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Investigating the Asymptotic Stability in The Ozone Layer Depletion for Pakistan Atmospheric Region

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Abstract

In this communication we present stochastic approach that finds application in atmospheric sciences such as ozone concentration variability. In order to investigate the asymptotic stability in the ozone layer depletion (OLD), the stochastic analysis will be utilized.
We propose to study the effectiveness of asymptotic stability. Developing a quantitative treatment for data covering a specific period, this piece of information will report for Pakistan an explorative assessment of the ozone layer depletion so that we would be able to predict future changes in the concentration of ozone on the basis of the CFC emissions and possibly forecast the UV-B radiation reaching the sea surface. We will also examine the parametric evaluation of autoregressive (AR) models reaching up to eight order of magnitude that is, from AR(1) to AR(8). The parameters values depict that the process of OLD is asymptotically stable. This study will also appraise results in terms of physical interpretation and the methodology used which could be helpful in obtaining predictions of use to various organizations in the private, the public, and the government sectors.

Keywords and Phrases: Asymptotic stability, atmospheric phenomenon, ozone layer depletion, explorative assessment, autoregressive modeling.

AMS Classification: 20D05.

1 Introduction

The evolution of science has increased the understanding of various aspects of the environment and consequently the predictability of many naturally occurring events. It is surprising to find that there do not exist, in general, quantitative studies of the well known problem of ozone layer depletion (OLD), this being particularly true for Pakistan. However, in the past few decades, there has emerged a widespread awareness of the importance of ozone from a new perspective. Environmental studies in 1970s indicated that the emission of Chlorofluorocarbons (CFCs) (anthropogenic factors) and several naturally occurring events like solar activity (solar cycle, solar flares, solar proton events, solar wind), aurora, and volcanic eruptions are responsible for the ozone hole. However, atmospheric scientists started taking serious notice of the development of ozone hole in early 1980s [1-8].

Due to a serious reduction of O_3 content in the atmosphere, life-forms on the earth are exposed to a new hazard in the form of an increase of harmful solar UV radiation intensity at the sea level. Obviously, this situation calls for an assessment, monitoring and prevention of the incidence of decrease in the O_3 concentration. Man-made causes of OLD are providing a threshold for OLD. The atmosphere is an open system, therefore, ozone layer depletion could be regarded as a non-linear process. However, linear models are simple to understand so as a first step we prefer to construct linear models (autoregressive) to study the phenomenon of OLD but we do not discard a possible non-linear representation of the OLD process.

The randomness of the phenomenon of OLD does lead to a consideration of the asymptotic theory of distribution of stochastic processes for modeling environmental phenomena. Now much of the asymptotic theory is predicted upon the assumption that the stochastic process consists of a sequence of independent random variable. This independence helps to establish the central limit theorem (CLT), establishing that under very general conditions the sum of a sequence of random variables, suitably normalized, has a normal distribution asymptotically [9-10].

Under the stationarity condition CLT holds for the first-order AR-processes. CLT also holds for higher-order AR-processes, with a slightly more complicated expression for the variance normalising factor. In the case of AR-processes of order greater than one, this CLT has been applied to perform tests of significance for atmospheric processes [11].

This theorem has a crucial role in AR-processes. For stationary stochastic processes CLT requires that the process should possess a certain type of asymptotic independence (i.e. observations separated by a large amount of time should be nearly independent)[12].

The validity of CLT ensures that the phenomenon in question is represented by the normal distribution. This, in turn, validates an application of stochastic and time series analysis. In particular, we attempt herein to look into the stochastic aspect of the phenomenon for Pakistan's atmosphere using the relevant time series modeling (via fluctuating dynamics of ozone layer). Obviously, such a modelling could be utilised to work out sundry appropriate correlation structures in making predictions of use to various organisations in Pakistan [13].

2 Autoregressive (AR) Models

For the sake of simplicity we have adopted to use linear autoregressive models rather than the nonlinear autoregressive models to illustrate a more vivid situation of ozone layer dynamics in the atmosphere. AR models belong to wider classes of ARIMA and ARMA models. In fact, a suitable AR model approximates an ARIMA model arbitrarily. Though in some cases, ARIMA models may be parsimonious (that is require fewer parameters) than the AR models, however, AR models have the advantage that the estimation of parameters by least-squares regression techniques, Box-Jenkins methodology and computer programs, all become simpler [14].

In AR$(n+1)$ models for stationary time series, a variable X_t can be written in terms of its past values X_{t-1}, X_{t-2}, $\cdots\cdots$, X_{t-k} so that

$$X_t = \alpha_t + \beta_1 X_{t-1} + \beta_2 X_{t-2} + \cdots\cdots + \beta_s X_{t-1} + \gamma_t \tag{1}$$

γ_t, the residuals of this fit are the white noise that are uncorrelated and assumed to be distributed normally with almost zero mean, and constant variance $\hat{\sigma}_a^2$. Expression (1) involves both the deterministic and stochastic terms. Fitting models of successive higher order can be obtained by applying the strategy to both the deterministic and the stochastic terms.

Now we come to the construction of models mentioned above for OLD and investigate the implications of such a modeling. Successive increments in the value of n leads towards fitting AR$(n+1)$ models to AR(n) models. The parameters are estimated for each order of the model and their final and initial values are recorded. The wider class of ARIMA (n, d, m) models are conditional regression models, their parameters can be estimated by the least squares method that minimizes the sum of squares of the residuals $\sum a_t$'s. In particular, when there are no moving average parameters as we are investigating in this thesis, we have pure AR$(n+1)$ models, where in this case we estimate $n = 0, 1, 2, 3, 4, 5, 6, 7$. This is worth mentioning that we will use two OLD data sets, one consisting of 296 data points X_i (i ranging from 1 to 296) (January 1970-August 1994) and the other consisting of 480 data points X_i (i ranging from 1 to 480) (January 1960-December 1999).

For the parameters, the least squares estimates are

$\hat{\beta} = (X'X)^{-1}X'Y$,

for which the residual sum of squares (RSS) are given by

$$\hat{\sigma}_a^2 = \sum_{t=2}^{n} \alpha_t^2 \tag{2}$$

where

$$Y = \begin{bmatrix} X_{n+1} \\ X_{n+1} \\ \vdots \\ X_N \end{bmatrix}, \quad X = \begin{bmatrix} X_n & \mathrm{X_{n-1}} & \dots & X_1 \\ X_{n+1} & \mathrm{X_n} & \dots & \mathrm{X_2} \\ \vdots & \vdots & & \vdots \\ X_{N-1} & \mathrm{X_{N-2}} & \dots & \mathrm{X_{N-n}} \end{bmatrix} \quad and \ \ \beta = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{bmatrix} \tag{3}$$

The formulation of AR (1)-AR(8) models and computation of values of the associated parameters for data $I(N = 296)$ follows as under:

AR (1) model

$$X_t = \beta_1 X_{t-1} + \alpha_0 \tag{4}$$

$$\hat{\beta}_1 = \frac{\sum_{t=2}^{\mathrm{N}} X_t X_{t-1}}{\sum_{\mathrm{t}=2}^{\mathrm{N}} X_{t-1}^2}$$

AR(2) model

$$X_t = \alpha_0 + \beta_1 X_{t-1} + \beta_2 X_{t-2} + \gamma_t \tag{5}$$

In this model we could find that it is stationary at $-1 < \beta_2 < 0$. Compute the values of auto-covariance and autocorrelation for a specified values of the parameters (β)

$$\hat{\beta}_1 = \frac{\sum_{t=2}^{\mathrm{N}} X_t X_{t-1}}{\sum_{\mathrm{t}=2}^{\mathrm{N}} X_{t-1}^2}, \quad \hat{\beta}_2 = \frac{\sum_{t=3}^{\mathrm{N}} X_{t-1} X_{t-2}}{\sum_{\mathrm{t}=3}^{\mathrm{N}} X_{t-2}^2} \tag{6}$$

$\hat{\beta}_1 = 0.793(\pm 0.061)$, $\hat{\beta}_2 = -0.129(\pm 0.060)$.

We have observed that the value of β_2 which measures the dependence of X_t on X_{t-2} is negative.

AR(3) model

$$X_t = \alpha_0 + \beta_1 X_{t-1} + \beta_2 X_{t-2} + \beta_3 X_{t-3} + \gamma_t \tag{7}$$

The parameter values are calculated along with their standard errors. This model is non-stationary for both of β_2 and β_3

$$\hat{\beta}_1 = \frac{\sum_{t=2}^{N} X_t X_{t-1}}{\sum_{t=2}^{N} X_{t-1}^2}, \quad \hat{\beta}_2 = \frac{\sum_{t=3}^{N} X_{t-1} X_{t-2}}{\sum_{t=3}^{N} X_{t-2}^2}, \quad \hat{\beta}_3 = \frac{\sum_{t=4}^{N} X_{t-2} X_{t-3}}{\sum_{t=4}^{N} X_{t-3}^2} \tag{8}$$

$\hat{\beta}_1 = 0.787(\pm 0.060)$, $\hat{\beta}_2 = -0.064(\pm 0.075)$, $\hat{\beta}_3 = -0.088(\pm 0.060)$

We have observed that the value of β_3 which measures the dependence of X_t on X_{t-3} is negative.

AR(4) model

$$X_t = \alpha_0 + \beta_1 X_{t-1} + \beta_2 X_{t-2} + \beta_3 X_{t-3} + \beta_4 X_{t-4} + \gamma_t \tag{9}$$

The computed parametric values along with their standard errors are as follows:

$$\hat{\beta}_1 = \frac{\sum_{t=2}^{N} X_t X_{t-1}}{\sum_{t=2}^{N} X_{t-1}^2}, \quad \hat{\beta}_2 = \frac{\sum_{t=3}^{N} X_{t-1} X_{t-2}}{\sum_{t=3}^{N} X_{t-2}^2}, \quad \hat{\beta}_3 = \frac{\sum_{t=4}^{N} X_{t-2} X_{t-3}}{\sum_{t=4}^{N} X_{t-3}^2}, \quad \hat{\beta}_4 = \frac{\sum_{t=5}^{N} X_{t-3} X_{t-4}}{\sum_{t=5}^{N} X_{t-4}^2} \tag{10}$$

$\hat{\beta}_1 = 0.789(\pm 0.061)$, $\hat{\beta}_2 = -0.062(\pm 0.075)$, $\hat{\beta}_3 = -0.112(\pm 0.075)$,
$\hat{\beta}_4 = 0.328(\pm 0.061)$

We have observed that the value of β_4 which measures the dependence of X_t on X_{t-4} is positive.

AR(5) model

$$X_t = \alpha_0 + \beta_1 X_{t-1} + \beta_2 X_{t-2} + \beta_3 X_{t-3} + \beta_4 X_{t-4} + \beta_5 X_{t-5} + \gamma_t \tag{11}$$

The computed parametric values for AR(5) along with their standard errors are found to be

$$\begin{aligned} &\hat{\beta}_1 = \frac{\sum_{t=2}^{N} X_t X_{t-1}}{\sum_{t=2}^{N} X_{t-1}^2}, \quad \hat{\beta}_2 = \frac{\sum_{t=3}^{N} X_{t-1} X_{t-2}}{\sum_{t=3}^{N} X_{t-2}^2}, \quad \hat{\beta}_3 = \frac{\sum_{t=4}^{N} X_{t-2} X_{t-3}}{\sum_{t=4}^{N} X_{t-3}^2}, \\ &\hat{\beta}_4 = \frac{\sum_{t=5}^{N} X_{t-3} X_{t-4}}{\sum_{t=5}^{N} X_{t-4}^2}, \quad \hat{\beta}_5 = \frac{\sum_{t=6}^{N} X_{t-4} X_{t-5}}{\sum_{t=6}^{N} X_{t-5}^2} \end{aligned} \tag{12}$$

$\hat{\beta}_1 = 0.785(\pm 0.061)$, $\hat{\beta}_2 = -0.058(\pm 0.075)$, $\hat{\beta}_3 = -0.110(\pm 0.075)$,
$\hat{\beta}_4 = 0.009(\pm 0.075)$, $\hat{\beta}_5 = 0.033(\pm 0.062)$

We have observed that the value of β_5 which measures the dependence of X_t on X_{t-5} is positive.

AR(6) model

$$X_t = \alpha_0 + \beta_1 X_{t-1} + \beta_2 X_{t-2} + \beta_3 X_{t-3} + \beta_4 X_{t-4} + \beta_5 X_{t-5} + \beta_6 X_{t-6} + \gamma_t \tag{13}$$

Parameters values have been evaluated along with their standard errors,

$$\begin{gathered}\hat{\beta}_1 = \frac{\sum_{t=2}^{N} X_t X_{t-1}}{\sum_{t=2}^{N} X_{t-1}^2}, \quad \hat{\beta}_2 = \frac{\sum_{t=3}^{N} X_{t-1} X_{t-2}}{\sum_{t=3}^{N} X_{t-2}^2}, \quad \hat{\beta}_3 = \frac{\sum_{t=4}^{N} X_{t-2} X_{t-3}}{\sum_{t=4}^{N} X_{t-3}^2}, \\ \hat{\beta}_4 = \frac{\sum_{t=5}^{N} X_{t-3} X_{t-4}}{\sum_{t=5}^{N} X_{t-4}^2}, \quad \hat{\beta}_5 = \frac{\sum_{t=6}^{N} X_{t-4} X_{t-5}}{\sum_{t=6}^{N} X_{t-5}^2}, \quad \hat{\beta}_6 = \frac{\sum_{t=7}^{N} X_{t-5} X_{t-6}}{\sum_{t=7}^{N} X_{t-6}^2}\end{gathered} \tag{14}$$

$\hat{\beta}_1 = 0.788(\pm 0.061)$, $\hat{\beta}_2 = -0.057(\pm 0.075)$, $\hat{\beta}_3 = -0.119(\pm 0.075)$,
$\hat{\beta}_4 = 0.003(\pm 0.075)$, $\hat{\beta}_5 = 0.104(\pm 0.078)$, $\hat{\beta}_6 = -0.088(\pm 0.060)$

We have observed that the value of β_6 which measures the dependence of X_t on X_{t-6} is negative.

AR(7) model

$$\begin{aligned} X_t &= \alpha_0 + \beta_1 X_{t-1} + \beta_2 X_{t-2} + \beta_3 X_{t-3} + \beta_4 X_{t-4} + \beta_5 X_{t-5} \\ &+ \beta_6 X_{t-6} + \beta_7 X_{t-7} + \gamma_t \end{aligned} \tag{15}$$

This gives the following parameter values along with their standard errors,

$$\begin{gathered}\hat{\beta}_1 = \frac{\sum_{t=2}^{N} X_t X_{t-1}}{\sum_{t=2}^{N} X_{t-1}^2}, \quad \hat{\beta}_2 = \frac{\sum_{t=3}^{N} X_{t-1} X_{t-2}}{\sum_{t=3}^{N} X_{t-2}^2}, \quad \hat{\beta}_3 = \frac{\sum_{t=4}^{N} X_{t-2} X_{t-3}}{\sum_{t=4}^{N} X_{t-3}^2}, \\ \hat{\beta}_4 = \frac{\sum_{t=5}^{N} X_{t-3} X_{t-4}}{\sum_{t=5}^{N} X_{t-4}^2}, \quad \hat{\beta}_5 = \frac{\sum_{t=6}^{N} X_{t-4} X_{t-5}}{\sum_{t=6}^{N} X_{t-5}^2}, \quad \hat{\beta}_6 = \frac{\sum_{t=7}^{N} X_{t-5} X_{t-6}}{\sum_{t=7}^{N} X_{t-6}^2}, \\ \hat{\beta}_7 = \frac{\sum_{t=8}^{N} X_{t-6} X_{t-7}}{\sum_{t=8}^{N} X_{t-7}^2}\end{gathered} \tag{16}$$

$\hat{\beta}_1 = 0.798(0.061)$, $\hat{\beta}_2 = -0.070(0.075)$, $\hat{\beta}_3 = -0.119(0.075)$, $\hat{\beta}_4 = 0.017(0.075)$,
$\hat{\beta}_5 = 0.117(0.078)$, $\hat{\beta}_6 = -0.185(0.076)$, $\hat{\beta}_7 = 0.120(0.060)$

We have observed that the value of β_7 that measures the dependence of X_t on X_{t-7} is positive.

AR(8) model

$$\begin{aligned} X_t &= \alpha_0 + \beta_1 X_{t-1} + \beta_2 X_{t-2} + \beta_3 X_{t-3} + \beta_4 X_{t-4} + \beta_5 X_{t-5} \\ &+ \beta_6 X_{t-6} + \beta_7 X_{t-7} + \beta_8 X_{t-8} + \gamma_t \end{aligned} \tag{17}$$

This model generates the parameter values along with their standard errors are given as under:

$$\begin{aligned} &\hat{\beta}_1 = \frac{\sum_{t=2}^{N} X_t X_{t-1}}{\sum_{t=2}^{N} X_{t-1}^2}, \quad \hat{\beta}_2 = \frac{\sum_{t=3}^{N} X_{t-1} X_{t-2}}{\sum_{t=3}^{N} X_{t-2}^2}, \quad \hat{\beta}_3 = \frac{\sum_{t=4}^{N} X_{t-2} X_{t-3}}{\sum_{t=4}^{N} X_{t-3}^2}, \\ &\hat{\beta}_4 = \frac{\sum_{t=5}^{N} X_{t-3} X_{t-4}}{\sum_{t=5}^{N} X_{t-4}^2}, \quad \hat{\beta}_5 = \frac{\sum_{t=6}^{N} X_{t-4} X_{t-5}}{\sum_{t=6}^{N} X_{t-5}^2}, \quad \hat{\beta}_6 = \frac{\sum_{t=7}^{N} X_{t-5} X_{t-6}}{\sum_{t=7}^{N} X_{t-6}^2}, \\ &\hat{\beta}_7 = \frac{\sum_{t=8}^{N} X_{t-6} X_{t-7}}{\sum_{t=8}^{N} X_{t-7}^2}, \quad \hat{\beta}_8 = \frac{\sum_{t=9}^{N} X_{t-7} X_{t-8}}{\sum_{t=9}^{N} X_{t-8}^2} \end{aligned} \tag{18}$$

$\hat{\beta}_1 = 0.797(0.061)$, $\hat{\beta}_2 = -0.068(0.076)$, $\hat{\beta}_3 = -0.121(0.075)$, $\hat{\beta}_4 = 0.017(0.075)$,
$\hat{\beta}_5 = 0.118(0.078)$, $\hat{\beta}_6 = -0.184(0.077)$, $\hat{\beta}_8 = 0.113(0.077)$, $\hat{\beta}_8 = 0.008(0.060)$

We have observed the value of β_8 that measures the dependence of X_t on X_{t-8} is positive. As is obvious from the above expressions that for AR (1) model we have $|\beta_1| < 1$, so the system is asymptotically stable [15].

AR models of which our model (4) is a special case were first introduced by Yule to provide a more realistic description of the atmospheric phenomena like sunspot, based on the statistical analysis of Wolfer's sunspot numbers so that one would be able to predict the activity in the sun [16][17].

3 Assessing the adequacy of OLD modeling

For checking the adequacy of these models we have gone through the following procedural steps:

(i) We have checked the sign of each of the estimated coefficients of each order. For a model to be adequate the sign of the estimated coefficients must agree with the theoretical expectations. This agreement appeared in the case of AR(1) model only.

(ii) We examined the standard deviation of the coefficients. For an adequate model this must be less than 5%. We found that for it is so in the case of AR(1) model only.

(iii) We checked the t-statistics and p-values. If the t-statistics is close to zero and the p-value is less than .05, then the true population regression coefficient is nonzero and that particular n should be adequate. This is also true only for AR(1) model.

(iv) We examined S_E (Estimated standard errors of the regression). If S_E is small, the sample data tend to lie close to the estimated value and in this case the order of the model will be adequate. Only AR(1) model fulfils the condition in our case.

(v) We have checked the coefficient of determination (R^2) which should be greater than 50% for an adequate model. This is 53% in the case of AR(1) model confirming its adequacy.

Thus the we consider AR(1) to be the most appropriate model to study the OLD behaviour in the stratospheric region of Pakistan. However, as a representation of the real stratospheric atmosphere our model has serious temporal and spatial limitations in terms of stationarity, seasonality and cyclicity. [18-22].

Hence AR (1) model appears to be adequate at that required level of significance. In view of all the above discussion we adopt an AR (1) model for the study of stratospheric ozone fluctuation forecasts. Estimating regression coefficients of order one, using data sets I and II we get the model equations as

$$x_t = 81.772 + 0.716x_{t-1} \tag{19}$$

$$x_t = 127.971 + 0.549x_{t-1} \tag{20}$$

Using eq. 19 the O_3 depth for 297^{th} month between the year 1970 and 1994 (i.e. the month of September 1994) can be computed as follows

$x_{297} = 81.772 + 0.716x_{296}$

For $x_{296} = 268$ DU we obtain

$x_{297} = 81.772 + 0.716 \times 268 = 273.66$ DU

Thus the forecast for the September 1994 is 273.66 DU within the forecast accuracy of 2.8% as absolute percentage forecast error (APFE). Similarly, Using eq. 20 the O_3 depth for 481^{st} month between the year 1960 and 1999 (i.e. the month of January 1999) can be computed as follows:

$$X_{481} = 127.971 + 0.549x_{480}$$

$$\hat{x}_{481} = 127.971 + 0.549x_{480} = 127.971 + 0.549 \times 260 = 270.71 \text{ DU}$$

Forecast accuracy was computed as 4.12%, which is suitable for Pakistan's stratosphere.

4 Conclusion

The phenomenon of OLD is a potential source of high incidence of UV radiation on the sea level. To meet this immediate threat, one requires proper gauging and monitoring of the impacts of OLD. For a systematic handle on the problem one needs to understand the nature of variations in ozone concentration at the stratospheric region of any specific area. In this communication Calculations inferred the following:

(i) the process possesses a good degree of normality, which is reasonable from the viewpoint of further analysis, though raising the question of the performance of Dobson spectrophotometers being used for recording the events at the detection centers, on one hand, and of the actual configuration of the UV radiation penetrating through O_3 filter (probability distribution)

(ii) Utilizing the estimates of ozone layer depletion worked out above constructs and validates a linear self-regressive model, giving a forecast of this variation for Pakistan's stratospheric region with reasonable forecast accuracy.

(iii) We have observed the parametric values from AR(1-8) and found that for every model AR (1) model indicates $|\beta_1| < 1$, so the system is asymptotically stable.

AR models of which our model (4) is a special case were first introduced by Yule to provide a more realistic description of the atmospheric phenomena like sunspot, based on the statistical analysis of Wolfer's sunspot numbers so that one would be able to predict the solar activity.

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