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Forecasting Short Time Series using Rolling Grey Bayesian Framework

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Abstract

Agriculture is the backbone of Indian economy; it plays a crucial role in the overall socio-economic development of the nation. To address the needs of a growing global population and poverty eradication, appropriate policy measures in agriculture is required for economic planning and decision making. Statistical forecasting models as well as artificial intelligence approaches are generally used to predict the future pattern. Several forecasting models viz., auto regressive integrated moving average (ARIMA), auto regressive conditional heteroscedastic or generalized auto regressive conditional heteroscedastic (ARCH or GARCH), artificial neural network (ANN), support vector machine (SVM) are largely being used for forecasting production behavior of different agricultural crops using mainly time series data. These modelling techniques perform well with large amount of data or in case of situations where sufficient information is available for the crops under consideration. In real life scenario, most often the situation involves time series with comparatively short length and it is almost impossible to find parallel methods for modelling and forecasting with short time series data. Among many different available forecasting methods, grey system theory based model, rolling grey models, Bayesian (grey Bayesian models), approaches all assumed to be alternative choice for dealing with short time series data for prediction and limited number of such studies have been reported in the context of Indian agriculture. In this study, an approach has been considered by combining three different methods viz. rolling mechanism, grey model and Bayesian estimation techniques. The proposed approach is also compared with individual forecasting method. The study has been illustrated with the yearly production data of rice, wheat, maize, total oilseeds and total pulses production of Uttarakhand state and demonstrated that rolling grey Bayesian models are found to be better compared to other competitive models in accurately forecasting the production behaviors in all cases.

Keywords: Grey, AGO, Rolling, Bayesian, Short Time Series.

AMS Classification: 62M10.

1. Introduction

Agriculture is the most important sector of Indian economy and plays a significant role in the overall socio economic development of the country. Forecasting of production of different agricultural crops is an integral part of commodity trading and price analysis which require proper planning and efficient utilization of resources to meet sustainable development goals. Time series modelling is an active research area since few decades which has attracted attentions of researchers community specifically focused on finding the behavioral pattern present in a specific phenomenon over the time and to predict future values based on past information. Raichaoren *et al*. (2003) stated that time series forecasting can be termed as the act of predicting the future by understanding the past. Due to the indispensable importance of time series forecasting in numerous practical fields such as business, economics, finance, science and engineering, etc., Tong (1983) suggested, that proper care should be taken to fit an adequate model to the underlying time series. These processes can be related to variations of different agricultural commodity production. A lot of efforts have been done by researchers over many years for the development of efficient models to improve the forecasting accuracy. As a result, various important time series forecasting models have been evolved and are integral part of literature. In literature, time series models are generally classified into two broad categories namely linear and nonlinear models. Linear models include exponential smoothing, autoregressive integrated moving average (ARIMA), state space models, etc. Some of the non-linear time series models are Autoregressive Conditional Heteroscedastic (ARCH), Generalized Autoregressive Conditional Heteroscedastic (GARCH), Artificial Neural Networks (ANN), Support Vector Machines (SVM) model, etc.

In real life scenario, most often the situation involves time series with comparatively short length and it is almost impossible to find the pattern and complexities present using methods of statistical analysis. For example, commodity product life cycle analysis, analysis of agricultural item sales when there is a wide range of products and short product life expectancy, evaluation of experiment conducted in very short period of time, and so on. The amount of data depends on the type of statistical model being used and on the amount of random variation present in the data. Therefore, with more data, the better is the identification of structure and patterns of a time series that are used for forecasting (Hyndman and Kostenko, 2007). The number of data points required for any statistical model depends on at least two things: the number of model coefficients (parameters) to estimate and the amount of randomness in the data. Some guidance is available to determine suitable forecasting models based on the data requirements for several common forecasting models (Lin, 2013; Wilson and Allison, 1992). According to that, when the sample size is small, only a few models can be used. Therefore, for moving average, simple exponential smoothing, trend models, casual regression models, Box-Jenkins, neural network and grey models, the data requirement are 2-30, 5-10, 10-20, 10 observations per independent variable, 50, large number and 4 respectively. There are many time series analysis methods but the uses of these methods are not applicable when the system is characterized by complex, non-linear and having limited amount of data. Therefore, the emphasis is to find suitable approaches that would solve the problem of short time series analysis with relatively fewer observations. Among different available modelling and forecasting approaches, grey system theory based model, Bayesian approaches can be a good choice for short time series data analysis and prediction.

In recent years grey system theory has become a useful method of solving uncertainty problems under discrete data and incomplete information. Grey system theory based model was introduced by Deng (1982), is specially effective for processing data that are uncertain, discrete, insufficient and focusing on small samples, has been adopted in many areas such as managerial decision making, socio-economic research, forecasting yields of grain, weather forecasting, analyzing agricultural economy, etc. Dang *et al.* (2016) utilized modified Lotka-Volterra model (L.V. model) and compared it with exponential smoothing model and the grey model GM(1,1) with limited data of foreign patients from 2001 to 2013 in six destinations. The various application of grey model, it's modification and their comparison with other methods can also be found from various literatures viz., Yang *et al.* (2018), Muqtadir *et al.* (2016), Kayacan *et al.* (2010) and Yu and Schwartz (2006). Besides grey model, the capabilities of rolling grey forecasting model (RGM) is also reported by Chang *et al.* (2005), Akay and Atak (2007) in their respective studies.

In literature, hybrid approach comprising individual model capabilities has been developed for time series forecasting. By combining different methods, the problem of model selection can be eased with little extra effort as it increases the chance to capture different patterns in the data and improve forecasting performance (Sinha, 2011). In addition, the combined model is more robust with regard to the possible structural change in the data. Hsu and Wang (2007) evaluated original grey model and Bayesian grey forecasting models for the integrated circuit (IC) industry based on the characteristics of short period life cycle of IC product, rapidly processing technology and scarce historical time series data and compared the forecasting capability of both the methods.

In this study, an effort has been made to evaluate the forecasting ability of conventional grey system theory based model and its hybrid architecture for agricultural crop production. The hybrid model is proposed by combining rolling mechanism, grey model and Bayesian technique to deal with short time series agricultural production data. The prediction ability of these techniques is assessed using yearly production of rice, wheat, maize, total oilseeds and total pulses of Uttarakhand state, India. The rest of the article is formulated in the following manner. Section two discussed about materials and methods, section three on results and discussion followed by conclusion and finally references.

2. Materials and Methods

Data: Yearly production (in '000 tonnes) data of rice, wheat, maize, total oilseeds and total pulses of Uttarakhand state are considered in this study. The state was formed recently, during the year 2000 and as such there had been lesser number of data points available for modeling and forecasting of different crop production behavior. The time series data on yearly production of Uttarakhand has been collected from the website of *https://dbie.rbi.org.in/*, Govt. of India. There are a total of 17 years data points starting from the year 2000 and ending in 2016.

2.1. Grey forecasting model (GM(1, 1))

GM(1, 1) model is commonly used in statistical time series forecasting model when usage of large dataset and statistical assumptions are violated with a limited number of data points for construction of time series forecasting model. GM(1, 1) type of grey model is the most widely used in the literature, pronounced as ''Grey Model First Order One Variable". GM(1, 1) model has a simple form of representation and can only be used in positive data sequences. Assume the original data sequence as $x^{(0)} = \left(x_1^{(0)}, x_2^{(0)}, \dots, x_n^{(0)}\right)$ be provided by one system and consist of *n* samples. A new sequence $x^{(1)} = \left(x_1^{(1)}, x_2^{(1)}, \dots, x_n^{(1)}\right)$ can then be generated from $x^{(0)}$ by the accumulated generating operation (AGO) as follows:

$x_1^{(1)} = x_1^{(0)}$ and

$$x_k^{(1)} = \sum_{j=1}^{k} x_j^{(0)} \quad k = 1,2,\dots,n \tag{i}$$

$x_1^{(1)}, x_2^{(1)}, \dots, x_n^{(1)}$ can then be approximated as a first order differential equation.

$$\frac{dx^{(1)}(t)}{dt} + \beta x^{(1)}(t) = \alpha \tag{ii}$$

where β and α are the developing coefficient and control variable respectively. The AGO is used because it can identify potential regularities hidden in the data sequences even if the original data are finite, insufficient, and chaotic. The grey derivative for the first-order grey differential equation with 1-AGO data as the intermediate information is conventionally represented as

$$\frac{dx^{1}(t)}{dt} = \lim_{\Delta t \to 0} \frac{x^{(1)}(t + \Delta t) - x^{(1)}(t)}{\Delta t}$$

and $\frac{dx^{1}(t)}{dt} = \frac{\Delta x^{(1)}(t)}{\Delta t} = x^{(1)}(t+1) - x^{(1)}(t) = x^{(0)}(t+1)$ (iii)

where $\Delta t \to 0$ roughly. The background value of $\frac{dx^{1}(t)}{dt}$, $x^{(1)}(t)$ is taken as the mean of $x^{(1)}(t)$ and $x^{(1)}(t+1)$. The solution of eq. (ii) with system parameters determined by least square method and initial condition $x_1^{(1)} = x_1^{(0)}$. The predicted value $\hat{x}_k^{(1)}$ of $x_k^{(1)}$can be obtained by solving the grey difference equation with initial condition as

$$\hat{x}_k^{(1)} = \left(x_1^{(0)} - \frac{\alpha}{\beta}\right) e^{-\beta(k-1)} + \frac{\alpha}{\beta} \ \forall\ k = 2,3,4,\dots n \tag{iv}$$

β and α can be estimated by means of a grey difference equation as

$$x_k^{(0)} + \beta z_k^{(1)} = \tag{v}$$

where the background value of $z_k^{(1)}$ is formulated as follows

$$z_k^{(1)} = \delta x_k^{(1)} + (1-\delta) x_{k-1}^{(1)} \tag{vi}$$

where δ is adjustment coefficient and usually specified as 0.5 for convenience, but this is not the optimal setting. By using *n-1* grey difference equations, β and α can be obtained by the ordinary least-squares method as

$$[\beta, \alpha]^T = \left(\boldsymbol{B}^T \boldsymbol{B}\right)^{-1} \boldsymbol{B}^T \boldsymbol{Y} \tag{vii}$$

where

$$\boldsymbol{B} = \begin{bmatrix} -z_2^1 & 1 \\ -z_3^1 & 1 \\ . & . \\ . & . \\ . & . \\ -z_n^1 & 1 \end{bmatrix} \tag{viii}$$

$$\boldsymbol{Y} = \left[x_2^{(0)}, x_3^{(0)}, \dots, x_n^{(0)}\right]^T \tag{ix}$$

Using the inverse AGO (IAGO), the predicted value, $\hat{x}_k^{(0)}$, of $x_k^{(0)}$ can be generated as follows:

$$\hat{x}_k^{(0)} = \hat{x}_k^{(1)} - \hat{x}_{k-1}^{(1)} \forall\, k = 2,3,4, \dots n$$

Therefore, the predicted value can be obtained as

$$\hat{x}_k^{(0)} = \left(1 - e^{\beta}\right)\left(x_1^{(0)} - \frac{\alpha}{\beta}\right) e^{-\beta(k-1)} \;\forall\, k = 2,3,4, \dots, n \tag{x}$$

where $\hat{x}_1^{(1)} = \hat{x}_1^{(0)}$.

The diagrammatic representation of grey model is shown in fig. 1.

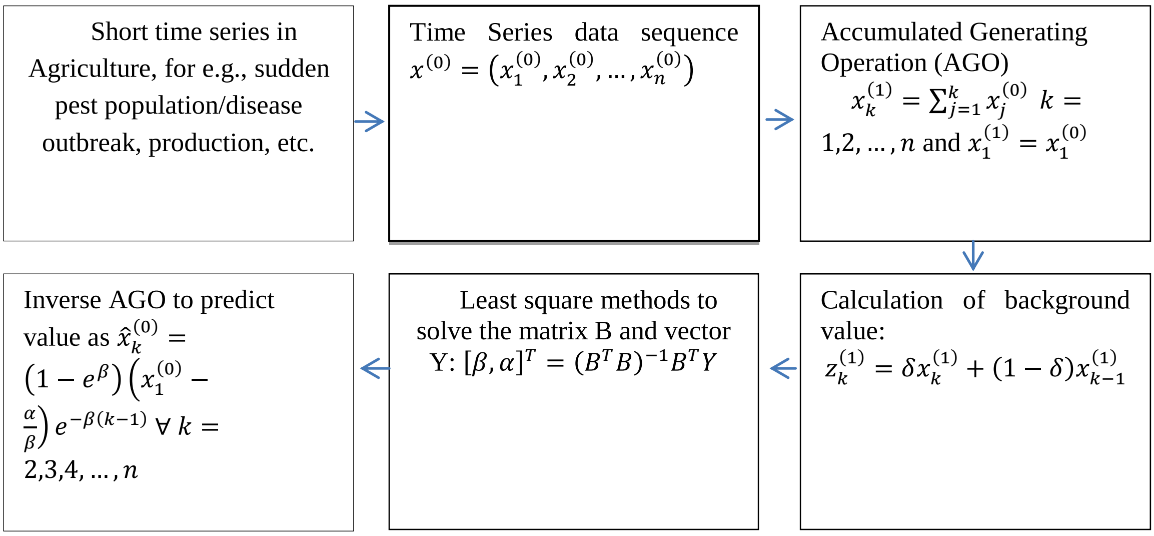

Figure 1: Flow chart of Grey system theory based model

2.2. Rolling Grey forecasting model

It has been reported that recent data can improve forecast accuracy in future prediction (Akay and Atak, 2007). Rolling mechanism in grey model is a metabolism technique that updates the input data by discarding old data for each loop in grey prediction. The purpose of rolling mechanism is that, in each rolling step, the data utilized for next step forecast is the most recent data. Rolling grey model is an efficient technique to increase the forecast accuracy in case of having noisy data. The data may exhibit different trends or characteristics at different times, so to address these differences, it is preferable to study such noisy data with the rolling grey model set up, and therefore for rolling grey model, the input data are always the most recent values. For instance, using $x^{(0)}(k), x^{(0)}(k+1), x^{(0)}(k+2)$ and $x^{(0)}(k+3)$, the model predicts the next value $x^{(0)}(k+4)$. In the next steps, the first point is always shifted to the second. It means that $x^{(0)}(k+1), x^{(0)}(k+2), x^{(0)}(k+3)$ and $x^{(0)}(k+4)$ are used to predict the value of $x^{(0)}(k+5)$. This procedure is repeated till the end of the sequence and this method is called rolling check. In rolling grey model, therefore identification of appropriate number of data points or model order along with varying δ value is an important concern for model construction. After generating and developing the model, further tests are necessary to understand the error between the forecasted value and the actual value. The methodology can be well described in the following way of fig. 2.

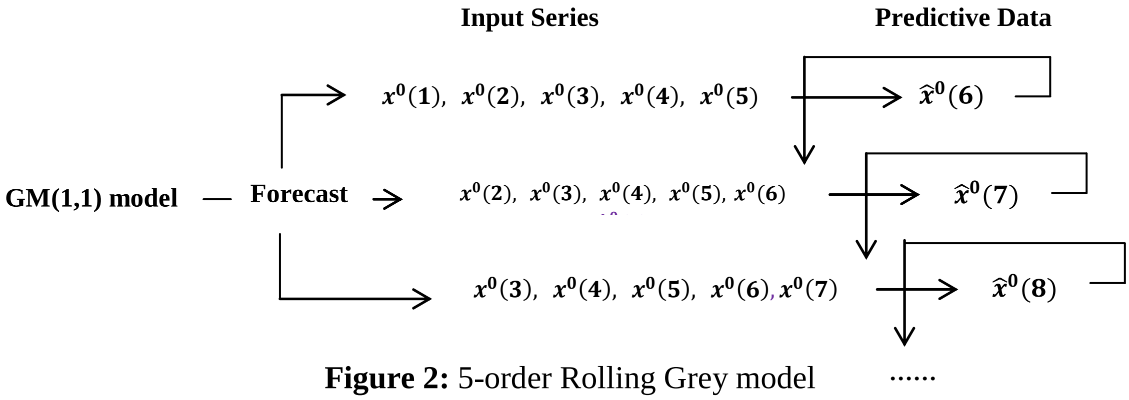

Figure 2: 5-order Rolling Grey model

2.3. Grey Bayesian Modelling

Bayesian method can be considered as an alternative to statistical analysis by which similar information is combined to produce a posterior probability distribution of one or more parameters of interest based on prior knowledge or information. In Bayesian methods, the parameters in the model may be simulated directly via methods for the exploration of posterior distributions. Therefore, asymptotic or approximate methods often relied upon using traditional inferences are not necessary. The most frequently used method is the Markov Chain Monte Carlo (MCMC) method and can be used to estimate posterior distributions by drawing sample values randomly from the full conditional distributions of each parameter conditional on all others and the data. A posterior probability is defined according to the precision of a priori and sample information by using Bayesian theorem. The task of Bayesian analysis is to build a model for the relationship between parameters (θ) and observable variables (y), and then calculate the probability distribution of parameters conditional on the data $p(\theta|y)$. In linear regression, the observations consist of a response variable in a vector $\boldsymbol{y}$ and one predictor variable in a matrix $\boldsymbol{X}$. The vector $\boldsymbol{y}$ has n elements corresponding to n observations. The matrix $\boldsymbol{X}$ has n rows corresponding to the observations, and 1 columns corresponding to the number of predictors. If the regression model consists an intercept, one of the columns of $\boldsymbol{X}$ is a column of one's. The parameters in a regression model to be estimated are regression coefficients β and the error variance of the fitted model, σ^2. In other words, this model states that the distribution of $\boldsymbol{y}$**,** given parameters β, σ^2 and predictors X, is a normal distribution with mean $\boldsymbol{X}B$ and variance $\sigma^2\boldsymbol{I}$. A normal distribution is completely specified by its mean and variance. A non-informative prior information that is commonly used for regression analysis is $p(\beta, \sigma^2) \propto {}^1/_{\sigma^2}$. This expression means that the joint probability distribution of β and σ^2 given X is a flat surface with a constant level proportional to ${}^1/_{\sigma^2}$. The posterior distribution of β given σ^2 is $\beta|\sigma^2$, $y{\sim}Normal(\beta_E, V_{\beta^{\sigma^2}})$ means that the probability distribution of β given σ^2 and y is normal with mean β_E and variance $V_{\beta^{\sigma^2}}$. The parameters of this normal distribution are computed as $\beta_E = (X'X)^{-1}X'y$, $V_\beta = (X'X)^{-1}$, the apostrophe $(')$ denotes matrix transposition. The marginal posterior distribution of σ^2 is $\sigma^2|y{\sim}$Inverse $\chi^2(n-k, s^2)$, and it says that probability distribution of σ^2 given y follows an inverse χ^2 distribution. After estimating the parameters using Bayesian technique, grey Bayesian forecasting model i.e., GM(1,1)-Bayesian can be obtained by substituting the estimated parameters in equation (x).

2.4. Rolling grey Bayesian modelling

A somewhat new approach has been formulated by combining the methodology of grey model, rolling mechanism and Bayesian approach as mentioned in the section 2.1, 2.2 and 2.3 respectively. Here, rolling grey model has been utilized to update the input data by discarding old data for each loop and consequently a Bayesian technique is implemented at each loop by taking the advantage of incorporating prior information for parameters. The estimated parameters are then placed to the grey forecasting equation and forecast value is obtained for the next period. The methodology is conducted by selecting a proper order of rolling grey model with an appropriate value of adjustment coefficient (δ) based on minimum value of mean absolute percentage error (MAPE). In the next, after selecting proper order of rolling grey model with δ value, Bayesian modelling is combined at each loop of rolling grey model and obtained the estimated values of parameters along with MC error. The forecasting equation of rolling grey Bayesian model is then generated by substituting the parameters obtain at each loop into eq. (x) of section 2.1. The steps for rolling grey Bayesian model can be written as follows:

Step i: Partitioning production time series data into training and testing part

Step ii: Appropriate grey model order and adjustment coefficient (δ) identification on training set

Step iii: Bayesian method implementation at each loop of 1-AGO based grey model

Step iv: Replace Bayesian parameter estimates in grey forecasting equation

Step v: Repeat from step i to iv up to last data point.

The combination of these methodologies is thought to be an ideal choice for modelling and forecasting of short time series agricultural data and hence has been applied in this study.

2.5. Forecast evaluation methods

Four different criteria based on error terms is used to make comparisons among the forecasting ability of different methods utilized in this study. The most commonly used accuracy measures whose scale depends on the scale of the data are root mean square error (RMSE) and mean absolute error (MAE) as shown by the Eq. (xi) and (xii) respectively.

$$RMSE = \sqrt{\frac{1}{n}\sum_{t}^{n}(y_t - \hat{y}_t)^2} \quad \text{(xi)}$$

$$MAE = \frac{1}{n}\sum_{t=1}^{n}|y_t - \hat{y}_t| \quad \text{(xii)}$$

where y_t is the actual value at time t, $\hat{y}_t$ is the predicted value at time t and n is the number of predictions.

Mean Absolute Percentage Error (MAPE) is used to measure about how much a dependent series varies from its model-predicted level and is computed as

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\frac{|y_t - \hat{y}_t|}{y_t} \times 100 \quad \text{(xiii)}$$

Along with these three criteria, to measure the closeness of the statement of forecast quantity and the actual value, precision rate has also been utilized in this study which is defined as

p=1-MAPE (xiv)

Table 1 represents different forecast grades criteria as proposed by Ma and Zhang, 2009 based on the value of mean absolute percentage error (MAPE) and has been utilized in this study. According to them, the four level precision rate include unqualified (≤85%), qualified (>85%≤90%), good (>90%≤95%) and excellent (>95%) for measuring forecasting ability of a model.

Table 1: MAPE forecasting accuracy reference criteria (Source: Ma and Zhang, 2009)

Range of MAPE	Forecasting accuracy
>95%	Excellent
>90%≤95%	Good
>85%≤90%	Qualified
≤85%	Unqualified

3. Results and Discussion

3.1. Descriptive statistics

Results reveal that there is a significant difference between average productions of rice and wheat with other crop production in this state mainly because of wheat is main crop followed by rice and other crops in this region. The coefficient of variation (CV) of production of different crops is also at per, revealing almost similar kind of variation in production throughout the study period. The amount of variation for total pulses production of Uttarakhand state as 29.92 percent whereas rice production has lowest amount of variation 6.22 percent. Therefore, rice production is stable in this region followed by wheat production as shown in table 2.

Table 2: Descriptive statistics of rice, wheat, maize, total oilseeds and total pulses production ('000 tonnes) in Uttarakhand

	Rice	**Wheat**	**Maize**	**Total oil seeds**	**Total pulses**
Data period	2000 to 2016 (17 years)				
Average	586.341	781.418	44.541	28.829	41.071
Variance	1330.813	6604.928	72.300	48.578	150.992
Skewness	-1.265	-0.519	1.709	-0.475	-0.473
Kurtosis	3.077	-0.920	2.833	-0.274	-1.304
CV (%)	6.22	10.40	19.09	24.18	29.92

3.2. Grey model

For grey model construction, the whole production series from the year 2000-16 is partitioned into training or in sample part and testing or out of sample part. Training set is for model building, consists of 15 data points (2000-2014) and testing set (2015-2016) is for out of sample forecast performance comparison, which consists of 2 data points. The grey modelling procedure for a particular crop viz., maize production ('000 tonnes) of Uttarakhand is presented here and similar kind of analysis has been performed for others in the following way.

Step 1: The original maize production data are denoted as $X^{(0)}$ = (59.6,51,…,50.9).

Step 2: Based on the initial sequence $X^{(0)}$, a new sequence is generated by the accumulated generated operation (AGO) as $X^{(1)}$ =(59.6,110.6,…,680.8) as discussed in section 2.1 by equation (i).

Step 3: Using $\delta = 0.5$, the B matrix and Y is calculated through the process as described in section 2.1 from eqs. (v) to (ix),

$$B = \begin{bmatrix} -85.1 & 1 \\ -129.6 & 1 \\ \vdots & \vdots \\ . & . \\ -655.35 & 1 \end{bmatrix}, \qquad Y = \begin{bmatrix} 51 \\ 38 \\ \vdots \\ . \\ 50.9 \end{bmatrix}$$

Using the formula, $\hat{\theta}$ is obtained as $\hat{\theta} = (B^T B)^{-1} B^T Y = \begin{bmatrix} 0.016 \\ 50.639 \end{bmatrix}$

Step 4: Taking $\hat{\theta}$ into the equation, grey forecasting model can be written as

$$\hat{x}_k^{(0)} = \left(1 - e^{\beta}\right)\left(x_1^{(0)} - \frac{\alpha}{\beta}\right) e^{-\beta(k-1)} \ \forall\ k = 2,3,4, \dots, n$$

The forecast equation of GM(1,1) model for maize production in Uttarakhand is obtained as follows,

$$\hat{x}_k^{(0)} = (1 - e^{0.016})\left(59.6 - \frac{50.639}{0.016}\right) e^{-0.016(k-1)} \ \forall\ k = 2,3,4, \dots, n$$

The parameter estimates of rice, wheat, maize, total oilseeds and total pulses production along with out of sample forecast values are reported in table 3 and it has seen that out of sample forecast values of maize production is as much as close to the actual values as compared to others. The training set and testing set RMSE, MAPE (percent) and MAE values for maize production are obtained as 6.976, 9.336, 4.435 and 1.076, 2.359, 0.881 respectively. The in sample and out of sample forecast performance are reported in table 4 based on RMSE, MAPE (percent) and MAE for all cases. The values of error measurements at testing part of the GM(1,1) model fell from 1.076 to 141.736 for RMSE value, from 2.359 to 22.391 for MAPE (percent) value and 0.881 to 115.676 for MAE value. Moreover, the precision rate of GM(1,1) model for different crop production in Uttarakhand state varies from 77.61 percent to 97.64 percent as shown in table 4. It has been realized that grey model has excellent forecast capability for maize production only and good forecast level for rice production with precision rate 94.04 percent. For other cases, the model has unqualified forecast performances with precision rate 82.62 percent for wheat, 80.64 percent for total oilseeds and 77.61 percent for total pulses production.

Table 3: Parameter estimates and out of sample forecast of GM(1,1) model in Uttarakhand state production

Rice			Out of sample forecast	
Parameter estimates		Year	Actual value	Predicted value
β	-0.00435	2015	639.1	595.891
α	556.7986	2016	631	598.486
Wheat				
β	-0.0078	2015	639.1	836.678
α	741.6785	2016	877	843.226

Maize				
β	0.016425	2015	39.4	39.136
α	50.6391	2016	37	38.498
Total oil seeds				
β	-0.02131	2015	35.6	34.577
α	25.07604	2016	26	35.322
Total pulses				
β	-0.06054	2015	51.6	62.677
α	24.73208	2016	54	66.589

Table 4: In sample and out of sample forecast performance of GM(1,1) model for different crop production in Uttarakhand state

	Evaluative items	Rice	Wheat	Maize	Total oil seeds	Total pulses
RMSE	Training	28.975	64.287	6.976	5.272	4.500
	Testing	38.237	141.736	1.076	6.631	11.857
MAPE (%)	Training	3.607	6.003	9.336	15.273	8.893
	Testing	5.957	17.383	2.359	19.363	22.391
MAE	Training	20.052	44.709	4.435	4.205	3.148
	Testing	37.862	115.676	0.881	5.172	11.833
	Precision Rate (%)	94.04	82.62	97.64	80.64	77.61
	Forecasting Grade	Good	Unqualified	Excellent	Unqualified	Unqualified

3.3. Grey Bayesian Modelling approach

Grey Bayesian model is constructed for all production dataset ('000 tonnes) for Uttarakhand state by following the procedure as described in section 2.3 and the parameters α and β are estimated by 3000. The study keeps the latter 2000 iterations because the former 1000 iterations were easily affected by initial value. The summary statistics of posterior mean for the parameters α and β, S.D. for the mean of α and β, 95 percent credible interval for the mean of α and β along with the extent of contribution of simulation error in the estimation of mean value for the parameters α and β are obtained and are stable. The resulting histogram of parameters α and β are obtained which are smoothed and the resulting marginal posterior for α and β shows a normal-like form for all the cases. After substituting the estimated parameters into eq. (x) of section 2.1, the grey Bayesian model using MCMC simulating approach is obtained as

$$x_B^{(0)}(k) = (x^0(1) + \frac{556.200}{0.004})(1 - e^{-0.004)})e^{0.004(k-1)}$$

The out of sample forecast performances as well as forecasted values are obtained by substituting $k = 2,3, \ldots, n$ into the above equation. The in sample and out sample forecast performance of the state for considered crops are reported in table 5 based on RMSE, MAPE (percent) and MAE value. The values of error measurements at training part of the GM(1,1)-Bayesian model fell from 4.499 to 64.295 for RMSE value, from 3.617 to 15.239 for MAPE (percent) value and 3.143 to 44.531 for MAE value. The values of error measurements at testing part of the GM(1,1)-Bayesian model fell from 1.101 to 142.321 for RMSE value, from 2.372 to 22.387 for MAPE (percent) value and 0.884 to 115.554 for MAE value. The precision rate of GM(1,1)-Bayesian model for different crop production in Uttarakhand state varies from 77.61 percent to 97.63 percent. The highest and lowest precision is obtained for maize and total pulses production. It has been realized that, grey Bayesian model has excellent forecast capability for maize production only with precision rate 97.63 percent followed by good forecast level for rice production with precision rate 94.10 percent. The forecast level for wheat, total oilseeds and total pulses production is unqualified with precision rate 82.61 percent, 80.62 percent and 77.61 percent respectively. It can be seen that the results of grey Bayesian model are at per with grey model and has a little or no improvement on forecasting performance.

Table 5: In sample and out of sample forecast performance of grey Bayesian model for different crop production in Uttarakhand

	Evaluative items	Rice	Wheat	Maize	Total Oilseeds	Total Pulses
RMSE	Training	28.976	64.295	6.976	5.273	4.499
	Testing	37.882	142.321	1.101	6.645	11.856
MAPE (%)	Training	3.617	5.977	9.316	15.239	8.867
	Testing	5.899	17.392	2.372	19.384	22.387
MAE	Training	20.120	44.531	4.428	4.201	3.143
	Testing	37.498	115.554	0.884	5.176	11.831
	Precision Rate (%)	94.10	82.61	97.63	80.62	77.61
	Forecasting Grade	Good	Unqualified	Excellent	Unqualified	Unqualified

3.4. Rolling Grey Bayesian Modeling

For all the crops under consideration, same method was followed; we present here the method considering rice production data. The first step is to identify appropriate order of grey model. After varying grey model order from 4 to 15, finally 6-order rolling grey model is selected with fix δ value of 0.5. In the next, after varying δ from 0.49 to 0.60, δ value of 0.55 is selected based on minimum mean absolute percentage error (MAPE) value. The final rolling grey model is therefore of order 6 with δ value 0.55. The Bayesian mechanism is combined with 6-order rolling grey model with δ value 0.55 and with a total of 11 loops to predict rice production in Uttarakhand. The estimated values of parameters along with MC error for both the parameters obtained at each loops are obtained and are shown to be stable. After, substituting the parameters obtained at each loop into eq. (x) of section 2.1, grey forecasting equation is obtained.

The RMSE, MAPE (percent) and MAE value are obtained as 23.105, 2.796 and 16.414 respectively which signify the excellent forecasting ability of rolling grey Bayesian model for forecasting rice production in Uttarakhand with precision rate 97.20 percent as shown in table 7. Similar procedure has been followed for other cases and reported in the table 6 which shows the appropriate number of rolling grey model order, efficient δ value and number of loops required for all production dataset.

Table 6: Summary of Rolling grey model order and δ value for different crops

	Rice	Wheat	Maize	Total oil seeds	Total pulses
RGM Order	6	6	15	7	9
δ value	0.55	0.55	0.51	0.70	0.58
Number of loop perform	11	11	2	10	8

The precision rates for the considered crops Uttarakhand state vary from 87.06 percent to 97.20 percent with highest precision rate is obtained for rice production and lowest precision rate is obtained for total oilseeds production. Overall, it can be observed that rolling grey Bayesian model has excellent forecasting capability for rice, maize and total pulses production with precision rate 97.20, 97.14 and 95.52 percent respectively. The model has obtained good forecasting level for wheat production and qualified forecasting level for total oilseeds production with precision rate 90.69 and 87.06 percent respectively as shown in table 7. It can be seen from earlier tables that both grey model and grey Bayesian model have unqualified forecasting level for wheat and total oilseeds production respectively.

Table 7: Out of sample forecast performance of rolling grey Bayesian model for rice, wheat, maize, total oilseeds and total pulses production ('000 tonnes) in Uttarakhand

	Rice	Wheat	Maize	Total oil seeds	Total pulses
RMSE	23.105	103.986	1.471	5.131	2.929
MAPE (%)	2.796	9.313	2.858	12.935	4.478
MAE	16.414	71.589	1.058	4.055	2.311
Precision Rate (%)	97.20	90.69	97.14	87.06	95.52
Forecasting grade	Excellent	Good	Excellent	Qualified	Excellent

According to the results as obtained, rolling grey Bayesian model has excellent forecasting capability for rice, maize and total pulses production and dominates over grey Bayesian and grey modelling for the dataset under consideration. There is not much difference of forecasting performance between grey model and grey Bayesian model based on forecast accuracy measures criteria. Rolling grey Bayesian model can be considered for forecasting purposes of yearly production of rice, wheat, maize, total oilseeds and total pulses production ('000 tonnes) of Uttarakhand state.

4. Conclusion

Time series modelling and forecasting has fundamental importance in various practical domains for accurately forecasting as well as for economic planning. Rice, wheat, maize, oilseeds and pulses are the most important crops cultivated not only in Uttarakhand but also throughout the country and play significant roles not only by providing food and nutritional security, also by generating revenue for Indian economy. Most of the statistical time series forecasting techniques depend on large amount of data or information and hence cannot be applicable for the time series having limited size of sample information. Among many different modelling approaches grey model, Bayesian estimation and rolling mechanism are ideal choice to deal with short time series data. Here, an innovative approach has been considered by combining these individual model capabilities to accurately forecast the production behavior of different crop in Uttarakhand state. Our results suggest that rolling grey Bayesian approach is performing better than grey model and grey Bayesian approaches in all cases. The modelling efficiency of grey model and grey Bayesian approaches are at per. This type of study is very helpful in understanding production behavior of different crops especially in situations where information for a very short period is available and where conventional time series approaches are not applicable. The study will provide guidance for government, researchers, planners and policy makers to forecast short time series data of different agricultural crops to meet the required demand of a nation. As forecasting of short time series in presence of random variation is not an easy task due to limitation in availability of data, the researcher needs to apply different short time series forecasting methodologies cautiously and can select the appropriate one.

References

[1] Akay, D. and Atak, M. (2007). Grey prediction with rolling mechanism for electricity demand forecasting of Turkey, Energy, 32(9), 1670-1675.

[2] Chang, S. C., Lai, H. C. and Yu, H. C. (2005).A variable P-value rolling grey forecasting model for Taiwan semiconductor industry production, Technological Forecasting and Social Change, 72, 623-640.

[3] Dang, H. S., Huang, Y. F., Wang, C. N. and Nguyen, T. M. T. (2016). Application of the short-term forecasting with limited data in the healthcare travelling industry, Sustainability, 8, 1037-1051.

[4] Deng, J. L. (1982). Control problem of grey system. Systems and Control Letters, 1, 288-294.

[5] Hsu, L. C. and Wang, C. H. (2007).Forecasting the output of integrated circuit industry using a grey model improved by the Bayesian analysis, Technological Forecasting and Social Change, 74, 843-853.

[6] Hyndman, R. J. and A. V. Kostenko (2007). Minimum sample size requirements for seasonal forecasting models, Foresight: the International Journal of Applied Forecasting, 6, 12–15.

[7] Kayacan, E., Ulutas, B. and Kaynak, O. (2010). Grey system theory based models in time series prediction, Expert Systems with Applications, 37, 1784-1789.

[8] Lin, C. S. (2013). Forecasting and analyzing the competitive diffusion of mobile cellular broadband and fixed broad-band in Taiwan with limited historical data, Economic Model, 35, 207-213.

[9] Ma, H. and Zhang, Z. (2009). Grey prediction with Markov chain for crude oil production and consumption in China, Advances in Intelligence Soft Computing, 56, 551-561.

[10] Muqtadir, A., Suryono and Gunawan, V. (2016). The Implementation of Grey Forecasting Model for Forecast Result's Food Crop Agricultural, Scientific Journal of Informatics,3(2), 159-166.

[11] Raichaoren, T., Lursinsap, C. and Sanguanbhoki, P. (2003). Application of critical support vector machine to time series prediction, Circuits and Systems.ISCAS '03.Proceedings of the 2003 International Symposium, 5, 741-744.

[12] Sinha, K. (2011). A study on combining ARIMA and artificial neural networks for time series forecasting, M.Sc. thesis, ICAR-Indian Agricultural Research Institute, New Delhi, India.

[13] Tong, H. (1983).Threshold Models in Non-Linear Time Series Analysis, Springer Verlag, New York.

[14] Yang, X., Zou, Kong, D. and Jiang, G. (2018). The analysis of GM (1, 1) grey model to predict the incidence trend of typhoid and paratyphoid fevers in Wuhan City, China, Medicine, 97(34), 1-5.

[15] Yu, G. and Chwartz, Z. (2006). Forecasting short time-series tourism demand with artificial intelligence models, Journal of Travel Research, 45, 194-203.

[16] Wilson, J. H. and Allison-Koerber, D. (1992). Combining subjective and objective forecasts improve results, Journal of Business Forecasting, 11, 3-8.