International Journal of Statistical Sciences *ISSN 1683-5603*
Vol. 22(2), November, 2022, pp 1-11

Four Ways to Compute Hypergeometric Probability

Anwar H. Joarder[1*] and M. Hafidz Omar[2]

[1]Department of Computer Science and Engineering, Faculty of Science and Engineering
Northern University of Business and Technology Khulna, Khulna 9100, Bangladesh

[2]Department of Mathematics and Statistics, King Fahd University of Petroleum and Minerals, Dhahran 31261, Saudi Arabia
Emails: omarmh@kfupm.edu.sa; ohmstat@gmail.com

[*]Correspondence should be addressed to Anwar H. Joarder
(Emails: anwar.joarder@nubtkhulna.ac.bd; ajstat@gmail.com)

[Received June 12, 2021; Accepted April 26, 2022]

Abstract

Hypergeometric probability arises in the context of sampling without replacement from a finite population. We describe four methods for computing hypergeometric probability, discus their relative merits and a connection between the two widely used methods. In turn, it has been transparent that the sample space is not unique though they result in the same probability model on the real line.

Keywords: Hypergeometric probability, finite sampling, sampling without replacement, binomial distribution

MSC (2020): 05-02, 05-08, 60C05.

1. Introduction

There are two major ways of calculating hypergeometric probabilities. One assumes that the items in the finite population are distinguishable, or, can be labelled to make them distinguishable. This is widely known as the Combinatorial Method (Method 3). At the beginning we present two other methods: Permutation Method (Method 1) and another method (Method 2) based on it. In these methods items are selected in a group which is an outcome in the sample space. Each outcome is equally likely guaranteeing the simple random sampling. The sample can also be drawn by a random number table or by any other method devised for simple random sampling.

In the other case, it is immaterial whether the items in the finite population are distinguishable or indistinguishable. Items are sequentially drawn one after another without having replaced the previous ones popularly known as sampling without replacement. This will yield a sample space where outcomes are based on dichotomous nature of the population. The sample space has lesser number of elements compared to the first two methods and calculation of probability transparently shows the sequential nature of the change in probability, it is getting popularity. This will be discussed in Method 4 called Sequential Sampling Method.

In this note, we derive the probability of an event $\{X = x\}$ in an instructive way by all four methods available, try to pinpoint the way they are connected through the sample space, and discuss their relative merits. Finally we present an algorithm for ease of calculation. The examples have been presented in a way that shows insights into the fundamentals so that undergraduate students from broad spectrum of areas can easily grasp the proof.

2. Distinguishable Items

Suppose that an urn contains K items of one kind (say non-defective items) and $N-K$ items are of a different kind (say defective items). The items may be distinguishable or can be made distinguishable by labelling in case they are indistinguishable. Let X denote the number of non-defective items, generally called successes, selected at a time. The probability of x successes in n trials is derived by 3 methods in this section.

Method 1 (Permutation Method)

Theorem 1 Let an urn contain K items of one kind (say non-defective items) and $N-K$ items are of a different kind (say defectives). The items may be distinguishable or indistinguishable. The probability of x non-defective items in a sample of size n is given by

$$P(X=x)=C_x^n\times\frac{P_x^K P_{n-x}^{N-K}}{P_n^N},\quad \max\{0,n-(N-K)\}\le x\le\min\{n,K\}. \tag{1}$$

where P_x^n and $\binom{n}{x}=C_x^n$ are usual notations for permutation and combination.

Proof. Let $P_x^K=K(K-1)\cdots(K-x+1)$ be the number of permutations of K elements taking x elements at a time. Then there are $P_x^K P_{n-x}^{N-K}$ sequences in the sample space that have x consecutive successes, a typical sequence of which is $G^{i_1}G^{i_2}\cdots G^{i_x}D^{x+1}\cdots D^n$, say, where $i_1=1,2,...,K$; $i_2=1,2,...,K$; …, $i_x=1,2,...,K$ and $\max\{0,\ n-(N-K)\}\le x\le\min\{n,\ K\}$. Then the probability of a sequence of x successive successes, say, $G^1G^2\cdots G^xD^{x+1}\cdots D^n$, in a sample of size n selected at a time is given by

$$P(G^1G^2\cdots G^xD^{x+1}\cdots D^n)=\frac{P_x^K P_{n-x}^{N-K}}{P_n^N},\ \max\{0,\ n-(N-K)\}\le x\le\min\{n,\ K\}. \tag{2}$$

Each of the C_x^n outcomes in the sample space generates x successes with probability equal to (2). Hence, the probability of x successes in the sample is given by (1).

Joarder and Al-Sabah (2007) proved it without combinatorial arguments. In this method, elements of the sample space can be written out on P_n^N pieces of papers, thoroughly mixed and one can randomly be taken blindly. This will be the required sample. Note however that in this method, the *n* elements of the experiment are not sampled sequentially but selected as a combination.

Example 1 A sample of 3 digital voice recorders is selected at a time from a set of 5 of which 1 is defective.

a. What is the probability that only the first two voice recorders will be non-defective?

b. What is the probability that any 2 voice recorders in the sample will be non-defective?

Solution: The sample space of the experiment is provided below. Here the order of the arrangement is important.

$G^1G^2G^3(e_1)$	$G^1G^3G^2(e_1)$	$G^2G^1G^3(e_1)$	$G^2G^3G^1(e_1)$	$G^3G^1G^2(e_1)$	$G^3G^2G^1(e_1)$
$G^1G^2G^4(e_1)$	$G^1G^4G^2(e_1)$	$G^2G^1G^4(e_1)$	$G^2G^4G^1(e_1)$	$G^4G^1G^2(e_1)$	$G^4G^2G^1(e_1)$
$G^1G^2D^5(e_2)$	$G^1D^5G^2(e_3)$	$G^2G^1D^5(e_2)$	$G^2D^5G^1(e_3)$	$D^5G^1G^2(e_4)$	$D^5G^2G^1(e_4)$
$G^1G^3G^4(e_1)$	$G^1G^4G^3(e_1)$	$G^3G^1G^4(e_1)$	$G^3G^4G^1(e_1)$	$G^4G^1G^3(e_1)$	$G^4G^3G^1(e_1)$
$G^1G^3D^5(e_2)$	$G^1D^5G^3(e_3)$	$G^3G^1D^5(e_2)$	$G^3D^5G^1(e_3)$	$D^5G^1G^3(e_4)$	$D^5G^3G^1(e_4)$
$G^1G^4D^5(e_2)$	$G^1D^5G^4(e_3)$	$G^4G^1D^5(e_2)$	$G^4D^5G^1(e_3)$	$D^5G^1G^4(e_4)$	$D^5G^4G^1(e_4)$
$G^2G^3G^4(e_1)$	$G^2G^4G^3(e_1)$	$G^3G^2G^4(e_1)$	$G^3G^4G^2(e_1)$	$G^4G^2G^3(e_1)$	$G^4G^3G^2(e_1)$
$G^2G^4D^5(e_2)$	$G^2D^5G^4(e_3)$	$G^4G^2D^5(e_2)$	$G^4D^5G^2(e_3)$	$D^5G^2G^4(e_4)$	$D^5G^4G^2(e_4)$
$G^2G^3D^5(e_2)$	$G^2D^5G^3(e_3)$	$G^3G^2D^5(e_2)$	$G^3D^5G^2(e_3)$	$D^5G^2G^3(e_4)$	$D^5G^3G^2(e_4)$
$G^3G^4D^5(e_2)$	$G^3D^5G^4(e_3)$	$G^4G^3D^5(e_2)$	$G^4D^5G^3(e_3)$	$D^5G^3G^4(e_4)$	$D^5G^4G^3(e_4)$

Table 1

Permutation Sample Space

Note that $\{e_1,\ e_2,\ e_3,\ e_4\}$ is the sample space in Sequential Sampling Method which will be discussed in Section 3. Each of the 60 outcome has the same probability of 1/60.

a. The event A comprises of the following elements from the permutation sample space:
$\{G^1G^2D^5,\ G^1G^3D^5,\ G^1G^4D^5,\ G^2G^3D^5,\ G^2G^4D^5,\ G^2G^1D^5,\ G^3G^1D^5,\ G^3G^2D^5,\ G^3G^4D^5,\ G^4G^1D^5,\ G^4G^2D^5,\ G^4G^3D^5\}$

Then the probability is $P(A)=\dfrac{12}{60}=0.20.$

Note that we have selected 12 elements from a population of 60 elements. It can be solved by preparing the following table and then using formula (2).

	Non-defective items	Defective items	Total
Population	K	$N-K$	N
Sample	x	$n-x$	n

Table 2

Here $N=5, K=4, n=3, x=2$ (see (2) or preferably the above table), the required probability is $\dfrac{P_2^4 P_{3-2}^{5-4}}{P_3^5}=\dfrac{12}{60}=0.20.$

b. The event B comprises of the following elements from the permutation sample space:

$G^1G^2D^5$	$G^1D^5G^2$	$G^2G^1D^5$	$G^2D^5G^1$	$D^5G^1G^2$	$D^5G^2G^1$
$G^1G^3D^5$	$G^1D^5G^3$	$G^3G^1D^5$	$G^3D^5G^1$	$D^5G^1G^3$	$D^5G^3G^1$
$G^1G^4D^5$	$G^1D^5G^4$	$G^4G^1D^5$	$G^4D^5G^1$	$D^5G^1G^4$	$D^5G^4G^1$
$G^2G^4D^5$	$G^2D^5G^4$	$G^4G^2D^5$	$G^4D^5G^2$	$D^5G^2G^4$	$D^5G^4G^2$
$G^2G^3D^5$	$G^2D^5G^3$	$G^3G^2D^5$	$G^3D^5G^2$	$D^5G^2G^3$	$D^5G^3G^2$
$G^3G^4D^5$	$G^3D^5G^4$	$G^4G^3D^5$	$G^4D^5G^3$	$D^5G^3G^4$	$D^5G^4G^3$

Table 3

Then the probability is $P(B) = C_2^3 \times \frac{12}{60} = 0.60.$

By (2), or preferably by using the above table, the required probability is

$$P(X = 2) = C_2^3 \times \frac{P_2^4 P_{3-2}^{5-4}}{P_3^5} = 3 \times \frac{12}{60} = 0.60.$$

Method 2 (Sampled and Un-Sampled Decomposition)

The probability of a sequence of x successes in a sample of size n is given by

$$P(G^1G^2 \cdots G^x D^{x+1} \cdots D^n) = \binom{N-n}{K-x} \div \binom{N}{K}, \tag{3}$$

where $\max\{0, n-(N-K)\} \le x \le \min\{n, K\}$. The probability of x successes in a sample of size n is given by

$$P(X = x) = \binom{n}{x} \times \left[\binom{N-n}{K-x} \div \binom{N}{K}\right], \tag{4}$$

where $\max\{0,\ n-(N-K)\} \le x \le \min\{n,\ K\}.$

Joarder (2011) mentioned the above without a logical proof though it follows from (1) of the Permutation Method. Note however that in this method as well, all n elements of the experiment are sampled as a pre-sequenced group, not sequentially.

A closer examination of equation (4) reveals that the probability of obtaining x successes in n trials (samples) can be expressed as the product of combination of the un-sampled and the sampled successes divided by the combination of the total successes in the population. The example in Method 1 is solved by Method 2 below.

Solution:

	Sampled	Un-sampled	Total
Any item	n	$N-n$	N
Non-defective items	x	$K-x$	K

Table 4

Solution to Example 1

a. Here $N=5, K=4, n=3, x=2$ and then by looking at the formula (3) or preferably by the above table, the required probability is

$$\binom{N-n}{K-x} \div \binom{N}{K} = \binom{5-3}{4-2} \div \binom{5}{4} = \frac{1}{5} = 0.20.$$

b. Here $N=5, K=4, n=3, x=2$ and then by looking at the formula (4) or preferably by the above table, the required probability is

$$P(X=2) = \binom{3}{2} \times \left[\binom{5-3}{4-2} \div \binom{5}{4} \right] = \frac{3}{5} = 0.60.$$

The sample space contains the following 5 elements:

$$\{G^1G^2G^3G^4,\ G^1G^3G^2D^5,\ G^1G^2G^4D^5,\ G^1G^3G^4D^5,\ G^2G^3G^4D^5\},$$

each having the same probability (0.20).

Method 3: Combinatorial Method

This is the most widely used method and will be called Combinatorial or Popular Method. The probability of x items in a sample of size n is given by

$$P(X=x) = \left[\binom{K}{x} \binom{N-K}{n-x} \right] \div \binom{N}{n}, \quad \max\{0,\ n-(N-K)\} \le x \le \min\{n,\ K\}. \tag{5}$$

The above is obvious from Table 2. Again note however that in this method, all n elements of the experiment are sampled as a group rather than sampling sequentially. The proof is available in most books on elementary probability. The example in Method 1 is solved by Method 3 below.

Solution to Example 1

a. The sample space contains 10 elements. Let the non-defective voice recorders be distinguishable and denoted by G^1, G^2, G^3, G^4 and the defective one by D^5. In case, the items are not distinguishable, the items may be labelled to make them distinguished. The method does not write out the sample space nor calculate the probability of outcomes in it. And there is no obvious way to calculate the probability of the event of interest. For a sample of $3(=n)$ items, from the population of $5(=N)$ items, then the sample space can be written as

$$\{G^1G^2G^3,\ G^1G^2G^4, G^1G^2D^5, G^1G^3G^4, G^1G^3D^5,$$
$$G^1G^4D^5, G^2G^3G^4, G^2G^4D^5, G^2G^3D^5, G^3G^4D^5\}.$$

Since the order of the arrangement of outcomes in the sample space is not important, this part of the question does not have a solution by this method.

b. The event of interest is

$A = \{G^1G^2D^5,\ G^1G^3D^5,\ G^1G^4D^5,\ G^2G^4D^5,\ G^2G^3D^5,\ G^3G^4D^5\}$ which has probability $P(A) = 0.60$ since each of the 10 sample points has a probability of 0.10. Note that any column of Permutation Sample Space is the sample space in this method.

Alternatively by using (5), with $N = 5, K = 4, n = 3, x = 2,$ or preferably, by looking at the table in Method 1, the required probability is

$$P(X = 2) = \left[\binom{4}{2}\binom{1}{1}\right] \div \binom{5}{3} = \frac{6}{10} = 0.60.$$

Notice that the 10 elements of the sample space in this method has been permuted $3!(= n!)$ times resulting in $n!C_n^N = 6\times 10$ elements in the sample space in Method 1. For example, the first outcome $G^1G^2G^3$ can be permuted to

$\{G^1G^2G^3, G^1G^3G^2, G^2G^1G^3, G^2G^3G^1, G^3G^1G^2, G^3G^2G^1\},$

i.e., 6 ways which is the row 1 of the permutation sample space in Method 1.

3. Distinguishable or Indistinguishable Items

Method 4 (Sequential Sampling Method)

In this method, the ordering of arrangement in the sample space refers to the particular draw and is important. Suppose that a population containing K items of one kind (say non-defective items) and $N - K$ items are of different kind (say defective items). Let n items be drawn at random in succession, without replacement, and X denote the number of non-defective items selected. The quantity $G_1G_2 \cdots G_xD_{x+1} \cdots D_n$ denotes the x successive non-defectives and $n - x$ successive defective items. The probability is expressed by truncated factorial by Joarder and Al-Sabah (2007). It can also be expressed by permutations or combinations (Joarder, 2011).

Theorem 3.1 Let n items be drawn at random in succession, without replacement, and X denote the number of non-defective items, generally called successes, selected. Then the probability of x successes in n trials is given by the following:

$$P\left(X = x\right) = \binom{n}{x}\frac{P_x^K}{P_x^N} \times \frac{P_{n-x}^{N-K}}{P_{n-x}^{N-x}},,\ \max\{0, n-(N-K)\} \le x \le \min\{n, K\}. \tag{6}$$

Proof.

a. Let $\tau_1 = P(G_1G_2...G_x)$. Then by using conditional probability, we have

$\tau_1 = P(G_1)P(G_2 \mid G_1)\cdots P(G_x \mid G_1G_2 \cdots G_{x-1}),$

which is tabulated below:

Event	G_1	$G_2 \mid G_1$	…	$G_x \mid G_1 G_2 \ldots G_{x-1}$	$G_1 G_2 \ldots G_x$
Probability	$\frac{K+0}{K+(N-K)} = a_1$	$\frac{(K-1)+0}{(K-1)+(N-K)} = a_2$	…	$\frac{(K-x+1)+0}{(K-x+1)+(N-K)} = a_x$	$a_1 a_2 \ldots a_x$

Table 5

i.e., $\tau_1 = a_1 a_2 \ldots a_{x-1} a_x$,

$$\tau_1 = \frac{K+0}{K+(N-K)} \times \frac{(K-1)+0}{(K-1)+(N-K)} \times \cdots \times \frac{(K-x+1)+0}{(K-x+1)+(N-K)} = \frac{P_x^K}{P_x^N}. \quad (7)$$

b. Let $\tau_2 = P(D_{x+1} D_{x+2} \cdots D_{n-1} D_n \mid G_1 G_2 \cdots G_{x-1} G_x)$. Then by using conditional probability, we have

$$\tau_2 = P(D_{x+1} \mid G_1 G_2 \cdots G_x) P(D_{x+2} \mid G_1 G_2 \cdots G_x D_{x+1}) P(D_{x+3} \mid G_1 G_2 \cdots G_x D_{x+1} D_{x+2}) \ldots$$
$$\times P(D_n \mid G_1 G_2 \cdots G_x D_{x+1} \cdots D_{n-1}),$$

which equals

$$\tau_2 = \frac{0+(N-K)}{(K-x)+(N-K)} \times \frac{0+(N-K-1)}{(K-x)+(N-K-1)} \times \cdots \times \frac{0+[N-K-(n-x)+1]}{(K-x)+[N-K-(n-x)+1]}.$$

The above simplifies to

$$\tau_2 = \frac{(N-K)(N-K-1)\ldots(N-K-n+x+1)}{(N-x)(N-x-1)\cdots(N-n+1)} = \frac{P_{n-x}^{N-K}}{P_{n-x}^{N-x}}. \quad (8)$$

By (7) and (8), we have

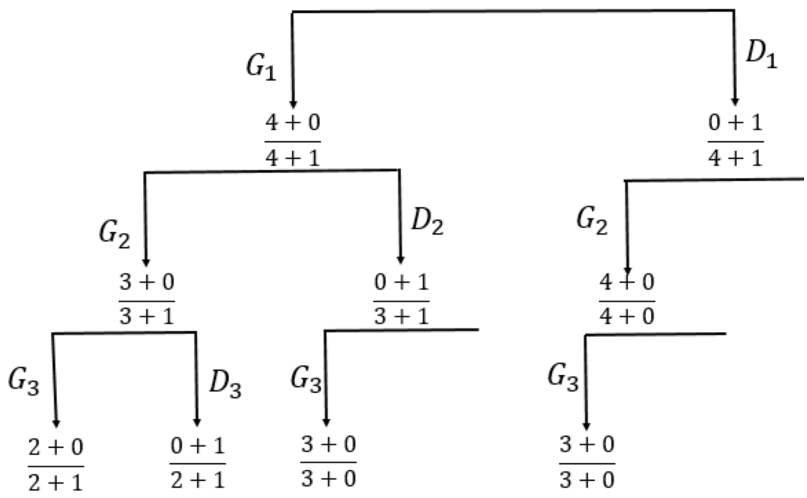
$$\tau = P(D_1 D_2 \cdots D_x D'_{x+1} \cdots D'_n) = \tau_1 \tau_2 = \frac{P_x^K}{P_x^N} \times \frac{P_{n-x}^{N-K}}{P_{n-x}^{N-x}}. \quad (9)$$

Each of the C_x^n outcomes in the sample space generates x successes with probability equal to (1). Hence, the probability of x successes in the sample is given by (6). Note that in this method, the *n* elements of the experiment are sampled sequentially rather than as a group. The example in Method 1 is solved by Method 4 below by making the fundamental arguments prominent.

Solution to Example 1

a. A tree diagram is drawn below to write out the sample space:

Figure 1



By following the branches of the tree diagram, the sample space is then given by $\{G_1G_2G_3(e_1), G_1G_2D_3(e_2), G_1D_2G_3(e_3), D_1G_2G_3(e_4)\}$, where $G_i(i=1,2,3)$ is the event that in the i-th selection, a non-defective recorder was obtained, and $D_i(i=1,2,3)$ is the event that a defective recorded was obtained in the i-th selection. In this method, for a sample of size $n=3$, the number of elements in the sample space should have been be $2^3=8$ but it has been only 4 because of smaller size of $N-K=1$. The probability of one sample outcome, say, $G_1G_2D_3$ is given by

$$P(G_1G_2D_3)=\frac{4+0}{4+1}\times\frac{3+0}{3+1}\times\frac{0+1}{2+1}=\frac{12}{60},$$

which can be done by the following table:

Event	G_1	$G_2 \vert G_1$	$D_3 \vert G_1G_2$	$G_1G_2D_3$
Probability	$\frac{4+0}{4+1}=a$	$\frac{3+0}{3+1}=b$	$\frac{0+1}{2+1}=c$	abc

Table 6

The number 60 in the denominator of the above probability is not obvious from the sample space of 4 elementary outcomes but it is clear from the permutation sample space in Method 1.

b. The other outcomes have probability

$$P(G_1G_2G_3)=\frac{4+0}{4+1}\times\frac{3+0}{3+1}\times\frac{2+0}{2+1}=\frac{24}{60},$$

$$P(G_1D_2G_3) = \frac{4+0}{4+1} \times \frac{0+1}{3+1} \times \frac{3+0}{3+0} = \frac{12}{60},$$

and

$$P(D_1G_2G_3) = \frac{0+1}{4+1} \times \frac{4+0}{4+0} \times \frac{3+0}{3+0} = \frac{12}{60},$$

respectively. Any of the above probability can simply be calculated by (2) or (3). The event of interest is $\{G_1G_2D_3, G_1D_2G_3, D_1G_2G_3\}$, has the probability

$$P(X=2) = \frac{12}{60} + \frac{12}{60} + \frac{12}{60} = \frac{36}{60},$$

which can also be calculated by $1 - P(G_1G_2G_3) = \frac{36}{60}$.

Each of the 4 elementary outcomes in the sample space of this method maps on to 24, 12, 12 and 12 elementary outcomes in permutation sample space in Method 1. The probability mass function simplifies to $P(X=2) = 0.60$ and $P(X=3) = 0.40.$

Though the method appears simple, we cannot do it let alone drawing a tree diagram in case sample size is large. Thus a method is required that provides insight, and yet amenable to calculate probability for large sample sizes.

4. Comparisons Among the Methods

The probability in (1) given by $\sum_x C_x^n \times \frac{P_x^K \times P_{n-x}^{N-K}}{P_n^N}$ is algebraically the same as $\sum_x \frac{P_x^K}{x!} \times \frac{P_{n-x}^{N-K}}{(n-x)!} \times \frac{n!}{P_n^N}$ which is the same as (5). That (4) and (5) are also probability mass functions can be easily checked by Vandermonde's identity. Since $P_x^N \times P_{n-x}^{N-x} = P_n^N$, from (2), we have

$$P(G_1G_2 \cdots G_x D_{x+1} \cdots D_n) = \frac{P_x^K}{P_x^N} \times \frac{P_{n-x}^{N-K}}{P_{n-x}^{N-x}}.$$

In case $K \to \infty, N \to \infty,$ such that $\frac{K}{N} \to p,$ then $\frac{P_x^K}{P_x^N} \to p^x$ and $\frac{P_{n-x}^{N-K}}{P_{n-x}^{N-x}} \to (1-p)^{n-x}$

proving that (1) converges to $P(X=x) = C_x^n p^x (1-p)^{n-x}, \quad 0 < p < 1, \quad x = 0,1,2,...,n,$ which is the mass function of binomial probability distribution. In case of sampling with replacement, the outcomes will be independent, probability of success at any trial will be constant but a connection to binomial distribution requires the binomial coefficient.

Because Method 1 (Permutation Method) has the binomial coefficient C_x^n, the particular form of hypergeometric probability function transparently provides connection with binomial probability function. But if the sample large is large, or even moderate, Method 1 will generate formidable number of outcomes in the sample space, and hence, we recommend using the sample space of Sequential Sampling Method and use (1) or (6) to calculate probability. We are hesitant to recommend Method 3 (the popular Combinatorial Method) as its probability mass function neither provides the probability of the sequential outcome directly nor shows any insight of its connection with the binomial probability mass function.

The probability mass function (1) or (6) given by Method 1 (Permutation Method) or Method 4 (Sequential Sampling Method) can also be extended naturally to multivariate hypergeometric distribution. If x_1 items are selected without replacement from N_1 items, x_2 items are selected from N_2 items and x_3 items are selected without replacement from N_3 items from a finite population of size $N = N_1 + N_2 + N_3$, then for a sample of size $n = x_1 + x_2 + x_n$, we have

$$P(X_1 = x_1, X_2 = x_2, X_3 = x_3) = \binom{n}{x_1, x_2, x_3} \frac{P_{x_1}^{N_1} P_{x_2}^{N_2} P_{x_3}^{N_3}}{P_n^N},$$

where $\binom{n}{x_1, x_2, x_3} = \frac{n!}{x_1! x_2! x_3!}$ The above is also exactly the same as an extension of (5) of Method 3 (Combinatorial Method) as

$$P(X_1 = x_1, X_2 = x_2, X_3 = x_3) = \left[\binom{N_1}{x_1}\binom{N_2}{x_2}\binom{N_3}{x_3}\right] \div \binom{N}{n},$$

for integers $0 \le x_1 \le N_1$, $0 \le x_2 \le N_2$, $0 \le x_3 \le N_3$, $1 \le n = x_1 + x_2 + x_n \le N$.

5. The Recommended Algorithm

We present our recommendation in the form of an algorithm:

STEP 1: Prepare Table 1

	Non-defective items	Defective items	Total
Population	$K = 4$	$N - K = 1$	$N = 5$
Sample	$x = 2$	$n - x = 1$	$n = 3$

Table 7

STEP 2: Calculate the probability of a sample point say, $G_1 G_2 \cdots G_x D_{x+1} \cdots D_n$, which is a part of the event of interest, say $\{X = x\}$ by Sequential Sampling Method if the sample size is small, say, not exceeding 5. Once having been efficient, or, for a large sample, do it by the formula:

$$P(G_1 G_2 \cdots G_x D_{x+1} \cdots D_n) = \frac{P_x^K P_{n-x}^{N-K}}{P_n^N},$$

which is given in (2). Observe that it is easy to write the above formula using information from Table 2.
For our example, by using Sequential Method (Method 4), or by formula in Method 1, we have $P(G_1G_2D_3)=\frac{P_2^4P_1^1}{P_3^5}=\frac{12}{60}$.

STEP 3: The probability of a compound event, say $P(X=x)$, is C_x^n times the probability in STEP 2.

For our example, the required probability is $P(X=2)=C_2^3\times\frac{12}{60}=0.60.$

6. Conclusion

We believe that the connection of sample spaces in Permutation Method, Combinatorial Method and the Sequential Sampling Method demonstrated in the paper provides insight into the issue and will inspire undergrad students and instructors. We conclude that sample space of an experiment is not unique though it generates the same probability model on the real line, in particular, on $\{x:x=2,3\}$ for our example. The Combinatorial Method and Sequential Sampling Method are well known but the connection between the sample space is never clearly explained in textbooks. Moreover, we feel, the way we decomposed the number of items in the population and the favorable number of ways in calculating the probability of a sample outcome in Sequential Sampling Method has less chance of making mistakes by students especially for small samples. We refer to an interesting paper by Trong (1993) that provides a method for calculating cumulative probability of hypergeometric distribution by appealing to prime numbers. We finally recommend that the four methods can be generalized to multivariate hypergeometric probability model.

Acknowledgements

The first author is grateful to Northern University of Business and Technology Khulna, Bangladesh for providing excellent research support. The authors are also grateful to the editor and the reviewers for constructive suggestions that have improved the quality of the original manuscript.

References

[1] Joarder, A. H. and Al-Sabah, W. S. (2007). Probability issues in without replacement sampling, International Journal of Mathematical Education in Science and Technology, 38(8), 823-831.

[2] Joarder, A. H. (2011). Hypergeometric distribution and its application in statistics. Published in International Encyclopedia of Statistical Science, 641-643. Edited by Miodrag Lovric. Springer.

[3] Trong, W. (1993). An accurate computation of the hypergeometric distribution function. ACM Transaction on Mathematical Software, 19(1), March 1993, 33-43.