques that are considered in the Section 3 along with some asymptotic results.

3 Improved Estimation Strategies

We will use the results of the preceding section to build some improved estimators of λ. Our interest here is the estimation of λ when it is suspected a priori that $\lambda = \lambda_o$. This natural origin λ_o could be any sort of a priori information about λ. In many applied problems usually the experimenter has some idea about the value of parameter λ based on the past experience or acquaintance with the problem under consideration. In practice, this information is readily available in the form of a realistic conjecture based on the experimenter's knowledge and experience with model and data. It is advantageous to utilize this information in the estimation process to construct improved estimation for the coefficient of dispersion parameter. In this investigation, a asymptotic theory of the suggested estimators are developed.

Having said above, it is reasonable then to move the classical estimator of λ close to λ_o. Hence we define a liner combination estimator as (also known as a shrinkage estimator)

$$\hat{\lambda}^S = \lambda\lambda_o + (1-\lambda)\hat{\gamma},$$

where $\lambda \in [0,1]$ denotes the shrinkage intensity. Noting that for $\lambda = 1$ the shrinkage estimate equals the shrinkage target λ_o whereas for $\lambda = 0$ the classical or unrestricted estimate (UE) is recovered. The key advantage of this construction is that it outperforms the UE in some part of the parameter space. However, the key question in this type of estimator is how to select an optimal value for the shrinkage parameter λ. In some situations, it may suffice to fix the parameter λ at some given value. The second choice, is to choose the parameter λ in a data-driven fashion by explicitly minimizing a suitable risk function. A common but also computationally intensive approach to estimate the minimizing λ by using cross-validation. On the other hand, from a Bayesian perspective one can employ the empirical Bayes technique to infer λ. In this case λ is treated as a hyper-parameter and that may be estimated from the data by optimizing the marginal likelihood. Here we treat λ as the degree of trust in the prior information λ_o. The value of $\lambda \in [0,1]$ may be assigned by the experimenter according to her/his prior belief in the prior value λ_o. Ahmed and Krzanowski (2004), Bickel and Doksum (2001) and other pointed out that such an estimator yields smaller mean squared error (MSE) when a priori information is correct or nearly correct, however at the expense of poorer performance in the rest of the parameter space induce by the prior information. In the present context, we will demonstrate that $\hat{\lambda}^S$ will have a smaller MSE than $\hat{\gamma}$ near the restriction, that is, $\lambda = \lambda_o$. However, $\hat{\lambda}^S$ becomes considerably biased and inefficient when the restriction may not be judiciously justified. As such, when the prior information is rather suspicious, it may be reasonable to construct a shrinkage preliminary test estimator (SPE) denoted by $\hat{\lambda}^{SP}$ which incorporates a preliminary test on $\lambda = \lambda_o$. Thus, the estimator $\hat{\gamma}$ and $\hat{\lambda}^S$ is selected depending upon the outcome of the preliminary test. The shrinkage preliminary test estimator (SPE) is defined as

$$\hat{\lambda}^{SP} = \hat{\gamma} I(\mathcal{T}_i \geq c_{n,\alpha}) + [(1-\lambda)\hat{\gamma} + \lambda\lambda_o] I(\mathcal{T}_i < c_{n,\alpha}), \tag{3.1}$$

where $\mathcal{T}_i$ is the test statistic for the null hypothesis $H_o : \lambda = \lambda_o$, which is defined in the previous section, and $I(A)$ is the indicator function of a set A. The critical value $c_{n,\alpha}$ converges to $\chi^2_{1,\alpha}$ as $n \to \infty$. Thus, the critical value $c_{n,\alpha}$ of T_i may be approximated by $\chi^2_{1,\alpha}$, the upper 100$\alpha\%$ critical value of the χ^2-distribution with 1 degree of freedom. If we substitute $\lambda = 1$ in (3.1) we get

$$\hat{\lambda}^P = \hat{\gamma} I(\mathcal{T}_i \geq c_{n,\alpha}) + \lambda_o I(\mathcal{T}_i < c_{n,\alpha}). \tag{3.2}$$

The estimator $\hat{\lambda}^P$ is known as the usual preliminary estimator (PE), due to Bancroft (1944). The SPE may be viewed as an improved PE which represents both UE and PE for $\lambda =$

0 and $\lambda = 1$ respectively. For a discussion about preliminary testing we refer to saleh (2006),Giles and Giles (1993), Magnus (1999), Ohanti (1999), Reif and VIcek (2002), Khan and Ahmed (2003), and other.

4 Asymptotic Properties

The asymptotic bias (AB) of an estimator γ^* of γ is defined as

$$AB(\lambda^*) = \lim_{n\to\infty} E\{\sqrt{n}(\lambda^* - \lambda)\}. \tag{4.1}$$

Under local alternatives $AB(\hat{\lambda}^S) = -\lambda\omega$. The expression of $AB(\hat{\lambda}^{SP})$ is obtained with the help of the following lemma.

Lemma 3: If the random variable Z is normally distributed with mean μ and variance 1, then

$$E\{ZI(0 < Z^2 < x)\} = \mu P\left(\chi^2_{3,\frac{\mu^2}{2}} < x\right),$$

where $\chi^2_{3,\frac{\mu^2}{2}}$ is random variable with the central chi-square distribution with 3 degrees of freedom and non-centrality parameter $\frac{\mu^2}{2}$. For proof of the lemma, we refer to Judge and Bock (1978).

By the virtue of Lemma 3, the following relation is established.

$$AB(\hat{\lambda}^{SP}) = -\lambda\omega\Lambda_3(\chi^2_{1,\alpha};\Omega),$$

where $\Lambda_q(.;\Omega)$ is the cumulative distribution of a noncentral χ^2-distribution with q degrees of freedom and non-centrality parameter Ω. Since $\lim_{\omega\to\infty} \omega\Lambda_3(\chi^2_{1,\alpha};\Omega) = 0$, it can be concluded that $\hat{\lambda}^{SP}$ is asymptotically unbiased, in the sense of ω, but $\hat{\lambda}^S$ is not so. The asymptotic bias of $\hat{\lambda}^P$ is $AB(\hat{\lambda}^P) = -\omega\Lambda_3(\chi^2_{1,\alpha};\Omega)$. The $AB(\hat{\lambda}^{SP})$ and $AB(\hat{\lambda}^P)$ are 0 when $\omega = 0$ which increases to maximum, then decreases towards 0 as ω increases.

Ahmed (1999) among others has pointed out that the estimators based on the preliminary test principle possess substantially smaller asymptotic mean square error $(AMSE)$ than $\hat{\gamma}$ in a shrinkage neighborhood of UPI in the parameter space. For this reason, a sequence $\{A_n\}$ of local alternatives is considered for asymptotic analysis. Now, under the local alternative in (2.1) we present the expressions for the $AMSE$ for the estimators under consideration. The $AMSE$ of the estimators are:

$$AMSE(\hat{\gamma}) = \nu^2;$$

$$AMSE(\hat{\lambda}^S) = \nu^2 - \nu^2\lambda(2-\lambda) + \nu^2\lambda^2\Omega;$$

$$\begin{aligned} AMSE(\hat{\lambda}^{SP}) &= \nu^2 - \nu^2\lambda(2-\lambda)\Lambda_3(\chi^2_{1,\alpha};\Omega) + \\ &\quad \nu^2\lambda\Omega\{2\Lambda_3(\chi^2_{1,\alpha};\Omega) - (2-\lambda)\Lambda_5(\chi^2_{1,\alpha};\Omega)\}. \end{aligned}$$

The expression of $AMSE(\hat{\lambda}^{SP})$ is obtained with the use of the following lemma.

Lemma 4: If the random variable Z is normally distributed with mean μ and variance 1, then

$$E\{Z^2 I(0 < Z^2 < x)\} = P\left(\chi^2_{3,\frac{\mu^2}{2}} < x\right) + \mu^2 P\left(\chi^2_{5,\frac{\mu^2}{2}} < x\right).$$

The $AMSE(\hat{\lambda}^S)$ is a straight line in terms of Ω which intersects the $AMSE(\hat{\gamma})$ at $\Omega = (2-\lambda)/\lambda$. At and near the null hypothesis the $AMSE$ of $\hat{\lambda}^S$ is less than the $AMSE$ of $\hat{\gamma}$. Comparing $AMSE(\hat{\lambda}^{SP})$ with $AMSE(\hat{\gamma})$,

$$\begin{array}{ccl} AMSE(\hat{\lambda}^{SP}) & \geq & AMSE(\hat{\gamma}) \text{ according as} \\ \Omega & \geq & (2-\lambda)\Lambda_3(\chi^2_{1,\alpha};\Omega)\left\{2\Lambda_3(\chi^2_{1,\alpha};\Omega) - (2-\lambda)\Lambda_5(\chi^2_{1,\alpha};\Omega)\right\}^{-1}. \end{array} \quad (4.2)$$

Thus, $\hat{\lambda}^{SP}$ dominates $\hat{\gamma}$ whenever

$$\Omega < (2-\lambda)\Lambda_3(\chi^2_{1,\alpha};\Omega)\left\{2\Lambda_3(\chi^2_{1,\alpha};\Omega) - (2-\lambda)\Lambda_5(\chi^2_{1,\alpha};\Omega)\right\}^{-1}.$$

Further, as α, the level of the statistical significance, approaches one $AMSE(\hat{\lambda}^{SP})$ tends to $AMSE(\hat{\gamma})$. Also, when Ω increases and tends to infinity, the $AMSE(\hat{\lambda}^{SP})$ approaches the $AMSE(\hat{\gamma})$. Broadly speaking, for larger values of Ω, the value of the $AMSE(\hat{\lambda}^{SP})$ increases, reaches its maximum after crossing the $AMSE(\hat{\gamma})$ and then monotonically decreases and approaches the $AMSE(\hat{\gamma})$.

The $AMSE$ of $\hat{\lambda}^P$ is

$$AMSE(\hat{\lambda}^P) = \nu^2 + \nu^2\Omega\{2\Lambda_3(\chi^2_{1,\alpha};\Omega) - \Lambda_5(\chi^2_{1,\alpha};\Omega)\} - \nu^2\Lambda_3(\chi^2_{1,\alpha};\Omega),$$

and

$$\begin{array}{ccl} AMSE(\hat{\lambda}^{P}) & \geq & AMSE(\hat{\gamma}) \text{ according as} \\ \Omega & \geq & \Lambda_3(\chi^2_{1,\alpha};\Omega)\left\{2\Lambda_3(\chi^2_{1,\alpha};\Omega) - \Lambda_5(\chi^2_{1,\alpha};\Omega)\right\}^{-1}. \end{array} \quad (4.3)$$

By comparing the right hand side of equation (4.2) to the right hand side of (4.3), we noticed that the range of the parameter space in (4.2) is smaller than that in (4.3).

it is seen that

$$\begin{array}{ccl} AMSE(\hat{\lambda}^{P}) & \leq & AMSE(\hat{\lambda}^{SP}) \text{ according as} \\ \Omega & \leq & (1-\pi)\Lambda_3((\chi^2_{1,\alpha};\Omega)\left\{2\Lambda_3(\chi^2_{1,\alpha};\Omega) - (1-\lambda)\Lambda_5(\chi^2_{1,\alpha};\Omega)\right\}^{-1}. \end{array}$$

Thus, $\hat{\lambda}^{SP}$ dominates $\hat{\lambda}^P$ whenever

$$\Omega > (1-\lambda)\Lambda_3(\chi^2_{1,\alpha};\Omega)\left\{2\Lambda_3(\chi^2_{1,\alpha};\Omega) - (1-\lambda)\Lambda_5(\chi^2_{1,\alpha};\Omega)\right\}^{-1}.$$

Based on above findings we suggest to use $\hat{\lambda}^{SP}$, since it has a good control on AMSE with respect to Ω.

In an effort to present the graphic analysis we consider the notion of asymptotic relative efficiency (ARE). The ARE of of $\tilde{\lambda}$ relative $\hat{\gamma}$ is defined as follows:

$$ARE(\hat{\gamma}, \tilde{\lambda}) = \frac{SMSE(\tilde{\lambda})}{SMSE(\hat{\gamma})}.$$

The ARE are plotted in Figures 1-2 versus noncentrality parameter, Ω. Figures 1 and 2 exhibit the ARE of the proposed estimators at two significance levels, α = 0.05 and 0.20, respectively. The ARE of $\hat{\lambda}^{SP}$ was investigated at different values of λ. Figures validated the behavior of the AMSE of the proposed estimators. The ARE is a function of $\alpha,\ \Omega$ and λ. For a fixed value of α, ARE decreases as Ω increases from 0, attains a minimum value at a point Ω_M and then increases asymptotically to 1. For $\alpha \neq 0$, this function has its maximum at $\Omega = 0$. The maximum relative efficiency is a decreasing function of α while the minimum efficiency is an increasing function of α. These properties are validated graphically in Figures. In addition, it may be seen that for smaller values of λ, when α is fixed, the variation in the ARE functions is greater. Also, it appears from Figure that the variation in the ARE functions is greater for smaller α. It is clear that the variation decreases as α increases. Thus, numerical results are in agreement with our analytical findings.

In summary, the graphical results illustrate our analytic asymptotic theory and are in agreement with the typical characteristics of the preliminary test and shrinkage estimation.

5 A Simulation Study

In this section, we conduct a Monte Carlo simulation study aimed to provide empirical outcomes to the theory developed in the early sections of this paper. The main objectives of this simulation experiment is to investigate the general trends of the relative efficiencies of the four estimators $\hat{\gamma}$, $\hat{\lambda}^{S}$, $\hat{\lambda}^{P}$ and $\hat{\lambda}^{SP}$. The Monte Carlo simulation study was carried out to investigate the relative efficiencies of the four estimators $\hat{\gamma}$, $\hat{\lambda}^{S}$, $\hat{\lambda}^{P}$ and $\hat{\lambda}^{SP}$. (Note that given $\pi = 0.5$, T_2 was used to compute $\hat{\lambda}^{P}$ and $\hat{\lambda}^{SP}$). We considered $H_o : \lambda = \lambda_o$ against $H_a : \lambda = \lambda_o + \omega$, where ω is a shift real number in the neighborhood domain of λ. We numerically calculated the relative efficiencies of the estimators relative to $\hat{\gamma}$ using $N = 5,000$ realizations of the alternative distributions for n = 30.

$$RE(\hat{\gamma}, \tilde{\lambda}) = \frac{SMSE(\tilde{\lambda})}{SMSE(\hat{\gamma})},$$

is the simulated relative efficiency of $\tilde{\lambda}$ relative $\hat{\gamma}$. Further, $SMSE(\tilde{\lambda})$ and $SMSE(\hat{\gamma})$ are the empirical mean squared errors of $\tilde{\lambda}$ and $\hat{\gamma}$, respectively. The empirical mean square error of

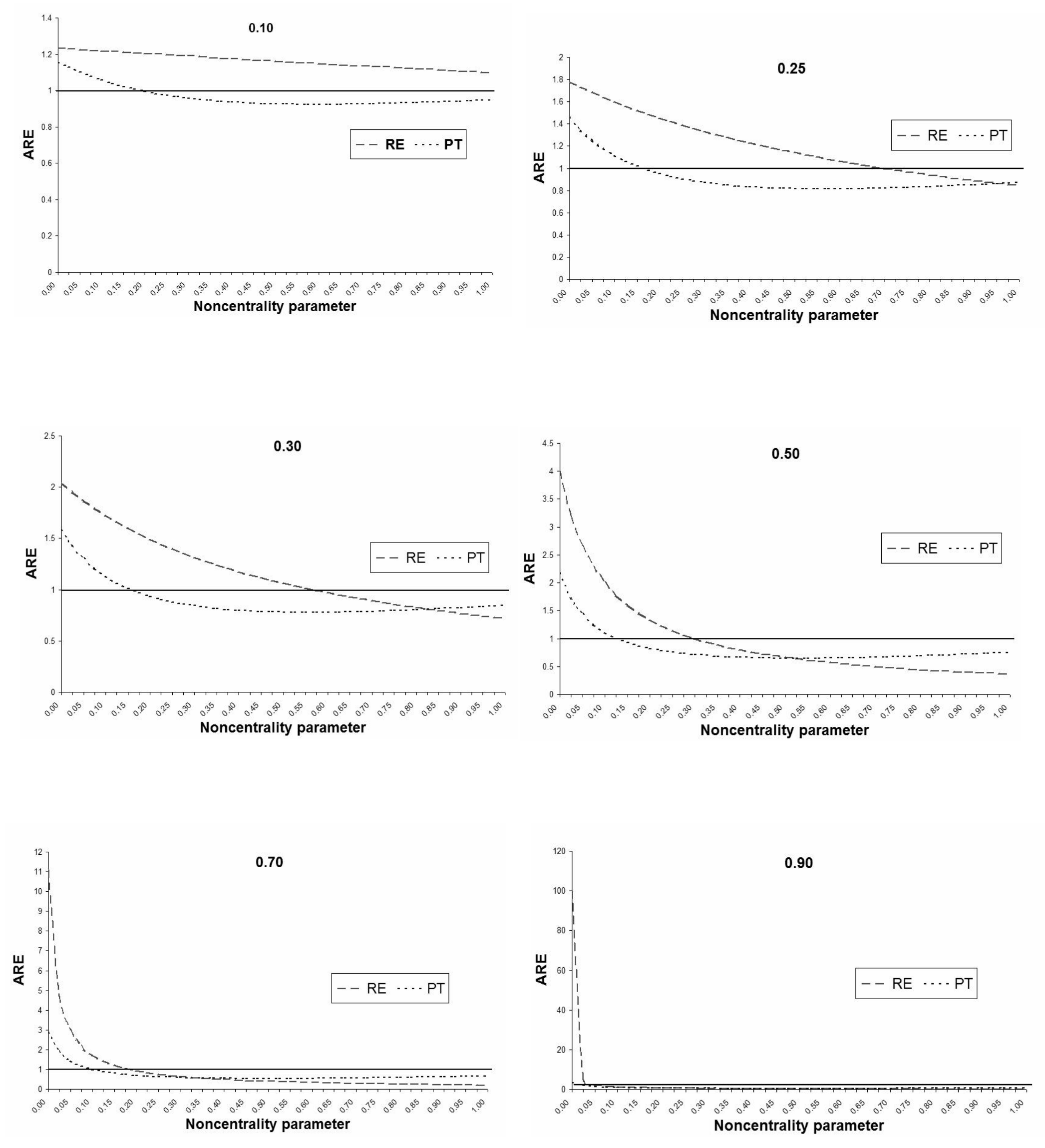

Figure 1: ARE of the estimators for $\alpha = 0.05$.

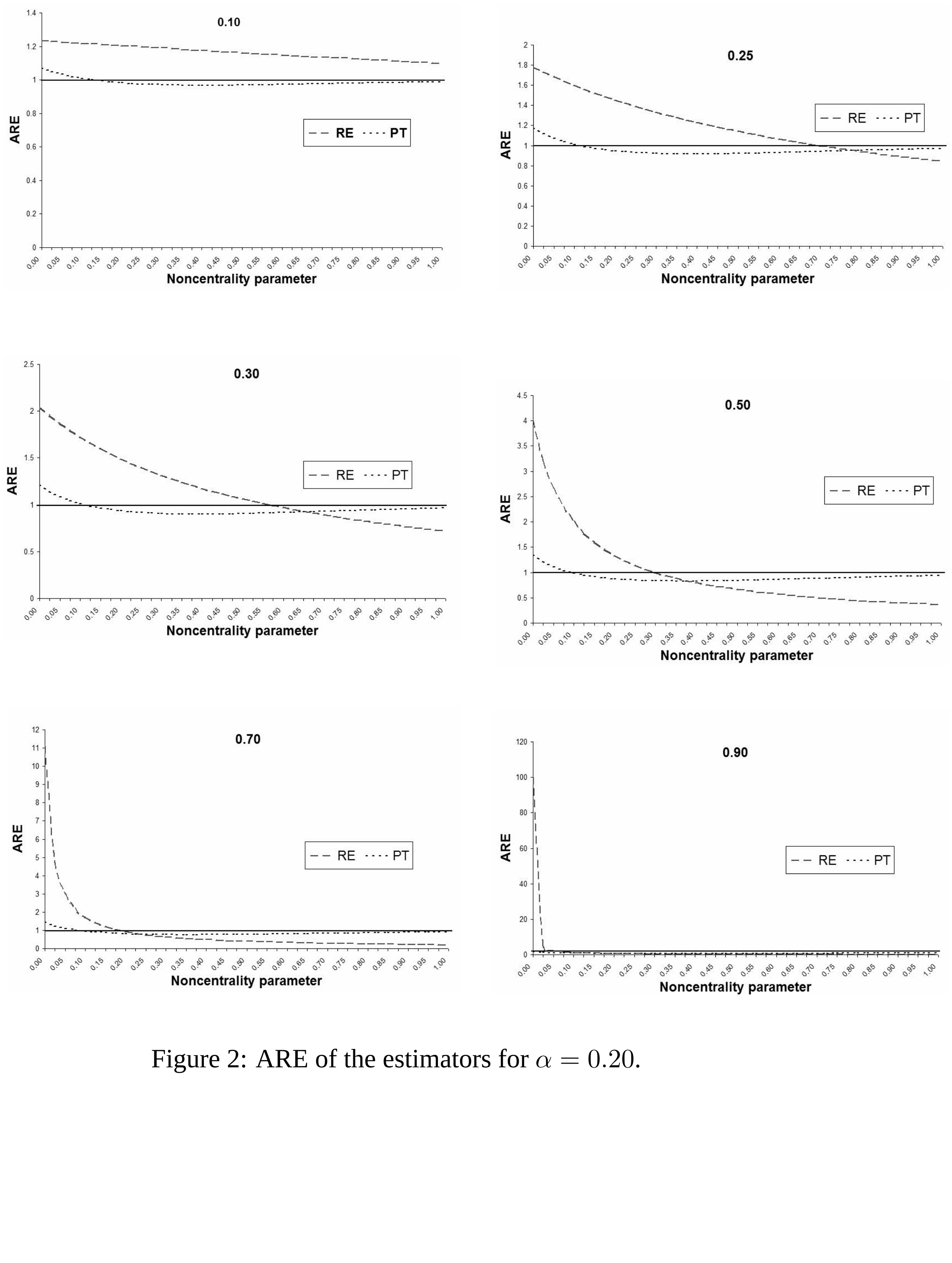

Figure 2: ARE of the estimators for $\alpha = 0.20$.

an estimator may be defined as follows. Let $\lambda_1^*, \lambda_2^*, \cdots, \lambda_M^*$ be a random sample of size m from the distribution of the estimator λ^*, then the MSE is estimated by

$$SMSE(\lambda^*) = \frac{1}{M} \sum_{h=1}^{M} (\lambda_h^* - \lambda)^2.$$

Our simulation study indicates that the maximum relative efficiency of all the proposed estimators relative to $\hat{\gamma}$ occurred at Ω (noncentrality parameter) = 0. It is apparent from Figures 1-2 that as ω increases, the relative efficiency of $\hat{\lambda}^{SP}$ becomes larger than that of $\hat{\lambda}^{S}$ and $\hat{\lambda}^{P}$, and approaches to one faster than that of $\hat{\lambda}^{P}$, while the relative efficiency of $\hat{\lambda}^{S}$ approaches zero.

We report in Table 1 the simulated relative efficiency of $\hat{\lambda}^{SP}$ to $\hat{\gamma}$ for a selected values of Ω.

Table 1: *Simulated efficiency of* $\hat{\lambda}^{SP}$ *relative to* $\hat{\gamma}$ *for* $\lambda = 0.5$.

α	$\Omega = 0$	$\Omega = 0.5$	$\Omega = 1$
0.01	3.88	2.29	1.08
0.05	2.42	1.90	1.01
0.10	2.00	1.58	1.04
0.15	1.83	1.42	1.02
0.20	1.81	1.33	1.01
0.25	1.78	1.39	1.05
0.30	1.49	1.19	1.02
0.40	1.13	1.05	1.01

In summary, the results illustrate our asymptotic theory and are in agreement with the typical characteristics of the preliminary test estimation.

6 Concluding Remarks

The goal of the paper was to succinctly develop the basic formulas, relationships and issues involved in estimation and testing of the dispersion parameter in a large sample setup. This task is accomplished by employing the asymptotic normal theory of the benchmark estimator.

We proposed three test statistics and studied their asymptotic null and nonnull distributions. For large n and under the null hypothesis, T_i follows a χ^2-distribution with one degree of freedom. Further, a class of point estimators are introduced when there is uncertainty concerning the the appropriate statistical model-estimator to use in representing data sampling process, we reinforced a basis for optimally combining estimation problems. We demonstrated a well-defined data-based shrinkage preliminary test estimator that combines estimation problem by shrinking the classical estimator to a plausible alternative quantity. Asymptotic and finite-sample mean squared error results are obtained. It was found that none of the four estimators is superior with respect to the other three, however, when the prior information regarding the parameter λ is nearly precise then these estimators can be ordered to the magnitude of their AMSEs as $\hat{\lambda}^{S} \succ \hat{\lambda}^{P} \succ \hat{\lambda}^{SP} \succ \hat{\gamma}$, where $\succ$ denotes domination.

We have developed our formulations and evaluations in a one-sample-problem context. The formulations can be extended to multiple estimation problems. Research on the statistical implications of these and other combining possibilities for a range of statistical models and distance measures is ongoing. We also conducted a simulation study to examine the behavior of our method for moderate sample sizes. Our analytical results are well supported by simulation findings. thus, the results are parallel with the theoretical proving.

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